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Child-allowances, fertility, and uncertain lifetime

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Abstract

We examine how child-allowance policies with pay-as-you-go systems affect fertility and growth rates. This study demonstrates that when a government initiates a child-allowance policy using some part of the pension budget, the fertility rate declines in aging economies.

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1 Introduction

Declining fertility rates threaten the feasibility of current social security systems, thereby posing a serious problem to many advanced countries. On the other hand, as the active life after retirement becomes longer through a higher life-expectancy, the importance of public pensions which stabilize a retired lifestyle is increasing.

This paper describes how child-allowance policies with PAYG systems affect fertility and growth rates by incorporating an uncertain lifetime. This model has an endogenous growth mechanism where the engine of growth is human capital accumulation. Groezen et al. (2003) show that a child-support policy stimulates fertility rates but disregard human capital accumulation. We show that a child-allowance policy increases the fertility rate, but decreases the growth rate.[1] In addition, the social security budget for elderly people is commonly much larger than that for young families in low-fertility countries, such as Japan, Italy, and Spain.[2] Therefore, we also analyze an effect when the government initiates a child-allowance policy by employing some portion of revenue which has been used for pensions. This policy change leads to a lower fertility rate in high life-expectancy economies with a larger opportunity cost of having children.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 explains effects of a child-allowance policy with PAYG-systems. Section 4 shows effects of the child-allowance policy by diminishing the size of pension systems. Concluding remarks appear in Section 5.

2 The economy

We consider an overlapping-generations model of endogenous growth, incorporating an uncertain lifetime. The life of a representative individual is divided into three periods: a childhood and a young-working period, both with fixed durations, and a retirement period. Each young individual has a probability $p \in (0,1]$ to survive to the retirement period. Competitive insurance companies promise individuals a payment of $\frac{(1+r_{t+1})}{p}a_t$ in exchange for having an estate a_t accruing to the companies.[3]

The government levies a tax $\tau \in [0,1)$ on labor wage and redistributes

[1] Zhang (1997) examines how a child-allowance policy affects fertility and growth rates in a simple model without savings, pensions, and an uncertain lifetime.

[2] The Japanese social security budget in 2006 for " children and families" is 3.4% in contrast to 50.1% for " old-age people".

[3] This is a simplified version of Blanchard's (1985) model.

μ part of the revenue to those young child-rearing individuals as child allowances, and $(1-\mu)$ part of it to retired individuals as pensions. The value of the allowance per child and the payment per retired at time t is described, respectively, as Q_t and A_t.

2.1 Households

In childhood, individuals only accumulate human capital. Individuals are endowed with one divisible unit of time in their young periods, reproduce asexually, and allocate their time toward labor and raising children. They receive labor income, which is taxed away, and allowances in the end of their young periods. They consume part of their income and invest the rest of it in annuities. Subsequently, living individuals obtain the principal and interest from their annuities and consume them with their pension benefits after retirement.

Each individual who is born at time t, and called generation $t+1$, accumulates human capital h_{t+1} according to

$$h_{t+1} = \theta e_t^{\gamma} h_t, \qquad \theta \geq 1,\ \gamma \in (0,1),$$

where e_t is parental-teaching time and h_t is parents' human capital.

The time constraint of generation t is

$$1 = l_t + n_{t+1}(q + e_t),$$

where l_t, n_{t+1}, and q respectively denote the labor time, the number of children, and the rearing time per child.

The budget constraints of a member of generation t when young and retired are given, respectively, by $(1-\tau)w_t h_t l_t + Q_t n_{t+1} = c_t^y + a_t$ and $\frac{(1+r_{t+1})}{p} a_t + A_{t+1} = c_{t+1}^o$, where c_t^y and c_{t+1}^o is the consumption when young and retirement.

The utility function of generation $t(\geq 0)$ is[4]

$$u_t = (1-\sigma)\varphi \ln c_t^y + p(1-\sigma)(1-\varphi)\ln c_{t+1}^o + \sigma \ln n_{t+1} h_{t+1}. \tag{1}$$

The parameter $\sigma \in (0,1)$ measures the taste for children's total human capital. The parameter $\varphi \in (0,1)$ describes the subjective discount factor. By solving individuals' optimization problems, the optimal values are given by:

[4]With an uncertain lifetime, $p \in (0,1]$, this utility form is employed by Pecchenino and Utendorf (1999) and Yakita (2001), among others.

$$
\begin{aligned}
n_{t+1} &= \frac{\kappa\sigma(1-\gamma)}{\{q(1-\tau)w_t h_t - Q_t\}} I_t, \\
l_t &= 1 - \kappa\sigma \frac{\{q(1-\tau)w_t h_t - \gamma Q_t\}}{\{q(1-\tau)w_t h_t - Q_t\}} \frac{1}{(1-\tau)w_t h_t} I_t, \\
e_t &= \frac{\gamma q}{1-\gamma} - \frac{\gamma Q_t}{(1-\gamma)(1-\tau)w_t h_t}, \\
c_t^y &= \kappa(1-\sigma)\varphi I_t, \\
c_{t+1}^o &= \kappa p(1-\sigma)(1-\varphi)\frac{(1+r_{t+1})}{p} I_t, \\
a_t &= \kappa p(1-\sigma)(1-\varphi) I_t - \frac{pA_{t+1}}{(1+r_{t+1})},
\end{aligned}
$$

where $\kappa \equiv \frac{1}{\{1-(1-p)(1-\sigma)(1-\varphi)\}}$ and $I_t \equiv (1-\tau)w_t h_t + \frac{pA_{t+1}}{(1+r_{t+1})}$.

2.2 Production

Competitive firms produce a single final good. The aggregate production function at time t is $Y_t = F(K_t, h_t l_t N_t) = K_t^\alpha (h_t l_t N_t)^{1-\alpha}$, where Y_t, K_t, N_t, and $\alpha \in (0,1)$ respectively denote the aggregate output, the physical capital which fully depreciates in the production process, the working-age population, and the share of physical capital.

The factor markets are presumed to be perfectly competitive. The factors are paid by their marginal products:

$$
\begin{aligned}
w_t &= (1-\alpha)\tilde{k}_t^{\alpha}, \\
(1+r_t) &= \alpha \tilde{k}_t^{\alpha-1},
\end{aligned}
$$

where $\tilde{k}_t \equiv \frac{K_t}{h_t l_t N_t}$ is the physical capital per effective-labor.

2.3 Equilibrium

The government faces the budget constraints of the policies:

child allowances; $\mu\tau w_t h_t l_t N_t = Q_t n_{t+1} N_t$,
public pensions; $(1-\mu)\tau w_t h_t l_t N_t = A_t p N_{t-1}$.

Using capital market-clearing conditions, $K_{t+1} = a_t N_t$, we can get the values at equilibrium as follows.[5]

$$n_{t+1} = \frac{[\mu\tau\Psi + \kappa\sigma\{1 + (1-\mu)\tau\frac{(1-\alpha)}{\alpha}\}\{(1-\gamma)(1-\tau) - \mu\gamma\tau\}]}{q\{1-(1-\mu)\tau\}\Psi} \equiv n_g,$$

$$l_t = \frac{(1-\tau)}{\{1-(1-\mu)\tau\}}[1 - \frac{\kappa\sigma\{1+(1-\mu)\tau\frac{(1-\alpha)}{\alpha}\}}{\Psi}] \equiv l_g,$$

$$e_t = \frac{\gamma q\kappa\sigma\{1-(1-\mu)\tau\}\{1+(1-\mu)\tau\frac{(1-\alpha)}{\alpha}\}}{[\mu\tau\Psi + \kappa\sigma\{1+(1-\mu)\tau\frac{(1-\alpha)}{\alpha}\}\{(1-\gamma)(1-\tau) - \mu\gamma\tau\}]} \equiv e_g,$$

where $\Psi \equiv [1 + \kappa\{1-(1-\sigma)(1-\varphi)\}(1-\mu)\tau\frac{(1-\alpha)}{\alpha}]$.

The capital per effective-labor becomes $\tilde{k_{t+1}} = \tilde{k_t} = \tilde{k^*}$ in the steady state. Consequently, the per-capita growth rate in the balanced-growth path depends only on the parental-teaching time. It is given by

$$(1 + g^*) = \theta e_g^{\gamma}.$$

3 Policy effects

The following proposition summarizes effects of an introduction of the child-allowance policy and the PAYG-pension system.

Proposition 1. *Introduction of a child-allowance policy and a PAYG-pension system increases the number of children and decreases the labor time. When the government introduces the child-allowance policy, the education time per child and the per-capita growth rate decrease. When the government introduces the PAYG-pension system only, the introduction has no effects on the education time per child and the per-capita growth rate.*

Proof. See Appendix A.

These results are explained as follows. The prices, in labor terms, of raising and educating a child are, respectively, $q - \frac{Q_t}{(1-\tau)w_t h_t}$ and n_{t+1}. The number of children increases because of income effects and price effects by the intervention. The per-child education time decreases because negative price effects dominate positive income effects. When only the pension system exists, the negative price effects are proportional to positive income effects. Therefore, there is no change in education time.

[5] Note that an increase in life expectancy lowers the fertility rate and education time.

4 Fertility and life expectancy

This section presents analysis of fertility effects when the government introduces a child-allowance policy using μ part of the revenue which has been spent on the PAYG-pension systems.[6] That is, the government partly diminishes the transfer from young to old and redistributes resources among young individuals to increase contributors in the future.[7]

The effect of such a policy change on the fertility rate can be recognized by the sign of the following formula.

$$\frac{\partial n_g}{\partial \mu}|_{\mu=0} = s^2p^2 - [\sigma(1-\gamma)(1-\tau)\frac{(1-\alpha)}{\alpha} - \{\sigma + 2(1-\sigma)\varphi\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}]sp$$
$$+(1-\sigma)\varphi(1-s)\{1+\tau\frac{(1-\alpha)}{\alpha}\}^2 \equiv f(p\,;\tau),$$

where $s \equiv (1-\sigma)(1-\varphi) > 0$.

This is a quadratic function of life expectancy, p. Herein, we shall see a case in which the policy change engenders a lower fertility rate.[8] The sign of $f(p\,;\tau)$ is either positive or negative depending on p, if both the labor-capital distribution ratio and the labor-tax rate are

$$\frac{(1-\alpha)}{\alpha} > \alpha_1 \quad and \quad 0 < \tau < \tau_1,$$

where $\alpha_1 \equiv \frac{1}{\sigma(1-\varphi)(1-\gamma)}$ and $\tau_1 \equiv \frac{-[\sigma(1-\gamma)s+(1-\sigma)\{\sigma(1-\varphi)+2\varphi\}]+\sqrt{\Upsilon}}{2(1-\sigma)\varphi(1-s)\frac{(1-\alpha)}{\alpha}}$.[9] The critical value of the old-age degree in the economy, p_1, which is an intersection of graph $f(p\,;\tau)$ with the x-axis, is the smaller solution of $f(p\,;\tau) = 0$:[10]

$$p_1 \equiv \frac{[\sigma(1-\gamma)(1-\tau)\frac{(1-\alpha)}{\alpha} - \{\sigma + 2(1-\sigma)\varphi\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}] - \sqrt{\Phi}}{2s}.$$

[6]When the government introduces the child-allowance policy by levying a new labor tax for the sole purpose of supporting the policy, the fertility rate always increases.

[7]Because individuals in each generation are assumed to be homogenous, this policy change exactly eases the tax burden of the young generation.

[8]See Appendix B for another case.

[9] $\Upsilon \equiv \sigma^2(1-\varphi)^2 + 4\sigma(1-\gamma)\varphi(1-\varphi)(1-s)\frac{(1-\alpha)}{\alpha}$
$+2(1-\sigma)\sigma(1-\gamma)s\{\sigma(1-\varphi)+2\varphi\} + \sigma^2(1-\gamma)^2s^2 > 0$

[10] $\Phi \equiv \{\sigma^2 + 4(1-\sigma)\varphi(1-s)^2(1+s)\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}^2 + \sigma^2(1-\gamma)^2(1-\tau)^2(\frac{1-\alpha}{\alpha})^2$
$-2\sigma\{\sigma+2(1-\sigma)\varphi\}(1-\gamma)(1-\tau)\{1+\tau\frac{(1-\alpha)}{\alpha}\}\frac{(1-\alpha)}{\alpha} > 0$

Therefore, we obtain the following proposition.

Proposition 2. *When the labor-capital distribution ratio is* $\frac{(1-\alpha)}{\alpha} > \alpha_1$ *and the labor-tax rate is* $0 < \tau < \tau_1$*, an introduction of the child-allowance policy using some part of pension revenue*
(2a) increases the fertility rate if $p \in (0, p_1)$*,*
(2b) decreases the fertility rate if $p \in (p_1, 1]$*,*
(2c) has no effects on the fertility rate if $p = p_1$*.*

In the event of a larger labor share, which implies higher labor income, or a lower labor-tax rate, the opportunity cost of having children becomes large. In aging economies, where working-age individuals face a high probability that they will be alive in their retirement periods, pension and annuity amount per capita is relatively small by the larger old-population. Therefore, if the pension benefit is decreased by the policy change, individuals have an incentive to increase the labor supply for their retired-age consumption. For that reason, even if individuals receive child allowances for having children, they decrease the number of children those they have.

5 Concluding remarks

We have presented the effects of child-allowance policies. Some empirical studies discuss whether child-allowance policies actually increase the fertility rate or not. It might be important that individuals are guaranteed sufficient pension benefits to stimulate the fertility rate in aging economies.

Appendix A.

The effect on the number of children is positive, as

$$\frac{\partial n_g}{\partial \tau}|_{\tau=0} = \frac{1}{q}[\mu(1-\kappa\sigma) + (1-\mu)\kappa^2 p\sigma(1-\sigma)(1-\varphi)(1-\gamma)\frac{(1-\alpha)}{\alpha}] > 0.$$

This sign is satisfied with any $\mu\in[0,1]$.

The effect on the labor time is negative, as

$$\frac{\partial l_g}{\partial \tau}|_{\tau=0} = -[\mu(1-\kappa\sigma) + (1-\mu)\kappa^2 p\sigma(1-\sigma)(1-\varphi)\frac{(1-\alpha)}{\alpha}] < 0.$$

This sign is also satisfied with any $\mu\in[0,1]$.

The effect on the education time per child is given by

$$\frac{\partial e_g}{\partial \tau}|_{\tau=0} = -\mu\frac{(1-\kappa\sigma)\gamma q}{\kappa\sigma(1-\gamma)^2} \leq 0.$$

This sign is negative when $\mu\in(0,1]$, and zero when $\mu = 0$.

Therefore, the effect on the per-capita growth rate in the balanced-growth path is given by

$$\frac{\partial(1+g_g^*)}{\partial \tau}|_{\tau=0} = \frac{\partial \theta e_g^\gamma}{\partial \tau}|_{\tau=0} \leq 0.$$

Appendix B.

The following proposition summarizes positive fertility effects of an introduction of the child-allowance policy.

Proposition. *An introduction of the child-allowance policy using some portion of pension revenue increases the fertility rate, when*
(a) the labor-capital distribution ratio is $\frac{(1-\alpha)}{\alpha}\leq\alpha_1$*,*
(b) the labor-capital distribution ratio is $\frac{(1-\alpha)}{\alpha} > \alpha_1$ *and the labor-tax rate is* $\tau_1 < \tau < 1$.
The introduction has no effects on the fertility rate when the labor-capital distribution ratio is $\frac{(1-\alpha)}{\alpha} > \alpha_1$ *and the labor-tax rate is* $\tau = \tau_1$. The following $f(p\,;\tau)$ expresses the effect on the fertility rate.[11]

$$f(p\,;\tau) = \frac{\partial n_{t+1}}{\partial \mu}|_{\mu=0} = s^2[p-\Gamma]^2 + \Lambda.$$

[11] $\Gamma\equiv\frac{[\sigma(1-\gamma)(1-\tau)\frac{(1-\alpha)}{\alpha} - \{\sigma+2(1-\sigma)\varphi\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}]}{2s}$,
$\Lambda\equiv -\frac{[\sigma(1-\gamma)(1-\tau)\frac{(1-\alpha)}{\alpha} - \{\sigma+2(1-\sigma)\varphi\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}]^2}{4} + (1-\sigma)\varphi(1-s)\{1+\tau\frac{(1-\alpha)}{\alpha}\}^2$.

The graph of $f(p\,;\tau)$ is convex downward and the value of $f(0\,;\tau)$ is positive.[12] The condition by which " the value of $f(1\,;\tau)$ is negative" allows the graph of $f(p\,;\tau)$ to intersect the x-axis once in $p\in(0,1]$.[13] .

The value of $f(1\,;\tau)$ is expressed by a function of τ:

$$
\begin{aligned}
f(1\,,\tau) = {} & (1-s)(1-\sigma)\varphi\{\frac{(1-\alpha)}{\alpha}\}^2\tau^2 \\
& +[(1-\sigma)\{\sigma(1-\varphi)+2\varphi\}+s\sigma(1-\gamma)]\frac{(1-\alpha)}{\alpha}\tau \\
& +[(1-\sigma)-s\sigma(1-\gamma)\frac{(1-\alpha)}{\alpha}].
\end{aligned}
$$

The sign of $f(1\,,1)$ is positive.[14] Because the first and second terms of $f(1\,,\tau)$ are positive, the sign of this function depends on the third term. When the labor-capital distribution ratio is

$$\frac{(1-\alpha)}{\alpha}\leq\frac{1}{\sigma(1-\varphi)(1-\gamma)}\equiv\alpha_1,$$

the third term is non-negative. Therefore, the sign of $f(1\,,0)$ is positive; thereby, that of $f(1\,;\tau)$ is positive for any $\tau(>0)$. Consequently, the value of $f(p\,;\tau)$ is always positive in $p\in(0,1]$.

When the third term is negative, as $\frac{(1-\alpha)}{\alpha}>\alpha_1$, the graph of $f(1\,,\tau)$ has an intercept in $\tau\in(0,1)$. The intersection is a larger solution of $f(1\,,\tau)=0$:[15]

$$\tau_1\equiv\frac{-[\sigma(1-\gamma)s+(1-\sigma)\{\sigma(1-\varphi)+2\varphi\}]+\sqrt{\Upsilon}}{2(1-\sigma)\varphi(1-s)\frac{(1-\alpha)}{\alpha}}.$$

The value of $f(1\,;\tau)$ is negative when $\tau\in(0,\tau_1)$ and positive when $\tau\in(\tau_1,1)$. Therefore, when $\frac{(1-\alpha)}{\alpha}>\alpha_1$ and $\tau_1<\tau<1$, the value of $f(p\,;\tau)$ is always positive in $p\in(0,1]$. When $\frac{(1-\alpha)}{\alpha}>\alpha_1$ and $0<\tau<\tau_1$, the value of $f(p\,;\tau)$ is positive or negative depending on p.

[12] $f(0\,;\tau)=(1-\sigma)\varphi(1-s)\{1+\tau\frac{(1-\alpha)}{\alpha}\}^2>0.$

[13] To evade intricacy, we eliminate the case in which the graph intersects the x-axis twice.

[14] $f(1\,,1)=(1-\sigma)[1+\{\sigma(1-\varphi)+2\varphi\}\frac{(1-\alpha)}{\alpha}+(1-s)\varphi(\frac{1-\alpha}{\alpha})^2]>0.$

[15] $\Upsilon\equiv\sigma^2(1-\varphi)^2+4\sigma(1-\gamma)\varphi(1-\varphi)(1-s)\frac{(1-\alpha)}{\alpha}$
$+2(1-\sigma)\sigma(1-\gamma)s\{\sigma(1-\varphi)+2\varphi\}+\sigma^2(1-\gamma)^2s^2>0$

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