Economics Bulletin

Volume 30, Issue 2

Product Innovation and Stability of Collusion

Sougata Poddar
Hanyang University, Seoul, Korea and Nanyang Technological University, Singapore

Bibhas Saha
University of East Anglia

Abstract

We study the nature of market competition in relation to stability of collusion in the infinitely repeated play of a two-stage game of product innovation and market competition, and show that cooperation in giving R&D efforts is more easily sustained when firms compete in quantity than in price.

We gratefully acknowledge the helpful comments from an anonymous referee. Sougata Poddar acknowledges financial support in the form of Research Grant from Hanyang University, Seoul, Korea. The usual disclaimer applies.
Citation: Sougata Poddar and Bibhas Saha, (2010) "Product Innovation and Stability of Collusion", *Economics Bulletin*, Vol. 30 no.2 pp. 1392-1400.
Submitted: Sep 01 2009. **Published:** May 16, 2010.

1. Introduction

There exists a wide literature which considers the effects of the degree of product differentiation on the stability of implicit collusion between firms (either in quantities or in prices) in the context of repeated interactions leading to a variety of conclusions (see Deneckere 1983, Rothschild 1992, Lambertini 1997, Albaek and Lambertini 1998, *inter alia*). On the other hand, very few studies have been made where the market game is preceded by another game in which the firm may choose a level of effort in order to differentiate its product from its rival's; and as a result, the firm's choice of the degree of differentiation becomes a costly commitment. Kesteloot and Veugelers (1995)considered a model of process R&D with spillover and showed that cooperation in R&D occurs in low spillover scenarios, due to lower incentive to free-ride. Lambertini and Rossini (1998) studied this problem through a binary model of strategic interaction in a symmetric duopoly where firms compete either on quantities or on prices, after having determined the degree of differentiation through R&D effort. They prove that regardless of quantity or price competition, firms may end up competing in perfect substitutes (i.e. undifferentiated goods) because of a prisoner's dilemma problem arising at the R&D stage. In other words, prisoner's dilemma is the main stumbling block for R&D cooperation. Given this result, it seems that one can develop a repeated version of this game and try to resolve the prisoners' dilemma and thereby achieve cooperation in R&D efforts. We precisely do that in this paper, and show how the nature of market competition affects the sustainability of R&D cooperation. Our main result is R&D cooperation is easier to sustain under quantity competition than under price competition.

2. The model

There are two firms characterized by constant symmetric marginal costs, normalized to zero. The market demand curve faced by each of the firm is

$$p_i = 1 - q_i - \gamma q_j, \quad i \neq j, \ i, j = 1, 2$$

where γ represents the degree of product differentiation; $\gamma \in [0, 1]$ (see Singh and Vives, 1984). Note that if $\gamma = 1$, the products of the two firms would be considered undifferentiated, and the two firms compete in the same market. At the other extreme, if $\gamma = 0$, two products will be fully differentiated and the firms will be enjoying monopoly. Crucially, γ in our model is endogenously determined by firm's decision (E) to spend on R&D or advertizing. Since, firms would prefer a differentiated market, partial or full, to an undifferentiated one, there arises a free-riding problem where a firm would like the other firm to give the R&D effort, but save on its own effort.

We assume that the R&D effort (or advertizing expenditure) is exogenously fixed denoted by $e > 0$. thus, firms have a binary choice, give no effort ($E = 0$) and give effort ($E = e$).[1] The differentiation implications of giving effort, unilateral or joint, are specified in the

[1]Fixed effort is assumed for simplification. Continuous effort does not alter the main conclusions. At this stage we do not specify the size of e; but later we will see what the range of e should be to make the analysis interesting.

following assumption (**Assumption 1**):

$$\begin{aligned}\gamma &= 0 \qquad \text{if} \quad E_i = E_j = e,\\ \gamma &= \frac{1}{2} \qquad \text{if} \quad E_i = e, E_j = 0, \quad i \neq j,\\ \gamma &= 1 \qquad \text{if} \quad E_i = E_j = 0.\end{aligned}$$

In the event of both giving efforts, the differentiation will be complete, i.e. $\gamma = 0$. On the other hand, if none of the firms gives effort, then no differentiation is achieved; then we have $\gamma = 1$. But if only one firm gives effort, the differentiation will be partial. We assume in this case γ takes the value $1/2$. It is noteworthy that if unilateral effort leads to a realization of $\gamma < 1/2$, then the incentive to free-ride will be stronger, and cooperation will be harder to come by; smaller the value of γ, greater the returns to free-riding. On the other hand, if the realized γ lies between $1/2$ and 1, returns to free-riding will be much less; cooperation in this case will be somewhat easier. Thus, specifying γ other than $1/2$ to represent partial differentiation introduces some sort of bias either in favor of free-riding or in favor of cooperation. $\gamma = 1/2$ seems to be a neutral choice.[2]

The interactions between the two firms are captured by a simple two-stage game. In the first stage, the firms simultaneously decide whether to give the effort or not. Then in the second stage, effort choices become common knowledge, and the firms engage in quantity competition (*a la* Cournot) or price competition (*a la* Bertrand). Here we assume, for simplicity, that production cost is zero.[3] Let π denote the second stage profit or revenue (when already the effort cost has been sunk), and Π the first stage profit that takes the effort cost into account. Thus, $\Pi = \pi - E$. Firm i's profit will be identified by a subscript i to the profit term, and the degree of differentiation associated with the profit will be indicated by a superscript such as UD (for the undifferentiated case), or PD (for partial differentiation), or FD (for full differentiation).

3. Firm interactions

3.1 Quantity competition

Under quantity competition, when the products are undifferentiated profits of firm i ($i = 1, 2$) are

$$\Pi_i^{UD} = \pi_i^{UD} = \frac{1}{9}, \tag{1}$$

[2]As will be evident, the analysis can be easily adapted to other values of γ. Assuringly, all of results hold over a range of γ around $1/2$; we comment on this later.

[3]The assumption of zero marginal cost is not essential. Even when the marginal costs are positive, and not too divergent, the entire analysis goes through.

and when the products are fully differentiated[4], profits are

$$\Pi_i{}^{FD} = \pi_i{}^{FD} - e = \frac{1}{4} - e. \tag{2}$$

Further, in the partial differentiation case, assuming firm i gives effort while firm j does not,

$$\begin{aligned} \Pi_i{}^{PD} = \pi_i{}^{PD} - e = \frac{4}{25} - e, \\ \Pi_j{}^{PD} = \pi_j{}^{PD} = \frac{4}{25}. \end{aligned} \tag{3}$$

That is only one firm gives the effort, but it 'may increase' profit for both of them.

It is straight forward to check that

$$\pi_i{}^{FD} > \pi_i{}^{PD} > \pi_i{}^{UD} > 0, \;\; i = 1, 2.$$

Now we state the prisoners' dilemma problem. This result is similar to Proposition 1 (Lambertini and Rossini 1998). A prisoners' dilemma problem in R&D effort arises if for $i = 1, 2$

$$Max[\pi_i{}^{FD} - \pi_i{}^{PD}, \pi_i{}^{PD} - \pi_i{}^{UD}] < e < \pi_i{}^{FD} - \pi_i{}^{UD}. \tag{4}$$

That is, not giving effort is a dominant strategy for both the firms. But the outcome is Pareto dominated by that of both giving efforts. This can be seen from the payoff matrix that can be easily constructed by assembling the first-stage payoffs.[5]

As noted earlier, $(\pi_i{}^{FD} - \pi_i{}^{PD}) = 1/4 - 4/25 = 9/100$ which is greater than $(\pi_i{}^{PD} - \pi_i{}^{UD}) = 4/25 - 1/9 = 11/225$. Therefore, for the occurrence of the prisoners' dilemma, e must be greater than $9/100 = 0.09$, but less than $(\pi_i{}^{FD} - \pi_i{}^{UD}) = 5/36 = 0.138$. That is, $e \in (0.09, 0.138)$.

3.1.1 Repeated interactions

Now we consider an infinite repetition of the game, in order to see under what condition the Pareto optimal outcome, namely where both the firms choose to give efforts, can be sustained. It is assumed that product differentiation lasts only one period. That is to say, R&D is not cumulative. It requires renewed effort. For example, if it is a knowledge-based product, or a service and if a new set of employees join the firm every period, then every period the effort must be given, say, in the form of training. Alternatively, this can be seen as a result of advertizing, that aims to influence the preferences of the representative consumer.

[4]The general expressions for quantity and profits for any $\gamma \in [0, 1]$ and positive (but constant) marginal costs are

$$q_i = \frac{(2-\gamma) - 2c_i + \gamma c_j}{4 - \gamma^2} \qquad \text{and} \qquad \pi_i = q_i{}^2, \;\; i \neq j, i, j = 1, 2.$$

Set $c_i = c_j = 0$ and $\gamma = 1, 0, 1/2$ to obtain (1), (2) and (3) respectively.

[5]If e is outside this range, various other possibilities arise, including giving effort being part of a Nash equilibrium, which is, however, not our main interest.

If the population is changing every period, then advertizing must be repeated to maintain the influence. Thus, either way the effort has to be repeated.

In order to sustain the 'repeated cooperation' (i.e. giving efforts forever), we follow the Friedman's (1971) trigger strategy implying that a single instance of 'noncooperation' (i.e. not giving effort) by any firm will be met with permanent noncooperation by both the firms. This gives us the standard condition on the discount factor for sustaining the cooperative solution:

$$\delta \geq \frac{{\Pi_i}^{PD} - {\Pi_i}^{FD}}{{\Pi_i}^{PD} - {\Pi_i}^{UD}}, \quad i = 1, 2$$

where δ is the common discount factor of the firms. ${\Pi_i}^{PD}$ is the one-shot deviation profit of firm i from not giving effort while the other firm gives effort, i.e. ${\Pi_i}^{PD} = {\pi_i}^{PD}$.

Note that the subgame perfectness of equilibrium requires the firm j to execute the trigger strategy (i.e. switch to no effort) from period t onwards, when firm i has deviated in period $t-1$. Assuming firm i will now play according to the trigger strategy (i.e. give no effort) from period t onwards, firm j will not deviate from the trigger strategy, if

$${\pi_j}^{PD} - e + \frac{\delta}{1-\delta}{\pi_j}^{UD} \leq \frac{{\pi_j}^{UD}}{1-\delta}.$$

This condition reduces to $e \geq {\pi_j}^{PD} - {\pi_j}^{UD}$ which will be satisfied, as we are going to consider e only from the interval specified in condition (4).

The condition on the discount factor can be simplified as

$$\delta \geq \frac{e - ({\pi_i}^{FD} - {\pi_i}^{PD})}{{\pi_i}^{PD} - {\pi_i}^{UD}}, \quad i = 1, 2. \tag{5}$$

The numerator on the right hand side is positive by (4).

Conditions (4) and (5) hold key to our analysis. The difference between the gains from matching the cooperation, $({\pi_i}^{FD} - {\pi_i}^{PD})$, and the gains from unilateral effort, $({\pi_i}^{PD} - {\pi_i}^{UD})$, together with the gains from collective cooperation, $(\pi_i^{FD} - \pi_i^{UD})$, determines the relevant range of effort (for the prisoners' dilemma to arise), as well as the critical size of the discount factor. In particular, the right hand side expression of eq. (5) tells us whether cooperation is easy or hard; if it is positive and relatively large then cooperation is hard. As is apparent now, this boils down to the comparison of the two terms $({\pi_i}^{FD} - {\pi_i}^{PD})$ and $({\pi_i}^{PD} - {\pi_i}^{UD})$, which, we shall see, depends on the nature of competition.

Under quantity competition, assuming $e \in (0.09, 0.138)$, the lowest level of δ necessary to sustain cooperation is given by,

$$\delta^q = -\frac{81}{44} + \frac{225}{11}e. \tag{6}$$

The relationship is linear and strictly increasing. Note that when the effort is at its lowest level (within the critical range), i.e. $e = 9/100$ any $\delta > 0$ can sustain cooperation. But as the effort level rises, the attractiveness of the fully differentiated payoffs decreases, and the discount factor has to rise proportionately to sustain cooperation. See Figure 1 for a visual illustration of δ^q.

3.2 Price competition

Now we consider the case of price competition (in the second stage). The first stage of the game remains unchanged. The second stage profit terms associated with price competition are:

$$\pi_i^{UD} = 0, \qquad \pi_i^{FD} = \frac{1}{4}, \qquad \text{for } \; i = 1, 2. \tag{7}$$

Firm 2's profits are symmetrically obtained. In the case of partial differentiation[6],

$$\pi_i^{PD} = \frac{4}{27}, \;\; i \neq j, i, j = 1, 2. \tag{8}$$

Once again, full differentiation is found to be strictly better than partial differentiation which in turn is strictly better than no differentiation. Under price competition, profits in the undifferentiated market become zero, but the relative profit gains from partial differentiation exceed that from full differentiation. That is, $(\pi_i^{FD} - \pi_i^{PD}) = 1/4 - 4/27 = 11/108 < (\pi_i^{PD} - \pi_i^{UD}) = 16/108 = 4/27$. So for the prisoners' dilemma to hold, according to the condition (4) it is necessary that $e \in (4/27, 1/4) = (0.148, 0.25)$.

Now compare this range of e with the range of e obtained under quantity competition. Two ranges of efforts are disjoint, implying that the prisoners' dilemma problem cannot hold simultaneously for both price and quantity competition. The effort level has to be much higher under price competition, for the prisoners' dilemma problem to occur. Since with price strategies, moving from no differentiation to full differentiation yields greater gains, effort costs will also have to increase significantly in order for the strategy of not giving efforts to be optimal.

This suggests that the occurrence of prisoners' dilemma may depend on the nature of competition. At a given level of e, the following can be said about the stage game. If R&D cooperation in not achievable under quantity competition due to prisoners' dilemma, it may be achieved under price competition (when $e \in (0.09, 0.138)$); see footnote 5. On the other hand, if price competition does not allow cooperation (due to prisoners' dilemma), then under quantity competition not only is cooperation unachievable, but it is not even Pareto optimal (when $e \in (0.148, 0.25)$).

Next, moving onto the infinitely repeated version of this game, we examine condition (5). Now, $e > (\pi_i^{FD} - \pi_i^{PD})$ within its critical range. This gives a strictly positive lower bound on δ. The minimum value of the discount factor necessary to sustain cooperation is given by:

$$\delta^p = -\frac{11}{16} + \frac{27}{4}e. \tag{9}$$

[6] The general expressions for outputs and profits for any given $\gamma < 1$ are as follows:

$$q_i = \frac{2 - \gamma(1+\gamma) + \gamma c_j - (2-\gamma^2)c_i}{(1-\gamma^2)(4-\gamma^2)}, \qquad \pi_i = (1-\gamma^2){q_i}^2, \;\; i = 1, 2.$$

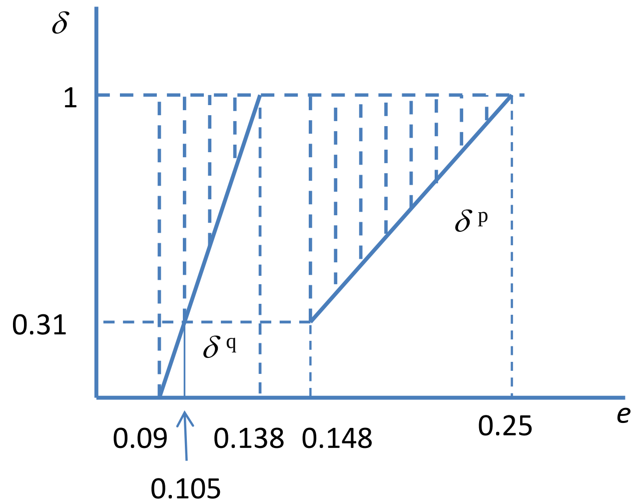

Figure 1. Minimum discount factors

In Figure 1, we illustrate δ^p and mark the feasible discount factors. δ^p, like δ^q, is positively sloped and linear. But note, within the relevant ranges, at the lowest value of e cooperation is more difficult under price competition. Under price competition at $e = 0.148$, the minimum δ needed is $\delta^p = 5/16 = 0.31$. In contrast, under quantity competition at $e = 0.09$, the minimum δ needed is $\delta^q = 0$. Clearly, in the latter case, cooperation can be sustained at any value of $\delta > 0$.

Moreover, for all $e \in (0.09, 0.105)$ (which constitutes 31.25% of the range of e), δ^q is strictly less than 0.31, which is the lowest value of δ^p. Since δ^q and δ^p are not directly comparable because of different ranges of e, we can compare δ^q and δ^p across differnt percentiles of the distribution. It can be checked that over the entire distribution (at least up to 90 % percentile of the distribution) δ^q is strictly smaller than δ^p.[7] Thus, it can be said that under quantity competition cooperation is more stable, which is somewhat contrary to conventional wisdom (applicable to homogenous goods).

This can be explained in the following way. Under quantity competition (for being a softer form of competition) gains to matching cooperation are greater than gains to unilateral effort; i.e. ${\pi_i}^{FD} - {\pi_i}^{PD} > {\pi_i}^{PD} - {\pi_i}^{UD}$. This creates greater stability. On the other hand, under price competition we get ${\pi_i}^{FD} - {\pi_i}^{PD} < {\pi_i}^{PD} - {\pi_i}^{UD}$. Because of fierce competition in the undifferentiated market, firms gain relatively much more from partial differentiation, than from matching their rivals' efforts. Put differently, moving from zero differentiation to some differentiation generates higher profit than moving from some differentiation to full differentiation. Hence, δ must be sufficiently high to induce cooperation to achieve full differentiation.

[7]For instance, consider the 50 percentile point, which corresponds to $e = 0.114$ under quantity competition and $e = 0.199$ for price competition. At $e = 0.114$ we get $\delta^q = 0.49$, while at $e = 0.199$ we get $\delta^p = 0.66$. Then the 80 percentile point corresponds to $e = 0.128$ for quantity competition and $e = 0.229$ for price competition. Here, we get $\delta^q = 0.79$ and $\delta^p = 0.87$, respectively. Finally, the 90 percentile point corresponds to $e = 0.133$ for quantity competition and $e = 0.239$ for price competition; we then get $\delta^q = 0.88$ and $\delta^p = 0.93$, respectively. So at these points and all other points in between we have $\delta^q < \delta^p$.

Proposition 1. *(a) For a given level of effort e, the prisoners' dilemma problem cannot hold simultaneously for both quantity and price competition. (b) In the infinitely repeated game, collusion in giving joint effort is more stable under quantity competition than under price competition.*

Do the results stated in Proposition 1 change if in **Assumption 1** $\gamma = 1/2$ is replaced by $\gamma > 1/2$ or $\gamma < 1/2$ for the partial differentiation case? The answer is mixed. Here we do not offer a complete formal analysis of that, but report the main results, which can be easily verified by using the profit expressions provided in footnotes 4 and 5. First of all, for all γ belonging to the interval $[0.35, 0.55]$ Proposition 1 remains fully in tact; both parts (*a* and *b*) go through. So we could deviate from $\gamma = 1/2$ either way, to some extent. Outside this interval, some aspects of the proposition get modified. For example, for $\gamma \in (0.55, 0.67)$ prisoners' dilemma *can* hold under both quantity and price competition, as the two ranges of e overlap. That is, part *a* of Proposition 1 does not hold. Next, we get $\delta^q = 0$ at all $\gamma \geq 0.35$, and we also get $\delta^p = 0$ at all $\gamma \geq 0.61$. That is to say, at all $\gamma \geq 0.61$ stability of collusion does not vary between the two types of competition, and collusion can be sustained at all $\delta > 0$; part *b* of proposition 1 does not hold. But as stated earlier, when we move too far away from $\gamma = 1/2$, we create bias in one or the other direction, and therefore, it is unwise to consider γ too high or too low.

4. Conclusion

In this analysis, we extended the analysis of Lambertini and Rossini's (1998) static game of incentive for product innovation and market competition in a repeated game framework. First, we characterized the discount factor associated with sustaining a collusion as a function of the magnitude of effort level in the relevant ranges of prisoners' dilemma problem. Then, we showed that collusion is more likely to be sustainable under quantity competition as opposed to price competition.

References

Albaek, S and L. Lambertini (1998) "Collusion in differentiated duopolies revisited" *Economics Letters* **59**, 305-08.

Deneckere, R. (1983) "Duopoly supergames with product differentiation" *Economics Letters* **11**, 37-42.

Friedman, J. (1971) "A noncooperative equilibrium for supergames" *Review of Economic Studies* **38**, 1-12.

Kesteloot, K and R. Veugelers (1995) "Stable R&D cooperation with spillovers" *Journal of Economics and Management Strategy* **4**, 651-72.

Lambertini, L. (1997) "Prisoners' dilemma in duopoly supergames" *Journal of Economic Theory* **77**, 181-91.

Lambertini, L and G. Rossini (1998) “Product homogeneity as a prisoner’s dilemma in a duopoly with R&D” *Economics Letters* **58**, 297-301.

Rothschild, R. (1992) “On the sustainability of collusion in differentiated duopolies” *Economics Letters* **40**, 33-37.

Singh, N and X. Vives (1984) “Price and quantity competition in a differentiated duopoly” *Rand Journal of Economics* **15**, 546-54.