E C O N O M I C S B U L L E T I N

Purchasing power parity: A nonlinear multivariate perspective

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Abstract

The goal of this paper is to disentangle the respective contributions of the nominal exchange rate and the price differential to the adjustment towards the Purchasing Power Parity relation. To this end, we estimate a multivariate threshold vector equilibrium correction model, whose dynamics is consistent with the PPP in presence of trading costs. European data support the relevance of this model for Belgium, France and Italy, but this is not the case for the G7 data against the US Dollar. Furthermore, the adjustment in European countries seems to have been achieved only through nominal exchange rate changes.

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1. Introduction

Many recent studies find evidence in favor of a nonlinear stationary dynamics for real exchange rates, especially for European currencies (see for instance Bec, Ben Salem and Carrasco (2004) and citations therein). This finding is rather striking since the nominal exchange rate adjustment should be hindered under managed or fixed exchange rate systems, like for instance the European Monetary System. Consequently, the prices adjustment should play a dominant role in countries belonging to this kind of exchange rate agreements unless the fixed parity itself is managed to target PPP, e.g. through successive realignments.

The main contribution of this paper is to exploit recent advances in multivariate nonlinear cointegrated time series theory (see e.g. Bec and Rahbek (2004) or Saikkonen (2005)) so as to disentangle the roles of the nominal exchange rate and the international price differential in the adjustment towards PPP.

2. The Threshold Vector Equilibrium Model (TVECM)

Denote $X_t = (e_t, z_t)'$, where z_t is the log real exchange rate, defined by $z_t = e_t + p_t^* - p_t$, e_t the log nominal exchange rate, p_t^* and p_t the foreign and home log price indices. According to the PPP relationship, z_t should be stationary, i.e. X_t should be cointegrated with cointegrating vector $\beta = (0, 1)'$. Following Bec and Rahbek (2004), we consider the TVECM for ΔX_t :

$$\Delta X_t = (s_t\alpha + (1 - s_t)a)\,\beta' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t \tag{1}$$

where

$$s_t = 1(|\beta' X_{t-1}| \geq \lambda). \tag{2}$$

The parameters α, a and β are 2×1 matrices while the short-run parameters $(\Gamma_i)_{i=1,\dots,k-1}$ are 2×2 matrices. The sequence ε_t is assumed to be *i.i.d.* $(0, \Omega)$. The transition function s_t depends on $\beta' X_{t-1}$ and λ, a positive scalar: s_t is zero-one valued and hence not continuous.

This TVECM is motivated by theoretical models which take into account the presence of trading costs. As shown in e.g. Obstfeld and Rogoff (2000), if such proportional costs exist, they impede arbitrage when the international price differential is lesser than these costs: the adjustment towards PPP may be discontinuous, active for large PPP deviations only. This is captured by the outer regime $s_t = 1$, which is triggered when $|\beta' X_{t-1}| \geq \lambda$. So, the trading costs are reflected by the threshold parameter λ.

3. Estimation and testing

Economic theory suggests $\beta = (0, 1)'$. By contrast, λ has to be estimated. We follow Hansen and Seo (2002) by doing a grid search over a set of threshold values, say $\Lambda = [\lambda_L, \lambda_U]$: for each value of λ in Λ, TVECM parameters are estimated by OLS regressions of ΔX_t on ΔX_{t-i}, $i = 1, .., k-1$, $s_t(\lambda)\beta' X_{t-1}$ and $(1 - s_t(\lambda))\beta' X_{t-1}$. The threshold estimator, $\hat{\lambda}$, is the value of λ which maximizes the likelihood. The boundaries of Λ are defined so that λ_L is the 5% percentile of $\beta' X_{t-1}$ and λ_U the 95% percentile.

In this TVECM, one interesting question is whether the adjustment towards PPP is significantly different across regimes or not. Note that the linear VECM and the TVECM are nested by imposing the equality of the α and a across regimes. Formally, the linearity hypothesis corresponds to the following null:

$$\mathrm{H}_0: \quad \alpha_1 = a_1 \text{ and } \alpha_2 = a_2 \tag{3}$$

Rewrite model (1) as

$$Y_t = B' V_t + \varepsilon_t,$$

with $Y_t = \Delta X_t$ a $p \times 1$ vector, $V_t = (s_t \beta' X_{t-1}, (1 - s_t)\beta' X_{t-1}, \Delta X_{t-1}, \cdots, \Delta X_{t-k+1})'$ a $\ell \times 1$ vector and B' the $p \times \ell$ matrix of parameters such that the null becomes

$$\mathrm{H}_0: \quad B = H\Psi, \tag{4}$$

with H and Ψ of dimensions $\ell \times s$ and $s \times p$ respectively, with $s < \ell$. With such notation, if λ were known, one could test for a threshold by using the following LM-statistic, derived as the score test for the hypothesis (4) on B:

$$\mathrm{LM}(\lambda) = T tr \left\{ S_{VV}^{-1} H_\perp (H_\perp' S_{VV}^{-1} H_\perp)^{-1} H_\perp' S_{VV}^{-1} S_{VY} \hat{\Omega}^{-1} S_{YV} \right\} \tag{5}$$

where T is the number of observations, $S_{VV} = \frac{1}{T}\sum_1^T V_t V_t'$, $S_{YV} = \frac{1}{T}\sum_1^T Y_t V_t'$ and $H_\perp$ is the $\ell \times (\ell - s)$ orthogonal complement of H. This statistic depends on λ through the first two variables that enter V_t, since they are defined using s_t. The issue arising because λ is a nuisance parameter under the null may be circumvented using the statistic SupLM $= \sup_{\lambda \in \Lambda} \mathrm{LM}(\lambda)$ whose asymptotic distribution obtains from Hansen (1996). Consequently, the residual bootstrap method described in Hansen and Seo (2002) is used to compute the p-value.

Finally, this multivariate analysis allows to determine the variable(s) at work in the adjustment to PPP : nominal exchange rate and/or price differential. Specifically, the null hypothesis that the changes in the price differential — conditional on short-run parameters — do not adjust the PPP may be tested from the joint restriction $\alpha_1 = \alpha_2$ and $a_1 = a_2$ in (1). The corresponding LR statistics is χ^2 distributed.

4. Data

Two sets of post Bretton-Wood monthly data are considered[1]. The G7 data set includes nominal exchange rates from Canada, Japan, United-Kingdom, Germany, France and Italy vis-à-vis the U.S. Dollar from 1973:01 to 2006:12. The European set includes data spanning from 1973:01 to 1998:12 for seven European countries : Germany, France, Italy, Spain, Portugal, Finland and Belgium. This choice aims at distinguishing the role of the exchange rate agreements from the role of relative price adjustment in explaining the real exchange rate dynamics. Indeed, Germany, France, Belgium and Italy were among the original members of the European currency snake in March 1972 and of the EMS in 1979. The three remaining countries entered this monetary system quite late: in 1989 for Spain, 1992 for Portugal and 1996 for Finland. Germany is chosen as the benchmark country, so that the nominal exchange rates for this European set are defined vis-à-vis the Deutschmark. We used data up to December 1998, since the Euro was introduced in January 1999. All the variables are taken in logarithms and demeaned.

Since the variable governing the switching between regimes must be stationary, we first test for the presence of a unit root in the real exchange rate by using the SupWald statistic proposed by Bec, Guay and Guerre (2008). Contrary to standard unit-root tests, it is consistent and powerful against any stationary alternative, even the nonlinear ones. According to the results of this test — available upon request — the unit root null cannot be rejected for the G7 currencies *vis-à-vis* the US Dollar. In Europe, it is rejected at the 5% level for France, Italy and Belgium, at the 10% level for Portugal and at the 15% level for Spain. Finally, the Finnish real exchange rate does not reject the unit-root null. These last three countries have entered the European Monetary System quite late compared to the three others.

5. TVECM estimates

For all European countries but Finland, the TVECM (1)-(2) was estimated. The autoregressive lag order was chosen so as to eliminate residuals autocorrelation, which lead to retain $k = 2$ except for France and Portugal where $k = 7$. The results obtained for the countries belonging to the "hard-core" of Europe are quite different from the ones found for Spain and Portugal. Regarding the French, Italian and Belgian data, the results reported in table 1 basically tell the same story: small departures from PPP do not influence neither the nominal nor the real exchange rate. Furthermore, the coefficient α_1 is significantly negative and of comparable magnitude in these three models — ranging from -0.080 in Belgium to -0.093 in Italy. This supports the view that the nominal exchange rate changes adjust only the large PPP deviations. However, departures from PPP exert no significant influence on the price differential: as can be seen in table 1, the LR test of the null $\alpha_1 = \alpha_2$ and $a_1 = a_2$ cannot reject this hypothesis for these countries. So, from these three models it appears that until the Euro creation, *i)* the adjustment conditional on short-run parameters was made through the nominal exchange rate changes only, and *ii)* only large deviations from PPP were adjusted.

[1] The consumer price indices and nominal exchange rates (monthly averages) data come from Datastream.

Regarding the Spanish and Portuguese data, the linear adjustment hypothesis can hardly be rejected, with SupLM $p-$values of 86.4% and 96.3% respectively. The estimated equilibrium correction parameters[2] suggest a linear adjustment toward PPP from both the nominal exchange rate and the price differential.

6. Conclusion

Our findings suggest that the PPP relationship is better supported by European data than by other major currencies outside this area, even when allowing for nonlinearity. Another important result is that the countries belonging to the hard-core of the European Community support the view of a nonlinear adjustment towards PPP. This adjustment takes place through the nominal exchange rate and for large departures from the PPP only.

Altogether, our results do not rule out the possibility that the European exchange rate agreements have at least partly aimed at not departing too much from the PPP relation.

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[2]The estimation results are available upon request.

Table 1: Estimates of the Threshold Vector Error Correction Models

			France	
$\Delta e_t=$	$\Gamma_{11}(L)\Delta e_{t-1}$	$+\Gamma_{12}(L)\Delta z_{t-1}$	$\mathbf{-0.091} s_t z_{t-1}$	$-0.006(1-s_t)z_{t-1}$
	F_{11}=0.00	F_{12}=0.14	(-4.23)	(-0.37)
$\Delta z_t=$	$\Gamma_{21}(L)\Delta e_{t-1}$	$+\Gamma_{22}(L)\Delta z_{t-1}$	$\mathbf{-0.093}\ s_t z_{t-1}$	$-0.011(1-s_t)z_{t-1}$
	F_{21}=0.02	F_{22}=0.00	(-4.29)	(-0.64)
$\hat{\lambda}$=0.07444; Loglik=-5731.9; $Q_1(12)$=0.19; $Q_2(12)$= 0.50; SupLM=2.5%; LR=71.0%				
			Italy	
$\Delta e_t=$	$\gamma_{11}\Delta e_{t-1}$	$+\gamma_{12}\Delta z_{t-1}$	$\mathbf{-0.093} s_t z_{t-1}$	$+0.000(1-s_t)z_{t-1}$
	(6.51)	(-3.28)	(-4.38)	(0.04)
$\Delta z_t=$	$\gamma_{21}\Delta e_{t-1}$	$+\gamma_{22}\Delta z_{t-1}$	$\mathbf{-0.095} s_t z_{t-1}$	$-0.005(1-s_t)z_{t-1}$
	(0.85)	(1.84)	(-4.37)	(-0.55)
$\hat{\lambda}$=0.1941; Loglik=-5163.0; $Q_1(12)$=0.33; $Q_2(12)$= 0.36; SupLM=2.3%; LR=13.9%				
			Belgium	
$\Delta e_t=$	$\gamma_{11}\Delta e_{t-1}$	$+\gamma_{12}\Delta z_{t-1}$	$\mathbf{-0.080} s_t z_{t-1}$	$+0.001(1-s_t)z_{t-1}$
	(3.70)	(-0.52)	(-4.44)	(0.13)
$\Delta z_t=$	$\gamma_{21}\Delta e_{t-1}$	$+\gamma_{22}\Delta z_{t-1}$	$\mathbf{-0.089} s_t z_{t-1}$	$+0.001(1-s_t)z_{t-1}$
	(0.69)	(1.78)	(-4.32)	(0.07)
$\hat{\lambda}$=0.0866; Loglik=-5953.5; $Q_1(12)$=0.32; $Q_2(12)$= 0.13; SupLM=2.7%; LR=64.0%				

t statistics in parentheses.

F_{ij}= p-value for the null that all coefficients of Γ_{ij} are equal to 0.

$Q_i(12)$= p-value of Ljung-Box Q-stat for residuals autocorrelation test in equation i.

SupLM= bootstrap p-value for the null of linear adjustment.

LR= p-value for the null $\alpha_1 = \alpha_2$ and $a_1 = a_2$.