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Detrending and the Distributional Properties of U.S. Output Time Series

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Abstract

We study the impact of alternative detrending techniques on the distributional properties of U.S. output time series. We detrend GDP and industrial production time series employing first-differencing, Hodrick-Prescott and bandpass filters. We show that the resulting distributions can be approximated by symmetric Exponential-Power densities, with tails fatter than those of a Gaussian. We also employ frequency-band decomposition procedures finding that fat tails occur more likely at high and medium business-cycle frequencies. These results confirm the robustness of the fat-tail property of detrended output time-series distributions and suggest that business-cycle models should take into account this empirical regularity.

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1. Introduction

In the recent years, different detrending techniques (e.g. linear trend, first-differencing, Hodrick-Prescott and bandpass filters) have been proposed in macroeconomics to study the business-cycle properties of aggregate time series (see e.g. Stock and Watson, 1999; Christiano and Fitzgerald, 2003). However, the choice of the detrending procedure is anything but neutral: different filtering methods can affect both the qualitative and the quantitative stylized facts of the business cycle (Canova, 1998) and can bias the determination of business-cycle turning points (Canova, 1999). Furthermore, the choice of the filter has also significant implications for macroeconomic theory: Delle Chiaie (2009) shows that different detrending techniques affect the estimation of structural parameters in Dynamic Stochastic General Equilibrium (DSGE) models (e.g., Woodford, 2003; Galí and Gertler, 2007), altering in turn the magnitude and persistence of model responses to shocks.

Following this line of research, this work explores whether detrending has any relevant impact on the distributional properties of filtered output time series. More specifically, we filter U.S. GDP and industrial production (IP) time series employing a number of different detrending techniques and we study the shape of the resulting time-series distributions using a parametric approach.

Our results show that: (i) detrended U.S. output time-series distributions can be well proxied by Expontial-Power densities with tails much fatter than those of a Normal distribution; (ii) fat-tailed distributions emerge irrespectively of the particular filtering technique employed to detrend the series (e.g. first-differencing, Hodrick-Prescott, bandpass filters).

Furthermore, we investigate whether there exist frequencies that are more conducive to fat-tail behaviors. Our exercises suggest that fat tails do not seem to be a distinctive feature of all classes of output fluctuations, as they can be more likely associated with high and medium business-cycle frequencies.

The foregoing results have implications for both theoretical and empirical research. First, they add further support to the widespread emergence of fat tails in the distributions of country output fluctuations and growth shocks. Indeed, Fagiolo, Napoletano, and Roventini (2008) show that fat-tailed, Exponential-Power densities can well proxy time-series distributions of GDP and IP growth rates – computed employing a first-differencing filter – for the U.S. and several other OECD countries[1]. Therefore, the findings presented in this note complement such previous results suggesting that non-Gaussian fat tails are a distinctive and robust feature of all distributions that proxy the deviations of business fluctuations around the trend.

Second, the fact that fat tails are more likely to appear at relatively high frequency bands associated to business-cycle periodicities implies that short-run models such as the DSGE should, on the one hand, try to replicate this additional stylized fact and, on the other hand, aim at delivering implications that are robust to non Gaussian shocks. Finally, our results indicate that short-run phenomena (e.g. business inventory management) might be possible candidates to provide convincing theoretical explanations of output fat-tail distributions.

[1] Fagiolo, Napoletano, and Roventini (2008) also show that fat tails in output growth-rate distributions emerge independently of the measures of the aggregate output used, the densities employed in the estimation, the length of time lags used to compute growth rates, and the presence of outliers, autocorrelation and heteroschedasticity. See also Castaldi and Dosi (2009) for similar results from a cross-country perspective.

The paper is organized as follows: Section 2 describes the data and the methodology employed. Section 3 presents the results. Section 4 concludes.

2. Data and Methodology

The main objects of investigation are the (natural) logarithms of U.S. GDP and industrial production (IP) time series. Quarterly GDP ranges from 1947Q1 to 2005Q3 (234 observations), monthly IP ranges from 1921M1 to 2005M10 (1017 observations). All series are seasonally-adjusted and are provided by the St. Louis Federal Reserve Economic Data (FRED) database[2].

In order to observe the behavior of the series at business-cycle frequencies it is common practice to detrend the series employing different filters. In this paper we take an agnostic approach to this problem: none of the filtering methods employed is supposed to be the correct one. Instead, following Canova (1998), we assume that all procedures are approximations that isolate different aspects of the secular and cyclical components of the series. So, the question is not which method is the more appropriate, but whether the statistical properties of the distributions of filtered time series are invariant to the filter employed. In what follows, we shall employ four filters methods: first-differencing filter (FD), the Hodrick and Prescott (1981) filter (HP), the Baxter and King (1999) bandpass filter (BK), and the Christiano and Fitzgerald (2003) bandpass filter (CF).

After having removed the trend from output time series, we take a parametric approach to the detection of fat-tail behavior by fitting detrended output time series with the Exponential-Power (EP) family of densities, also known as Subbotin distribution (for details, see Bottazzi and Secchi, 2003a,b). The EP probability density function reads:

$$f(x;b,a,m) = \frac{1}{2ab^{\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)} e^{-\frac{1}{b}\left|\frac{x-m}{a}\right|^{b}}, \tag{1}$$

where $\Gamma(\cdot)$ is the Gamma function.

The exponential-power density is a flexible statistical tool, which is characterized by a scale parameter a, a location parameter m, and a shape parameter b. The latter determines the fatness of the tails. Since the EP encompasses both Gaussian ($b = 2$) and Laplace ($b = 1$) densities, the estimate of b (together with its standard deviation) can be employed to measure how far the empirical distribution is from these benchmarks: the lower the estimate for b, the fatter the tails. We jointly estimate the three parameters of the Subbotin density via maximum likelihood (ML), employing the package SUBBOTOOLS[3]. In what follows, we indicate by $\hat{x}$ the ML estimate of the parameter $x \in \{a, m, b\}$.

3. Results

We start presenting the first four moments of U.S. output time series for the different filters employed (cf. Table I). The mean levels are near zero, with the exception of FD filter.

[2]Freely available online at `http://research.stlouisfed.org/fred2/`.

[3]Available online at `http://cafim.sssup.it/giulio/software/subbotools/`. See Bottazzi (2004) for details. For other theoretical and computational issues concerning this procedure, we refer to Bottazzi and Secchi (2006).

Table I: Summary Statistics of U.S. Output Time Series

Series	Filter	Mean	Std. Dev.	Skewness	Kurtosis
GDP	FD	0.0083	0.0098	-0.0500	4.3614
GDP	HP	0.0000	0.0165	-0.6071	3.8007
GDP	CF	-0.0001	0.0158	-0.2505	3.4900
GDP	BK	0.0007	0.0156	-0.3896	3.0602
IP	FD	0.0031	0.0191	0.3594	14.6537
IP	HP	0.0000	0.0707	-0.4691	6.4696
IP	CF	0.0005	0.0716	-0.1029	6.0671
IP	BK	0.0001	0.0655	-0.5437	7.2340

Moreover, the variance levels of HP, BK and CF filters are very large, relatively to the FD filter. Furthermore, skewness levels are generally small, implying a marked symmetry of the underlying distributions. Finally, the relatively large levels of kurtosis (especially for IP) suggest that output distributions exhibit fat tails.

In order to better explore the leptokurtosis present in the data, we fit U.S. output distributions with the Exponential-Power density. Maximum likelihood estimates, standard errors and Cramér-Rao confidence intervals (for $\hat{b}$ only) are reported in Table II[4]. We find that filtered GDP and IP series display tails fatter than Gaussian ones[5]. The only exception is BK filtered GDP series. Note also that for all filtered IP time series $\hat{b}$ is smaller than 1, implying tails even fatter than those of a Laplace distribution[6]. These results are statistically confirmed by Cramér-Rao confidence intervals (CI), which show that all CIs are below 2 (again, BK filtered GDP is the only exception). The CI for IP series remains entirely to the left of 1. The foregoing results seem to confirm the findings of non-normality presented in Fagiolo, Napoletano, and Roventini (2008) for the case of first-differencing filter (i.e., output growth rates). In addition, the considerable fatter tails exhibited by IP distributions as compared to GDP ones suggest that there might exist smoothing mechanisms at work (related to e.g. services industry, automatic stabilizers, etc., cf. Zarnowitz, 1992) dampening the effects of idiosyncratic firm shocks.

Note also that estimates for the shape parameters of BK, CF and HP time series are greater than those of the FD filter. A possible explanation is that HP, BK and CF filters wash away high-frequency fluctuations, which could be responsible for the emergence of fat tails. In order to explore this hypothesis, we try to understand which frequencies in the spectrum of the series are more conducive to the leptokurtosis observed in the data. We address this issue by isolating different frequency bands in the spectrum of the series and estimating the parameters of the Exponential-Power density in each single band. More precisely, we bandpass filter output growth rates employing rolling bandwidths of 1 and 6.5

[4]Cramér-Rao intervals in Table II are given by $\hat{b} \pm 2\sigma(\hat{b})$, where $\sigma(\hat{b})$ is the standard error of b. Estimates of the parameters of the symmetric EP density and their standard errors can be obtained by setting the appropriate value of the "verbosity" parameter in the command `subbofit`, available in the SUBBOTOOLS package. For more information we refer to the SUBBOTOOLS package user's manual (see Bottazzi, 2004).

[5]Goodness-of-fit analysis, performed through several tests, do not typically reject the null hypothesis that filtered output time series are Exponenial-Power distributed.

[6]The comparison between GDP and IP $\hat{b}$ can be biased by the different econometric sample sizes employed. Nevertheless, in line with Fagiolo, Napoletano, and Roventini (2008), we find that (not shown) estimates of b rise but are still lower than GDP ones if the same sample period is employed.

Table II: Estimated Exponential-Power Parameters

Series	Filter	$\hat{b}$	Std.Err.	Cramér-Rao CIs	$\hat{a}$	Std.Err.	$\hat{m}$	Std.Err.
GDP	FD	1.1757	0.1454	0.8849 , 1.4665	0.0076	0.0005	0.0079	0.0005
GDP	HP	1.3956	0.1798	1.0360 , 1.7552	0.0141	0.0010	0.0008	0.0011
GDP	CF	1.5189	0.2001	1.1186 , 1.9191	0.0140	0.0010	0.0002	0.0012
GDP	BK	1.8180	0.2656	1.2868 , 2.3492	0.0149	0.0011	0.0006	0.0017
IP	FD	0.7026	0.0376	0.6274 , 0.7778	0.0095	0.0004	0.0032	0.0002
IP	HP	0.7839	0.0428	0.6983 , 0.8695	0.0413	0.0017	0.0023	0.0011
IP	CF	0.7210	0.0388	0.6434 , 0.7986	0.0402	0.0017	0.0025	0.0011
IP	BK	0.7765	0.0438	0.6889 , 0.8641	0.0371	0.0016	0.0019	0.0010

years and we plot the ensuing $\hat{b}$ together with their CI in Figure 1.

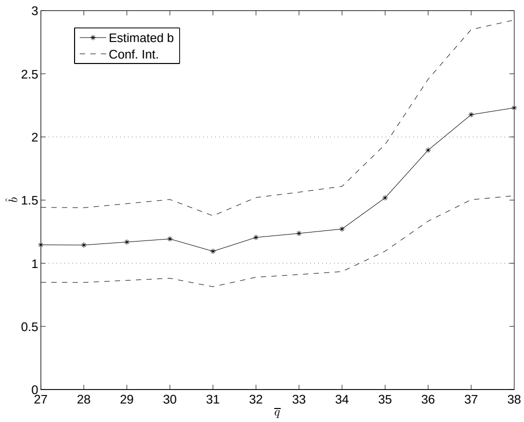
Let us begin with GDP. For the wider bandwidth of 6.5 years, $\hat{b}$ are initially almost constant and then start growing as the band moves towards lower frequencies. In particular, filtered series display tails fatter than Gaussian ones until the shifting band takes on board periodicities higher than 35 quarters. The picture changes if we employ a one-year bandwidth. In this case, $\hat{b}$ remains initially constant, then starts growing and finally, after a slight decrease, stays constant again. Anyway, leptokurtosis is present in the data up to 13 quarters. More generally, the analysis of filtered GDP seems to point out that relatively higher frequencies in the business-cycle spectrum are more conducive to fat-tail distributions. Filtered IP series distributions appear to be non Gaussian whatever band and bandwidth we consider. In particular, shape parameters appear to be lower than one. Moreover, similarly to GDP, fat tails are more marked at frequencies belonging to the relatively more irregular part of the spectrum.

4. Concluding Remarks

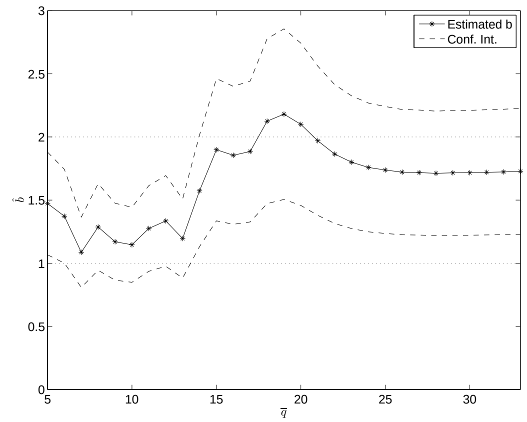
In this note, we have investigated the statistical properties of the distributions of U.S. filtered output time series. We have shown that fat tails in the distributions of output are robust to different filtering methods employed for isolating the cyclical component in the series. Moreover, we have performed a detailed frequency analysis finding that high and medium frequencies in the spectrum (associated to relatively short business-cycle fluctuations) are also the ones more conducive to fat tails in the data.

Our results bring further support to the claim that fat tails are a very robust feature of detrended output distributions at the aggregate level, as they emerge also independently from the particular filter applied to data. In addition, they suggest that business-cycle models such as those belonging to the DSGE family, should try to reproduce this empirical regularity, but their empirical performance should also be robust to the inclusion of shocks drawn from fat-tail distributions. Finally, they appear to indicate that possible theoretical explanations of output fat-tail distributions could be obtained studying short-run phenomena such as business inventory management.

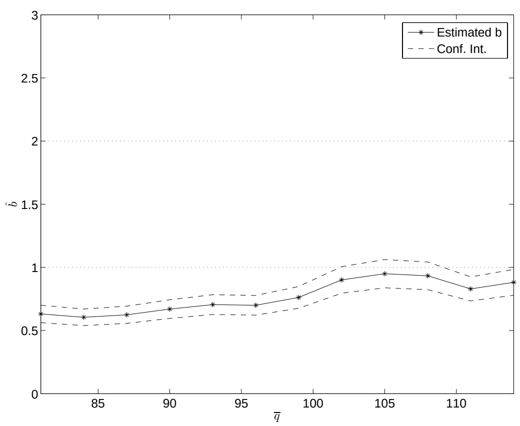
The present work could be extended in many ways. To begin with, one could perform the same exercises in other countries taking into account different business-cycle chronologies in the specification of bandpass filter parameters. Moreover, one could study the distributional properties (and the relatively robustness as to different detrending techniques) of other important macroeconomic time series, such as consumption, investment, monetary aggregates, etc. In particular, the presence of fat tails in filtered inflation and unemployment time-series distributions may be of particular interest for policy makers. Finally, one could examine the distribution of disaggregated industrial production time series in order to try to understand which sectors are more responsible for the fat-tail property observed in IP series.



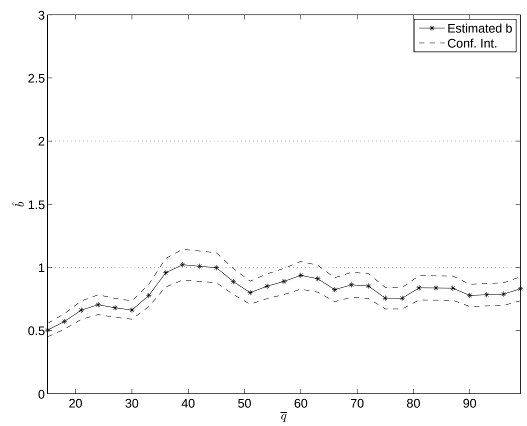
(a) GDP, 6.5 year band



(b) GDP, 1 year band



(c) IP, 6.5 year band



(d) IP, 1 year band

Figure 1: Estimated Exponential-Power $\hat{b}$ Parameters vs. Shifting Frequency Bands ($\overline{q}$)

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