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Is admiration a source of indeterminacy when the speed of habit formation is finite?

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Abstract

In an economy where the time preference rate is sufficiently decreasing in individual consumption, Chen and Hsu (2007) find that a consumption admiration effect can be a source of local indeterminacy, whereby average consumption flows exert a positive external effect on an individual's utility. In our paper, average consumption habits externally increase an individual's utility. The increase in average consumption habits is the difference between average consumption flows and existing average consumption habits adjusted for by the speed of the consumption habit formation. The model in Chen and Hsu (2007) is a special case that emerges only when the speed of habit formation is infinite. In our general model, an admiration effect is no longer a source of equilibrium indeterminacy unless the speed of consumption habit formation is infinite.

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1 Introduction

Dynamic models with indeterminacy are valuable as they explain economic fluctuations without relying on exogenous shocks, and question interpretations of simple estimations obtained by pooling data, among others. Local indeterminacy is established in existing one-sector, neoclassical growth models based on the assumption of increasing social returns or externalities in production, pioneered by Benhabib and Farmer (1994) and Farmer and Guo (1994). The assumption of increasing social returns, however, has been challenged by empirical studies (e.g., Burnside, 1996; Basu and Fernald, 1997). As a result, much effort has been devoted to searching out an economic environment that may generate local indeterminacy without relying on the assumption of increasing social returns.[1]

In a recent paper, Chen and Hsu (2007) studied a standard growth model with the Uzawa (1968) time preference rate. They found that if the time preference rate is sufficiently decreasing in individual consumption, a consumption admiration effect or a positive external utility effect of average consumption flows can establish local indeterminacy. Their work is important and valuable as local indeterminacy is established on the assumption of externalities in preference without relying upon the assumption of increasing returns to production.

In this paper, we generalize the model of Chen and Hsu (2007). We postulate that average consumption habits in a society externally increases an individual's utility. The average consumption habit in our paper is a weighted average of past average consumption flows in a society as proposed by Ryder and Heal (1973). While theoretical literature in macroeconomics and finance has widely used utility functions with consumption habit,[2] there is a growing body of empirical work that confirms the importance of consumption habits and the hypothesis of consumption habit formation. Van de Stadt et al. (1985) provided evidence in support of the hypothesis of consumption habit formation by using panel data in the Netherlands, and Fuhrer (2000) strongly supported the hypothesis of consumption habit formation by using time series data in the U.S. More recently, Ravina (2005) and Korniotis (2008) provided support for external habit formation by employing panel data from the U.S.

Specifically, in this paper we follow Ryder and Heal (1973) and assume that a consumption habit in a society is formed by past average consumption flows with weights declining exponentially in the distant past. In terms of the law of motion, an increase in average consumption habits is determined by the difference between average consumption flows and existing average consumption habits adjusted for by the speed

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[1] See the survey by Benhabib and Farmer (1999).

[2] For macroeconomics, see papers on business cycles by Boldrin et al. (2001), on saving and growth by Carroll et al. (2000), on monetary shocks by Fuhrer (2000), on optimal tax policy by Ljungqvist and Uhlig (2000), and on different economic growth rates by Chen (2007) and Doi and Mino (2008). For finance, see Abel (1990), Constantinides (1990) and Campbell and Cochrane (1999).

of the consumption habit formulation. Our setup is general and renders the setup in Chen and Hsu (2007) as a special case that emerges only when the speed of the consumption habit formulation is infinite. Under this generalization, we expect that an admiration effect is not a source of indeterminacy unless the speed of the habit consumption formation is infinite.

The intuition for the different result is as follows. When the speed in the consumption habit formation is infinite, the level of external consumption habits is equal to the level of external consumption flows. This is the case in Chen and Hsu (2007). In this case, decreasing impatience causes intertemporal substitution so that intratemporal inefficiency that is produced by an external consumption flow results in re-optimization in the capital demand. Under a sufficiently large degree of decreasing impatience, anticipations of higher average consumption flows can lead to higher individual consumption and capital demand so that initial sunspot-driven expectations are self-fulfilling.

However, when the speed of the consumption habit formation in a society is finite, the level of average consumption habits is different from the level of average consumption flows in transitions. In this circumstance, a household's optimal consumption is decreasing in the shadow price of capital and increasing in the level of external habits. Suppose that the economy initially stays at a steady state. Suppose further that a sunspot-driven shock hits the economy and that the households anticipate that the rate of return to capital increases. A higher rate of return to capital raises the shadow price of capital, which reduces the demand for consumption. A fall in consumption makes the level of consumption lower than the level of habits, so that habits start declining. As optimal consumption decreases in the level of habits, a fall in habits yields a further decline in consumption. Thus, current investment increases and the rate of return to capital will fall. As a consequence, the initial expectations are not self-fulfilled.

The reminder of the paper is organized as follows. Section 2 sets up the model and studies the steady state. Section 3 is the main body that examines the issue of local dynamics. Finally, some concluding remarks are made in Section 4.

2 The Model

There is a representative household with an infinite life who supplies labor in-elastically, owns shares of firms, and decides the amount of consumption and savings at each instant of time. The individual's lifetime preference is

$$U = \int_0^\infty u(c_t, H_t) X(t) dt, \tag{1}$$

where c_t is individual's consumption in time t, H_t is the consumption habit in the society in time t and X_t is the discount factor in time t.

As in Uzawa (1968), the discount factor is $X_t = exp\{-\int_0^t \rho(c_s) ds\}$ and depends on an individual's consumption. The discount factor evolves as follows.

$$\dot{X} = -\rho(c_t) X_t, \text{ with initial } X_0 \text{ given,} \tag{2}$$

where $\rho(c_t)$ is the instantaneous discount rate in t.

As in Ryder and Heal (1973, p. 2), the consumption habit is $H_t=H_0e^{-\gamma t}+\gamma\{\int_0^t e^{-\gamma(t-s)}C_s ds\}$, where C_t is average consumption in the society in time t and $\gamma\geq0$ is a parameter. The relation says that the stock of consumption habits is the weighted sum of past average consumption in the society, with the weights declining exponentially into the distant past.[3] The consumption habit evolves as follows.

$$\dot{H}=\gamma(C_t-H_t), \quad \text{with initial } H_0>0 \text{ given.} \tag{3}$$

The parameter γ is the speed that social consumption habits adjust to a change in average consumption flows in the society with a larger value implying a higher speed of adjustment. Two special cases are in order. First, if $\gamma=0$, then H_t is fixed and is given by H_0 for all t. Then, the felicity function depends on private consumption alone. In this case, our model is a typical Uzawa (1968) model. Second, if $\gamma\to\infty$, then $C_t=H_t$ for all t. In this case, our model is the Chen and Hsu (2007) model.

The felicity and the discount function are assumed to be twice differentiable with

Assumption 1. (*i*) $u_1(c,H)>0>u_{11}(c,H)$ for any c and H, and $\rho(0)\equiv\rho_0>0$;
(*ii*) $u_2(c,H)>0$ and $u_{12}(c,H)>0$ for any c and H,;
(*iii*) $\rho'(c)<0$ and $-\frac{\rho''}{\rho}<-\frac{u_{11}}{u}$ for any c.

In Assumption 1-(*i*), we assume a standard concave utility. The effect of external consumption habits on utility may be positive or negative, i.e., an *admiration* effect or a *jealousy* effect (e.g., Dupor and Liu, 2003). In Assumption 1-(*ii*) we follow the assumption made in Chen and Hsu (2007) and assume an admiration effect. We also assume that average consumption exerts a positive effect on an individual's marginal utility, referred to as *keeping up with the Joneses* (Dupor and Liu, 2003). In Assumption 1-(*iii*), we also follow Chen and Hsu (2007) and assume an impatience that is decreasing in an individual's own consumption. Finally, like Chen and Hsu (2007), the remaining assumption in Assumption 1-(*iii*) is technical and demands that the curvature of the felicity be larger than that of the discount with respect to consumption. This assumption assures a positive intertemporal elasticity of substitution (henceforth *IES*) in consumption.

The representative firm is endowed with a neoclassical technology $y=f(k)$, where y is output per capita and k is capital stock per capita with initial k_0 given. For simplicity, there is no depreciation on capital. We assume that the technology satisfies the same conditions as those imposed in Chen and Hsu (2007, Assumption 3) as follows.

Assumption 2. $f'(k)>0>f''(k)$, $f''(k)<\rho'(c)f'(k)$, $f(0)=0$, $f'(0)=\infty$, and $f'(0)>\rho(0)$ for any k and c.

Let us comment on the assumptions of $f''(k)<\rho'(c)f'(k)$ and $f'(0)>\rho(0)$. The former assumption is necessary in order to satisfy the Correspondence Principle (Samuelson,

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[3] A similar form is $H_t=\gamma\{\int_{-\infty}^t e^{-\gamma(t-s)}C_s ds\}$, as used in Alvarez-Cuadrado et al. (2004). This form also leads to (3).

1948). This assumption amounts to a variant of the Brock-Gale condition that requires the increase in the discount rate to dominate the increase in the marginal product of capital in the steady-state equilibrium, $f''(k)/f'(k)<\rho'(c)$. The latter assumption $f'(0)>\rho(0)$ is for the existence of steady state that requires the steady-state Keynes-Ramsey condition to start with a positive capital when the level of consumption is zero.

The representative household allocates consumption and savings. Thus,

$$\dot{k} = f(k_t) - c_t. \tag{4}$$

The representative household's problem is to maximize the lifetime utility in (1), given the constraints in (2) and (4), taking as given the consumption habits in the society in (3). The Hamiltonian associated to the program is

$$\mathcal{H}(c,k,X,\theta,\lambda) = X\{u(c,H) + \eta[f(k)-c] - \lambda\rho(c)\},$$

where η and λ are the co-state variables associated with capital and discounting, respectively.

The necessary optimal conditions are

$$\eta = u_1(c,H) - \lambda\rho'(c), \tag{5a}$$

$$\dot{\eta} = \eta[\rho(c) - f'(k)], \tag{5b}$$

$$\dot{\lambda} = -u(c,H) + \lambda\rho(c), \tag{5c}$$

along with the transversality conditions, $\lim_{t\to\infty} \eta_t k_t=0$ and $\lim_{t\to\infty} \lambda_t X_t=0$. In these optimal conditions, (5a) equates the marginal costs and the net discounted marginal utility of consumption, while (5b) and (5c) are the Euler equations for capital and discounting, respectively.

Definition 1. Under Assumptions 1 and 2, for k_0 and H_0, a symmetric equilibrium is a path $\{C_t, k_t, \eta_t, \lambda_t, H_t\}$ with $c_t=C_t$, which solves (3), (4), (5a)-(5c) and the transversality conditions.

Definition 2. A steady state is a symmetric equilibrium with $\dot{k} = \dot{\eta} = \dot{\lambda} = \dot{H} = 0$ and a constant C^*.

The steady state is determined as follows.

$$\eta^* = u_1(C^*,H^*) - \lambda^*\rho'(C^*), \tag{6a}$$

$$u(C^*,H^*) = \lambda^*\rho(C^*), \tag{6b}$$

$$f'(k^*) = \rho(C^*), \tag{6c}$$

$$f(k^*) = C^*, \tag{6d}$$

$$H^* = C^*. \tag{6e}$$

Obviously, under Assumptions 1 and 2 there exists a unique interior steady state in (6a)-(6e). To see this, the steady state is determined in the (k, C) plane by the goods market clearance condition (6d) and the steady-state Keynes-Ramsey condition (6c). While (6d) is positively sloping as is usual, the steady-state Keynes-Ramsey condition (6c) would have been vertical if ρ is constant but is now also positively slopping as $\rho'(c)<0$. The condition $f''(k)<\rho'(c)f'(k)$ in Assumption 2 assures that the locus (6c) be steeper than the locus (6d). As the locus (6c) starts from (k, C)=(0, 0), the locus (6d) needs to start from $(k, C)=(k_1, 0)$, $k_1>0$, in order assure the intersection of the locus (6d) with the locus (6c). The assumption $f'(0)>\rho(0)$ guarantees this. Therefore, under Assumptions 1 and 2, there is a unique steady-state pair of (k^*, C^*).

Unique values for H^*, λ^* and η^* are obtained by substituting the unique pair of (k^*, C^*) into (6e), (6b) and (6a), respectively.

3 Local Dynamic Stability

This section investigates the dynamic stability properties in the neighborhood of the unique steady state. Differentiation of equation (5a), with the use of (3) and (5b), leads to

$$\dot{C} = \tfrac{1}{\sigma}[(f' - \rho) + \tfrac{u_{12}}{u_1 - \lambda\rho'}\dot{H} - \tfrac{\rho'}{u_1 - \lambda\rho'}\dot{\lambda}], \tag{7}$$

where $\sigma = -\frac{u_{11} - \lambda\rho''}{u_1 - \lambda\rho'} > 0.$ is the reciprocal of the IES in consumption, which is positive under Assumption 1-(*iii*). The dynamic system consists of (3), (4), (5c) and (7).

To analyze the stability property, we take the linearization of the dynamic system in a neighborhood of the unique steady state. The result is as follows.

$$\begin{bmatrix} \dot{C} \\ \dot{k} \\ \dot{\lambda} \\ \dot{H} \end{bmatrix} = \begin{bmatrix} \frac{\gamma u_{12}}{\sigma(u_1 - \lambda\rho')} & \frac{f''}{\sigma} & \frac{-\rho\rho'}{\sigma(u_1 - \lambda\rho')} & \frac{u_2\rho' - \gamma u_{12}}{\sigma(u_1 - \lambda\rho')} \\ -1 & \rho & 0 & 0 \\ -(u_1 - \lambda\rho') & 0 & \rho & -u_2 \\ \gamma & 0 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} C - C^* \\ k - k^* \\ \lambda - \lambda^* \\ H - H^* \end{bmatrix}. \tag{8}$$

We remark that in the case $\gamma \longrightarrow \infty$, then $C_t=H_t$ for all t. In this case, $\dot{H} = 0$ holds for all t and (3) is not a part of the conditions in the dynamic system. Moreover, as $C_t=H_t$ for all t, the dynamics of H_t is captured by the dynamics of C_t. As a result, the evolution of C_t is no longer in (7) above, but is in the form expressed in Chen and Hsu (2007, eq. (7)). The linearization of the dynamic system in this case is as analyzed in Chen and Hsu (2007, eq. (8)). Alternatively, in all other cases when $\gamma<\infty$, $\dot{H} = 0$ holds only in steady state. The dynamics of H_t is different from the dynamics of C_t and is in the form as presented in (3) above. Therefore, the linearization of the dynamic system is the one represented in (8) above.

Let ω denote the corresponding eigenvalue of the Jacobean matrix in (8). Then the corresponding characteristic polynomial is

$$P(\omega) = (\omega - \rho)G(\omega) = 0,$$

where $G(\omega) = \omega^3 - (\rho - \gamma - \frac{\gamma u_{12}}{u_{11} - \lambda\rho''})\omega^2 - [\gamma\rho + \frac{\rho\rho' - f''}{\sigma} - \gamma\frac{\rho' u_2 - \rho u_{12}}{u_{11} - \lambda\rho''}]\omega + \frac{\gamma}{\sigma}(f'' - f'\rho').$

As the economic system involves two state-like variables whose initial values are predetermined at k_0 and H_0, local indeterminacy requires at least three roots with negative real parts. Obviously, there is a positive eigenvalue, $\omega=\rho>0$.

We now investigate $G(\omega)=0$ to understand the sign for the remaining three roots. First, it is easy to see that the reciprocal of the IES is positive ($\sigma>0$). Next, $(f''-f'\rho')<0$ under Assumption 2. These conditions imply $(\gamma/\sigma)(f''-f'\rho')<0$. The product of remaining three roots is equal to $-G(0)$, which is $-[(\gamma/\sigma)(f''-f'\rho')]>0$. This indicates that the dynamic system has either (1) four positive roots or (2) two positive roots and two negative roots. In either case, the result is that local indeterminacy will not emerge in this model.

Proposition 1. *In a standard growth model with endogenous discount and with felicity affected by average consumption habits, under Assumptions 1 and 2, local indeterminacy cannot arise if the speed of habit formation is finite.*

Finally, it is worth noting that our above determinacy result is maintained even if we depart from the assumption made in Chen and Hsu (2007) regarding decreasing impatience and an admiration effect on felicity. This is so because if instead we allow for the assumptions of either increasing impatience ($\rho'>0$), or a jealousy effect ($u_2<0$), or both, the condition $-[(\gamma/\sigma)(u_1-\lambda\rho')(f''-f'\rho')]>0$ remains hold true. As a consequence, local indeterminacy will not emerge.

Our model leads to local determinacy for reasons as follows. When the speed of the habit formation is finite and the habit stock then evolves gradually in transitions. The habit stock is thus a state variable rather than a control variable. Therefore, the equilibrium system now has two state variables, k and H. In this situation, the condition with a sufficiently large absolute value of the negative effect of the consumption habit on the discount rate that generates two stable roots, now can exactly pin down the unique equilibrium path. It is thus impossible for local indeterminacy to emerge.

Intuitively, the results my be understood if we use (5a) and obtain the relationship of optimal consumption $c=c(\eta, H)$, with $c_\eta(\eta, H)=\partial c/\partial\eta=1/(u_{11}-\lambda\rho'')<0$ according to Assumption 1 and $c_H(\eta, H)=\partial c/\partial H=(-u_{12})/(u_{11}-\lambda\rho'')>0$. Suppose that the economy is initially at a steady state. Suppose further that a sunspot-driven shock hits the economy and that households anticipate an increase in the rate of return to capital. An anticipated higher rate of return to capital raises the shadow price of capital and because of $c_\eta(\eta, H)<0$, the demand for consumption is reduced. In addition, a fall in consumption makes the level of consumption lower than the level of habits, so that habits start declining. As $c_H(\eta, H)>0$, a fall in habits yields a further decline in consumption. Thus, current investment increases and the rate of return to capital will fall. As a consequence, the initial expectations are not self-fulfilled.

4 Concluding Remarks

This paper generalizes Chen and Hsu (2007) and studies the indeterminacy issue. In Chen and Hsu (2007), a consumption admiration effect can be a source of local indeterminacy when the rate of time preference is sufficiently decreasing in individual consumption. In Chen and Hsu (2007), average consumption flows externally exert a positive effect on an individual's utility. In our paper, average consumption habits externally increase an individual's utility. The increase in average consumption habits is the difference between average consumption flows and existing average consumption habits adjusted for by the speed of the consumption habit formation. The consumption habit formation renders the model in Chen and Hsu (2007) as a special case that emerges only when the speed of habit formation is infinite. In our general model, an admiration effect is no longer a source of equilibrium indeterminacy unless the speed of consumption habit formation is infinite.

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