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The empty core in the Coase theorem: a critical assessment

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Abstract

The purpose of this paper is to give some insights about the debate between Aivazian and Callen (1981, 2003) and Coase (1981) regarding the empty core problem. In particular our analysis concerns the role played by transaction costs in the debate. By maintaining the Aivazian-Callen transaction cost structure, we introduce in the analysis the new category "transaction costs in changing the state" which is linked to reputation and which may occur in situations of endless bargain. We argue, under our particular assumptions, that although Aivazian and Callen (2003) analysis is correct, Coase's intuition regarding the relationship between transaction costs and the empty core (Coase 1981) may be supported within Aivazian and Callen (2003) framework, if we introduce reputational concerns.

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1 Introduction

About twenty-five years ago, Aivazian and Callen (1981) found an interesting result on the Coase Theorem: They show that "with more than two participants the Coase theorem can not always be demonstrated" (Aivazian and Callen 1981, p. 175).

The Coase Theorem, as it is known, asserts that in the absence of transaction costs and other frictions, the final outcome of bargaining (the resource allocation among individuals) is efficient and does not depend on the initial distribution of rights (or liability rules)[1]. In their work Aivazian-Callen found that in the absence of transaction costs, if there are more than two parties and at least two externalities (as pointed out by Mueller 2003, p. 32), then the process of bargaining may give rise to an empty core. In other terms, to an unstable situation leading to an endless re-negotiation cycle[2].

Subsequently, in his rejoinder to the Aivazian-Callen's analysis, Coase (1981) stresses at the end of the paper, among other things[3], the important role played by the zero transaction costs assumption in the Aivazian-Callen's work in order to get their results. He seems to claim that with positive transaction costs the empty core will eventually disappear or may be mitigated and that a word of zero transaction cost is not interesting[4].

Aivazian and Callen (2003) show by a counter-example that Coase's claim is not generally true. In particular, they use transaction costs with a convex technology in the number of agents. In our analysis we maintain the Aivazian-Callen's transaction cost technology. We show that by introducing the *transaction costs to change the state* (TCCS) (see section 3), Coase's argumentation can be supported and we argue that this category of transaction costs could emerge in an endless bargain situation. We point out that our result only holds in the Aivazian-Callen's example with our particular form of transaction costs; a general conclusion can only be obtained from general core existence analysis such as balancedness (see Bondareva 1962; Shapley 1967).

In the next section we resume the empty core debate. In section 3 we introduce the TCCS category and we show the main result of the paper. Finally, section 4 concludes.

2 The empty core debate

The Aivazian-Callen's analysis (1981) can be stated as follows. Suppose there are two factories, A and B, producing a pollutant by product such as smoke. Suppose also that there is a third factory, a laundry C, whose costs are raised by the emission of smoke. Laundry C does not produce any kind of pollutant. We use the symbol Π to denote the characteristic function of the cooperative game. In particular Π_i stands for the profits for the firms in the absence of bargaining. As in Aivazian and Callen (1981), we suppose that they amount to: Π_A=3,000, Π_B=8,000, Π_C=24,000. There are no transaction costs

[1]See Coase (1960, 1988). It was Stigler (1966, p. 113) the first author to introduce the label "Coase Theorem". A commonly accepted definition of Coase Theorem may be found in Mueller (2003, p. 28) "In the absence of transaction and bargaining costs, affected parties to an externality will agree on an allocation of resources that is both Pareto optimal and independent of any prior assignment of property rights".

[2]In other worlds, the coalition agreements, in a super additive cooperative game, are not stable (see section 2).

[3]In this paper Coase expresses also other observations we are not concerned with.

[4]Other solutions, proposed in the literature of the empty core, are: *penalty clauses*, *binding contracts* (i.e., the fact that contracts once stipulated are binding for all participants and cannot be breached without the permission of all of them), and *bargaining restrictions* (Bernholz 1997, 1999; Mueller 2003).

and firms are free to bargain, or to make coalitions with the following particular payoffs: $\Pi_{A,B}$=15,000 (where $\Pi_{A,B}$ stands for the joint profit of the coalition formed by A and B after having reached an agreement or after having merged); $\Pi_{A,C}$=31,000; $\Pi_{B,C}$=36,000; $\Pi_{A,B,C}$=40,000.

It can be shown that all the firms have an incentive to bargain, since $\Pi_{i,j} > \Pi_i + \Pi_j$ for all $i \neq j$ in $\{A, B, C\}$. Furthermore, it results that $\Pi_{A,B,C} > \Pi_{i,j} + \Pi_k$ for all $i \neq j \neq k$ in $\{A, B, C\}$, and therefore all the firms have the incentive to form the *grand coalition* which indeed represents the Pareto optimal outcome. In the coalition formed by AC (similarly BC), laundry (C) stops the production of A (B), by proper side payments. In the *grand coalition* ABC the laundry makes B and C interrupt their production, by side payments. Furthermore - denoting by X_i the amount of profit firm i gets when participating to the *grand coalition* - it results that X_A+X_B+X_C=$\Pi_{A,B,C}$

An interesting question concerns the final outcome of this situation. In order to know whether the firms will choose the *grand coalition*, we have to consider the liability to emit pollutant.

If A and B are liable for the pollution, then the final outcome will be $\Pi_{A,B,C}$=40,000 with X_A=X_B= 0 are X_C= 40,000. In this situation C will force the other factories not to produce. In other terms, A and B will not be able to pay adequately C in order to continue the production: the maximum sum A and B can pay amount to $\Pi_{A,B}$=15,000 which is less than 16.000=$\Pi_{A,B,C} - \Pi_C$, the damage they cause to C.

We may think that also in the case in which A and B are not liable for the pollution, the final outcome will be $\Pi_{A,B,C}$=40,000, since C will be able to compensate the other factories for their closure. However, it can be shown that in this case the grand coalition is unstable, i.e. *is not in the core*[5]. In order to be stable, the grand coalition should satisfy the following conditions[6]

$$X_A > \Pi_A, \quad X_B > \Pi_B, \quad X_C > \Pi_C \tag{1}$$

$$X_A + X_B > \Pi_{A,B}, \quad X_A + X_C > \Pi_{A,C}, \quad X_B + X_C > \Pi_{B,C} \tag{2}$$

$$X_A + X_B + X_C = \Pi_{A,B,C}. \tag{3}$$

By summing the inequalities in (2) we obtain:

$$X_A + X_B + X_C > \frac{1}{2}[\Pi_{A,B} + \Pi_{B,C} + \Pi_{A,C}]. \tag{4}$$

Then by substituting (3) in (4), we obtain *the non-empty core condition*:

$$\Pi_{A,B,C} > \frac{1}{2}[\Pi_{A,B} + \Pi_{B,C} + \Pi_{A,C}]. \tag{5}$$

Condition (5) is violated in the numerical example given above:

$$\Pi_{A,B,C} = 40,000 < \frac{1}{2}[\Pi_{A,B} + \Pi_{B,C} + \Pi_{A,C}] = 41,000.$$

[5]Regarding the core theory see, among others, Telser (1994, 1996).

[6]Aivazian and Callen (1981, p. 179).

Therefore, if A and B are not liable for the pollution, the process of forming coalition is intrinsically unstable[7].

In his rebuttal to Aivazian-Callen's analysis, Coase (1981, p. 187) stressed, among others, the absence of transaction costs:

"I would not wish to conclude without observing that, while consideration of what would happen in a world of zero transaction cost can give us valuable insights, these insights are, in my view, without value except as steps on the way to the analysis of the real world of positive transaction costs."

In other terms, he seemed to argue that with positive transaction costs the probability to observe an empty-core may be negligible. In order to solve this part of the Coase rejoinder, Aivazian and Callen (2003, pp. 290-92) introduce positive transaction costs in their framework. In particular they model transaction costs as an increasing function of the number of agents participating in the bargain. They show that the occurrence of an empty-core may be even wider.

Following Aivazian-Callen, consider a generic normalized characteristic function:

$\Pi_i = 0$ for all $i = A, B, C$; $\Pi_{A,B} = a$; $\Pi_{A,C} = b$; $\Pi_{B,C} = c$; $\Pi_{A,B,C} = d$

where a, b, c, d are positive constants with $d > a, b, c$. It can be shown that the core of the game is empty if and only if

$$d < (a + b + c)/2.$$

Now introduce a transaction cost $C(x)$ equal to

$$C(x) = \begin{cases} x^k, & \text{if } x > 1 \\ 0, & \text{if } x = 1, \end{cases} \tag{6}$$

where x is the number of agents in the coalition and $k > 1$ is a parameter. In other terms, $C(x)$ is a convex function in the number of agents.

With such transaction cost the characteristic function becomes $\Pi_i = 0$ for all $i = A, B, C$; $\Pi_{A,B} = a - 2^k$; $\Pi_{A,C} = b - 2^k$; $\Pi_{B,C} = c - 2^k$; $\Pi_{A,B,C} = d - 3^k$.

This implies that the core is empty if

$$d < 3^k + (a + b + c - 3 \cdot 2^k)/2.$$

Note that increasing k will cause the inequality to eventually be true since 3^k increase more quickly than 2^k. Therefore Aivazian and Callen (2003, p. 291) may conclude that the introduction of positive transaction costs make the occurrence of the empty core even more likely.

Before introducing the TCCS category, we note that Aivazian-Callen restrict their attention to the case in which $k > 1$. We conclude this section by considering the other cases which are:

1) $k = 0$.

Under this assumption $C(x)$ becomes:

[7] Consider, for example the situation in which $X_A = 5,000$, $X_B = 10,000$ and $X_C = 25,000$. In this case there are more than one coalitions which could stop the grand coalition. For example A and C would have the incentive to form a new coalition by dividing $\Pi_{A,C} = 31,000$ in the following way: 5,500 to A and 25,500 to C.

$$C(x) = \begin{cases} 1, & \text{if } x > 1 \\ 0, & \text{if } x = 1. \end{cases} \tag{7}$$

In this case each bargain presents fixed transaction costs normalized to 1 which does not depend on the number of the members of the coalition. These costs may include, for example, notary and legal expenses for the bargain, and so on. Substituting $C(x)$ in the empty core condition, we get $d < (a+b+c)/2 - 1/2$. Therefore, in this case, the presence of transaction costs mitigates the occurrence of an empty core.

2) $k = 1$.
In this case $C(x)$ becomes:

$$C(x) = \begin{cases} x, & \text{if } x > 1 \\ 0, & \text{if } x = 1. \end{cases} \tag{8}$$

This means that each agent has fixed transaction cost normalized to 1 in order to join the coalition. Since $3^k = 3 \cdot 2^k/2$, with $k = 1$ we come back to the 'original' Aivazian-Callen's result (1981) as described in equation (5). Therefore we may conclude that the introduction of fixed transaction costs (for each agent) leaves the Aivazian-Callen's conclusion unaffected.

3) $k < 1$.
In this case $C(x)$ is a concave function in the number of agents forming the coalition. Since $3^k < 3 \cdot 2^k/2$ with $k < 1$, the introduction of transaction costs reduces the occurrence of the empty core giving some support to the aforementioned Coase's argumentation.

Although convexity in transaction costs is not "necessarily more reasonable than, say concavity" (Aivazian and Callen 2003, p. 297, note 13), we do not claim, obviously, that they are 'pathological' at all. Indeed we will continue to use the Aivazian-Callen's assumption. However, in what follows we will argue that in an empty core framework, the Aivazian-Callen's definition of transaction costs is not exhaustively defined.

3 Costs to change the state and transaction costs

Bernholz (1997, 1999) analyzes the Aivaizian-Callen empty core when contracts are binding for the agents. He shows that such contracts (both internal and external) break the endless bargain and thus avoid the empty core. Contrary to Bernholz we assume that contracts are not binding for the agents. This means that contracts may be freely breached but we assume that they leave a trace in the future contracts' costs in terms of reputation. Indeed it is quite reasonable to assume that an agent breaching a contract previously subscribed gives an image of unreliability which may create consequences in his future bargains.

In the following $N = \{A, B, C\}$ continues to be the set of economic agents and S is the set of all the possible coalitions. We define $C_i(s, n_i)$ the non productive costs for reaching the coalition s afforded by agent i, where $s \in S$, $i \in N$. The variable n_i represents the number n of transactions breached by agent i before to reach state s.

Now define $\Gamma_i(s)$ the 'information-independent' transaction cost component: i.e. the cost which depends only on the specific type of contract agent-i has to stipulate in order to reach s [8] and $\nabla_i(s, n_i) = C_i(s, n_i) - \Gamma_i(s)$, so that we can obtain the following transaction cost decomposition:

$$C_i(s, n_i) = \Gamma_i(s) + \nabla_i(s, n_i),$$

where $\nabla_i(s, n_i)$ is defined to be the additional transaction cost depending on reputation concerns. We call this component *transaction costs to change the state* (TCCS as defined in the introduction).

Now, transaction costs considered by Aivazian-Callen refer only to Γ. The main difference between Γ and ∇ is that while Γ are the 'standard' transaction costs, which are necessary for the negotiation or the *bargain*, ∇ are the cost depending also on the actual characteristics of the agent.

In the Aivazian-Callen's analysis the role of Γ is played by x^k (see eq. 6). In our analysis total transaction costs are equal to:

$$C_i(s, n_i) = \Gamma_i(s) + \nabla_i(s, n_i) = \theta_i x^k + \nabla_i(s, n_i),$$

where x^k has the same meaning of the Aivazian-Callen's framework, θ_i is a fraction[9] in $[0, 1]$ whereas, $\nabla_i(s, n_i)$ [10] is the additional cost previously defined.

To understand the role of $\nabla_i(n_i)$, let's assume that its first difference with respect to n is positive, that is:

Assumption 1 $\nabla_i(n_i) - \nabla_i(n_i - 1) > 0$.[11]

We also assume the following:

Assumption 2 The only way agents have to pass from a state (coalition) s to a state s' ($s \neq s_0$, where s_0 is the initial situation in which there is no coalition yet) without breaching the previous agreement, is by unanimous agreement.[12]

Assumption 1 implies that TCCS is increasing with the number of bargains occurred. As said above, a quite natural interpretation of this condition may be found in reputation concerns. Reputation, as it is known, depends on the past behavior of the individual and this information is contained in n_i, that is the number of coalitions breached by the agent i in the past. If an agent passes from s to s' (i.e., from one coalition to another one) she/he may give a signal of unreliability to other agents and this negative signal may involve an additional transaction cost for her/him regarding future transactions. For example she/he would experience more difficulty in persuading a potential partner about her/his reliability: this difficulty could lead to a TCCS increase[13]. For this reason we argue that reputational concerns could emerge quite naturally in an empty core situations with an endless bargain.

[8] As described above in section 2, in the Aivazian-Callen's framework (2003) it depends on the number of agents participating to the bargain and hence on s.

[9] Obviously, $\theta_A + \theta_B + \theta_C = 1$, and $\theta_i > 0$.

[10] In the following for notational convenience we will write $\nabla_i(n_i)$ instead of $\nabla_i(s, n_i)$.

[11] This obviously implies that also $C_i(s, n_i) - C_i(s, n_i - 1) > 0$. For expositional purposes we prefer to state Assumption 1 in terms of ∇ because we want to focus the attention on the role played by TCCS.

[12] There is a similarity between this assumption and Assumption 7 in Bernholz (1997, p. 428).

[13] Note that this issue is different from a penalty clause. While penalty clauses are explicitly fixed in contracts, reputation has an 'implicit' nature; it emerges quite naturally, 'endogenously'

As noted by Macaulay (1963, p. 58) quoted by Halonen (2002, p. 539): "Businessmen often prefer to rely on a 'a man's word' in a brief letter, a handshake, or 'common honesty and decency' - even when the transaction involves exposure to serious risks". The reason of this behavior may be found in reputation concerns[14]. Reputation is also an important issue in the asymmetric information literature since it is an important component of bankruptcy costs (Greenwald and Stiglitz 1993).

Under Assumptions 1 and 2 we are able to state the following result:

Proposition
Given Assumptions 1 and 2, there exist TCCS sufficiently high such that:
1) If a two agent coalition is reached, then the grand coalition is reached too.
2) The grand coalition is stable.

Proof:
In order to show the stability of the grand coalition (part two), we consider that the Aivazian-Callen's analysis changes as follows: the normalized characteristic function becomes $\Pi_i = 0$ [15] for all $i = A, B, C$;

$$\Pi_{A,B,C} = d - 3^k - \sum_{i \in N} \nabla_i(n_i)$$

$$\Pi_{A,B} = a - 2^k - \nabla_A(n_A + 1) - \nabla_B(n_B + 1)$$

$$\Pi_{A,C} = b - 2^k - \nabla_A(n_A + 2) - \nabla_C(n_C)$$

$$\Pi_{B,C} = c - 2^k - \nabla_B(n_B + 2) - \nabla_C(n_C + 1),$$

where

$$n_i \quad < \quad n_i + 1, \quad n_i + 2, \quad n_i + 3. \tag{9}$$

Since it is not important, in our analysis, to specify the formation order of two agent coalitions, we assumed, without loss of generality, that the coalition A, B is formed after the grand coalition, the coalition A, C is formed after the A, B one, and so on.

The empty core condition now changes in

$$d < 3^k + \frac{a + b + c - 3 \cdot 2^k}{2} - \sum_{i \in N} F_i + G, \tag{10}$$

where

$$G = \sum_{i \in N} \nabla_i(n_i)$$

$$F_A = \frac{\nabla_A(n_A + 1) + \nabla_A(n_A + 2)}{2}$$

[14] "[...] the one-shot gain from opportunistic behaviour is outweighed by the loss of trust in the future" (Halonen 2002, p. 539).

[15] The single coalition is not affected by transaction costs since it is a non-recurrent state in the bargaining loop.

$$F_B = \frac{\nabla_B(n_B+1) + \nabla_B(n_B+2)}{2}$$

$$F_C = \frac{\nabla_C(n_C) + \nabla_C(n_C+1)}{2}.$$

Under Assumption 1 it is straightforward to note that

$$\sum_{i \in N} F_i - G > 0, \tag{11}$$

and there exists

$$\sum_{i \in N} F_i,$$

sufficiently high such that the empty core condition (10) does not hold.

We have shown that there are TCCS which are sufficiently high such that the empty core condition disappears and thus the 'endless' bargain is interrupted.

In order to show the first part of the proposition, we note that there are TCCS sufficiently high such that it is not convenient to breach the two agent coalition (once it has been attained) in order to reach a different two agent coalition. In order to complete this part of the proof we have simply to invoke Assumption 2.‖

We note that our result does not concern with the shift from the initial situation s_0 to the grand coalition. Indeed, it is not sure that in an empty core situation, the grand coalition will be, actually attained. As noted by Avaizian-Callen (2003, p. 294): "One thing is clear, however, that when the core is empty, if an initial agreement is made between two (or among the three parties) it will violate individual (or sub-coalitional) rationality."

4 Conclusions

In this paper we analyze the role of transaction costs in the Aivazian-Callen and Coase debate about the empty core. We point out that our result only holds in the Aivazian-Callen's example with the introduction of TCCS. Moreover, it does not invalidate the Aivazian-Callen's analysis: it just shows that by introducing this type of friction in the bargain process, the Coase argumentation (1981) may be supported. We argue that this new friction could possibly emerge in an endless bargain, being linked to reputation aspects. In particular the result of our work is that with convex transaction costs (as in Aivazian and Callen 2003), there exist TCCS sufficiently high such that the empty core may disappear. We conclude by claiming again that if, on one hand TCCS seem to mitigate the empty core, on the other hand, we cannot say that the efficient outcome will be reached for sure.

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