E c o n o m i c s B u l l e t i n

Hours and effort variation in sunspot-based business cycle theory

Mark Weder
The University of Adelaide

Abstract

This paper analyzes the role of variable work effort in inducing sunspot equilibria in real business cycle models. Not only is it demonstrated that variable workers' work intensity reduces the degree of increasing returns that is needed to generate indeterminacy but it is also shown that this can be done without assuming a very elastic supply of labor.

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1 Introduction

This paper analyzes the role of variable work effort in inducing sunspot equilibria in real business cycle models. It stands in line with various recent attempts that move sunspot models' calibrating assumptions towards parametric zones that render them acceptable in terms of their plausibility. For example, Wen's (1998) model which is likely the current field's front runner pulls down the increasing returns to 1.104. Nevertheless, this success is only accomplished by postulating that labor supply is perfectly elastic. Survey data suggest fairly small labor supply elasticities usually around 1/2 or not much higher. If calibrated in par with these findings, Wen's model requires increasing returns of well above two which is implausibly high. In the present paper I assume that a minor alteration of utility and production technology. In particular, I assume work effort is a choice variable which affects utility negatively and output positively. Procyclical work effort is well established empirically (e.g. Basu, Fernald and Kimball, 2006, or Bewley, 1998). Hence, given the empirical scepticism for significant increasing returns to scale it appears worthwhile to consider alternative directions (for example, Burnside, Eichenbaum and Rebelo, 1996, for this issue).

Burnside and Eichenbaum (1996) and Cho and Cooley (1994) have demonstrated that introducing variable factor utilization in a real business cycle model improves the model's performance. Neiss and Pappa (2005) show that variable factor utilization in sticky price models moves theory into greater conformity with data. Here, I will apply Neiss and Pappa's (2005) framework of variable factor inputs. Not only can I demonstrate that variable workers' work intensity reduces the degree of externalities that is needed to generate indeterminacy but I can also show that this can be done without assuming a very elastic supply of labor. All variables are procyclical at impact and the model displays large endogenous persistence. The procyclicality brings about a larger impact of the external effects in the production technology. This parallels Wen (1998), however, here the effect is coming from the self enforcing, procyclical effects of effort and hours movements. As both effort and hours increase, technology in equilibrium exhibits a larger degree of scale than only coming from the externalities. Variable factor utilization takes on some of the externalities role, however, some imperfections in the form of positive external effects are still required to obtain nonunique equilibria.

2 The artificial economy

The present model augments the plain-vanilla real business cycles model in three directions. First, the intensity of capital utilization is endogenously set. Second, workers' work intensity is a choice as well. Third, the production technology exhibits small externalities.

All intertemporal decisions are conducted by the household sector. Households supply labor and capital services and purchase output from the firms. The *stand-in* household's preferences are ordered by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, e_t, h_t) \qquad u(.,.,.) = \ln c_t - \xi \frac{e_t^{1+\eta}}{1+\eta} - \vartheta \frac{h_t^{1+\zeta}}{1+\zeta}$$

where c_t, e_t, h_t and β stand for consumption, effort, hours worked and the discount factor. E_t is the expectations operator conditional on all information available in periods t and earlier. Parameters are restricted as follows: $0 < \beta < 1$, $\eta > 0$, $\zeta > 0$, $\xi > 0$ and $\vartheta > 0$. ζ and η denote the inverse of the supply elasticities of hours and effort and their restriction make the household's problem well defined. The specific functional form of periodic utility is adapted from Neiss and Pappa (2005).[1] Denote by k_t the stock of capital. Capital depreciation, δ_t, is an increasing convex function of capital utilization (or employment rate), u_t

$$\delta_t = \frac{1}{\theta} u_t^{\theta} \qquad \theta > 1.$$

Higher utilization causes faster depreciation because of wear and tear on the capital stock. Then, the capital accumulation equation

$$k_{t+1} = (1 - \delta_t)k_t + w_t e_t h_t + r_t u_t k_t - c_t.$$

Factor prices are taken as given by the household. The household's maximization can be summarized by

$$\xi e_t^{1+\eta} = \vartheta h_t^{1+\zeta} = \frac{w_t e_t h_t}{c_t} \tag{1}$$

$$\frac{1}{c_t} = E_t \frac{\beta}{c_{t+1}} \left(r_{t+1} u_{t+1} + 1 - \frac{1}{\theta} u_{t+1}^{\theta} \right) \tag{2}$$

$$u_t^{\theta - 1} = r_t. \tag{3}$$

[1] See also Cho and Cooley (1994, equation 47) for an equivalent setup. I considered the nonseparable period-utility of the form of $\xi \frac{e_t^{1+\eta} h_t^{1+\zeta}}{1+\eta}$. This then requires $\eta = \zeta$ for off-corner solutions making it somewhat less attractive – effort and hours move one-for-one. It can be shown that indeterminacy arises with nonseparable period-utility as well.

In addition, the budget constraint

$$k_{t+1} = (1 - \delta_t)k_t + y_t - c_t$$

and the usual transversality condition – given the initial stock of capital, $k(0) > 0$ – must hold. Equation (1) describes the household's preferred allocation of hours and effort given wages as well as the leisure-consumption trade-off. It states that the (negative) marginal rates of substitution between hours and consumption and effort and consumption must equal the real wage. It is easily seen that effort and hours enter the model economy in a parallel fashion. In a sense, the two margins will allow the household to adjust both hours and effort and consequently reduce the utility loss of, say, working more. (2) is the intertemporal Euler equation. (3) characterizes the optimal level of capital utilization (see also section 2.1.).

Competitive firms rent effective labor services, n_t, and services from effective capital, κ_t, at the competitive rates w_t and r_t. Capital services are defined as the employment rate by which the capital stock is operated times the amount of capital, i.e. the number of machines. Similarly, labor services are the product of hours worked and the effort levels that households exert at work. Firms produce the single final good by having access to an externally increasing returns to scale technology given by

$$y_t = \kappa_t^{\alpha} n_t^{1-\alpha} Y_t^{\frac{\gamma}{1+\gamma}} = (u_t k_t)^{\alpha} (e_t h_t)^{1-\alpha} Y_t^{\frac{\gamma}{1+\gamma}} \tag{4}$$

with $0 < \alpha < 1$, $\gamma > 0$. The externality is captured by the effect that aggregate output, Y_t, brings to bear to the individual technology. Then, γ measures the external effects. I assume that both utilization rates of capital and labor (i.e. effort) can vary. Evidence for this assumption is provided by Basu, Fernald and Kimball (2006). Each firm's profit maximization is given by the well-defined static problem

$$\max_{n_t, \kappa_t} y_t - w_t n_t - r_t \kappa_t \qquad \text{s.t. } (4)$$

which results in the factor demands:

$$r_t = \alpha y_t / (u_t k_t) \qquad \text{and} \qquad w_t = (1 - \alpha) y_t / (e_t h_t). \tag{5}$$

The reason for assuming that firms rent effective input factor units is the following. Given the variable intensity rates, technology would display a nonconvexity if the usual commodity point is employed (that is if handling u_t, k_t as well as e_t and h_t as separate inputs). To circumvent this problem, the alternative commodity points were selected and the production function is concave in κ_t and n_t and also homogenous of degree one in inputs κ_t and n_t.[2] The firms do not care how increases in capital services are realized; that decision is made by the households who own the capital stock and who can decide on the utilization rates. As per the factor labor, the household can either offer to work more hours or expand hourly effort. Of course, here I assume the existence of an (implicit) wage contract that links wage payments not only to the physical presence of workers but also to effort extended.

2.1 Equilibrium conditions

Note that the intertemporal Euler equation becomes (from 2 and 5)

$$\frac{1}{c_t} = E_t \frac{\beta}{c_{t+1}} \left(\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \frac{1}{\theta} u_{t+1}^{\theta} \right)$$

in the symmetric equilibrium. Hence, the alternative commodity point selection does not change the usual form of this equation. The household sets the utilization rate such that the marginal return to increasing this rate equals the marginal costs from higher depreciation of the existing capital stock (from 3 and 5):

$$\alpha y_t / u_t = r_t k_t = u_t^{\theta - 1} k_t.$$

2.2 Calibration and dynamics

After taking a log-linear approximation to the steady state which is fully presented in the Appendix, the model boils down to

$$\begin{bmatrix} E_t \widehat{c}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \widehat{c}_t \\ \widehat{k}_t \end{bmatrix}. \tag{6}$$

The number of eigenvalues of $\mathbf{M}$ that are located inside the unit circle controls the local stability of the steady state. Since consumption is nonpredetermined, there is only one initial condition represented by the initial capital stock. Thus, if both eigenvalues of $\mathbf{M}$ are positioned inside the unit circle, then the steady state is indeterminate and the economy is buffeted by belief-driven sunspot shocks. Indeterminacy will occur if and only if

$$-1 < \mathrm{Det}\mathbf{M} < 1 \qquad \text{and} \qquad -1 - \mathrm{Det}\mathbf{M} < \mathrm{Tr}\mathbf{M} < 1 + \mathrm{Det}\mathbf{M}.$$

[2] Using the factor prices, we can also check the adding-up constraint:

$$w_t e_t h_t + r_t u_t k_t = (1-\alpha) \frac{y_t}{e_t h_t} e_t h_t + \alpha \frac{y_t}{u_t k_t} u_t k_t = y_t.$$

Hence, competitive markets, external effects and seemingly four inputs are compatible.

If the production externality is zero, then Det$\mathbf{M} = 1/\beta > 1$ and indeterminacy can be excluded.

To understand the mechanism that delivers indeterminacy, it is worthwhile to notice a labor supply correspondence from the present model to the real business cycle model. The loglinearized model involves

$$(1+\zeta)\widehat{h}_t = (1+\eta)\widehat{e}_t$$

which allows eliminating the effort variable. The reduced-form of the leisure versus consumption trade-off is then given by

$$\frac{\zeta\eta - 1}{1+\eta}\widehat{h}_t = \widehat{w}_t - \widehat{c}_t$$

from which it is easy to see that the current model's labor supply performs like one with a standard indivisible labor market setup as long as $\zeta\eta = 1$. In fact, ζ and η both disappear from the eigenvalue expressions and while assuming no externalities, the model is saddle-path stable since the eigenvalues, $\mu_{1,2}$, split around the unit circle

$$0 < \mu_1 = \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)} < 1 < \frac{\alpha(1-\delta)+1/\beta+\delta-1}{\alpha} = \mu_2.$$

Then, the minimum increasing returns that delivers indeterminacy are the same as in Wen (1998): the equilibrium wage-hours locus must be positively sloped. If $\zeta\eta > 1$, the reduced-form hours supply curve is negatively sloped and indeterminacy becomes possible even when the equilibrium elasticity of output with respect to labor is negatively sloped as well. All that is required is that the equilibrium wage-hours locus remains steeper than the hours supply curve similar to Aiyagari's (1995) case 1f.

In what follows, I evaluate the model numerically. The calibration is standard and it brings the artificial economy in line with long-run averages of the U.S. economy. The capital share, α, is set to 30 percent. The discount factor β equals 0.99 so that the steady state risk free net return to capital is one percent per quarter (this again matches long run returns of essentially risk free U.S. bonds). Finally, the rate of deprecation, δ, is 2.5 percent. The specific numbers were primarily chosen to make results directly comparable to Farmer and Guo (1994) and Wen (1998).[3]

The calibration implies from the steady state conditions that $\theta = 1.40404$ which is consistent with the corresponding GMM estimate that is reported in

[3] My results are not greatly dependable on the specific value picked. For example, if I increase α to 40 percent, the minimum increasing returns increase from 1.002 to 1.045 (at $\beta = 0.99$, $\delta = 0.025$, $\zeta = 2$ and $\eta = 0.25$).

Burnside and Eichenbaum (1996). This branch of my calibration is identical to Wen (1998). What remains to be fixed is the calibration of the model's labor supply section. Microeconometric studies find this parameter to be at around 1/2. I therefore consider a version of the model in which I set $\zeta = 2$.

In Figure 1, I vary the elasticity of effort supply and report the minimum increasing returns to scale (i.e. the external effect) needed to generate indeterminacy. Zones showing a "I" ("II") denote determinacy (indeterminacy) constellations. Several conclusions can be drawn from the Figure.

First, by introducing variable effort, I am able to reduce the minimum degree of externality significantly. For example, if the labor supply side is described by $\zeta = 2$ and $\eta = 0.25$, increasing returns to scale of as low as 1.002 generate indeterminacy. Or phrased differently, Wen's (1998) minimum returns to scale are matched with $\zeta = 2$ and $\eta = 0.50$. Given the labor supply elasticity findings in microeconometric studies, this is a significant improvement from Wen's model or Benhabib and Farmer (1994). Unfortunately, there is no empirical work on how large η is. Neiss and Pappa's (2005) preferred calibration is $\zeta = 4$ and $\eta = 0.5$ and that puts the minimum increasing returns at 1.230. This seems a little bit too high, however, it cannot be rejected by Burnside et al. (1995) and is in line with estimates reported by Cooper and Johri (1997). Bils and Chang (2001) suggest to calibrate $(1+\eta)/(1+\zeta) = 1/3$ (i.e. their γ) which at $\zeta = 4$ would pin down η to 1/3. With this calibration, the minimum scale economies fall back to 1.148.

Second, if I decrease the elasticities of both effort supply or hours supply, then indeterminacy becomes harder to obtain. If effort supply is totally inelastic (i.e. $\eta \to \infty$, hence the level of effort is fixed), then the minimum increasing returns are 2.263 (with $\zeta = 2$) and 2.865 (with $\zeta = 4$) which replicates the number in Wen (1998, Figure 1). Moreover, if the supply of hours is totally inelastic (i.e. $\zeta \to \infty$), then the minimum increasing returns at $\zeta = 2$ are 2.263. Thus, unlike in the Wen model, indeterminacy is possible at a completely inelastic supply of hours.

Furthermore, if the elasticity of effort supply is too low, then the economy might be completely unstable, i.e. a source (zone denoted by III), in the presence of minute degrees of externalities. Some imperfections in the form of external effects are required to obtain nonunique equilibria.

Lastly, if I assume that capital utilization is constant the model (modelled by letting $\theta \to \infty$), it becomes amorphous to Benhabib and Farmer (1994) or Farmer and Guo (1994). However, variable effort reduces the increasing returns in this model too as these fall to 1.289 (given $\zeta = 2$ and $\eta = 0.25$). This value is significantly smaller that Benhabib and Farmer's degree and it cannot be rejected on empirical grounds by Burnside et. al (1995) or Cooper and Johri (1997).

The effect that drives the indeterminacy result is easily understood. With variable effort, the firm's marginal costs are less reactive when output expands even when labor supply is relatively inelastic. If paired with small increasing returns, expectations are self-fulfilling in the same sense as in existing indeterminacy models.

3 Dynamics

Next I will report the dynamic response of the economy to a one-time innovation to sunspots. The calibration involves very small external effects, $\gamma = 0.01$, $\zeta = 2$ and $\eta = 0.25$. Figures 2 and 3 plot the response of various key macroeconomic variables. All variables move in the same direction at impact. Persistence in the model is enormous, but can easily be reduced by choosing even smaller increasing returns to scale. Consumption is extremely smooth which is known from Wen (1998): models with endogenous utilization generate a very flat consumption profile since, in the short run, changes of capital input can be managed via changing utilization rates. As per the labor market, agents expand hours and effort at impact. Since $\zeta > \eta$, effort varies considerably more than hours (if the "effort supply curve" would have been steeper – i.e. $\zeta < \eta$ – then effort would react to a lesser degree). Agents are able to operate on two margins, they tend to smooth out their employment response by increasing both hours and work effort.

4 Conclusion

This paper has analyzed the role of variable work effort in inducing sunspot equilibria in real business cycle models. I show how variable workers' work intensity reduces the degree of externalities that is needed to generate indeterminacy and I also show that this can be done without assuming a very elastic supply of labor. The reason for the existence of sunspot equilibria is that effort is procyclical and comoves with hours. This makes labor supply more elastic and thus reduces the degree of market imperfections required. Variable factor utilization takes on some of the role of externalities, however, some imperfections in the form of external effects are still required to obtain (non-optimal) nonunique equilibria.

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5 Appendix – The linearized model

Let us denote $\widehat{y}_t \equiv (y_t - y)/y$ *et cetera*, then the linear model is given by

$$\widehat{y}_t = \alpha(1+\gamma)\widehat{u}_t + \alpha(1+\gamma)\widehat{k}_t + (1-\alpha)(1+\gamma)\widehat{e}_t + (1-\alpha)(1+\gamma)\widehat{h}_t \quad \text{(A1)}$$

$$(1+\zeta)\widehat{h}_t = (1+\eta)\widehat{e}_t \quad \text{(A2)}$$

$$(1+\zeta)\widehat{h}_t = \widehat{y}_t - \widehat{c}_t \quad \text{(A3)}$$

$$\widehat{\delta}_t = \widehat{y}_t - \widehat{k}_t \quad \text{(A4)}$$

$$\widehat{\delta}_t = \theta\widehat{u}_t \quad \text{(A5)}$$

$$\frac{c}{y}\widehat{c}_t + \frac{x}{y}\widehat{x}_t = \widehat{y}_t \quad \text{(A6)}$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \alpha\beta\frac{y}{k}\left(E_t\widehat{y}_{t+1} - \widehat{k}_{t+1}\right) - \beta\delta E_t\widehat{\delta}_{t+1} \quad \text{(A7)}$$

and

$$\widehat{k}_{t+1} = (1-\delta)\widehat{k}_t - \delta\widehat{\delta}_t + \frac{x}{k}\widehat{x}_t. \quad \text{(A8)}$$

The static equations (A1) through (A6) yield

$$\mathbf{\Pi}_1 \begin{bmatrix} \widehat{y}_t \\ \widehat{x}_t \\ \widehat{l}_t \\ \widehat{u}_t \\ \widehat{\delta}_t \\ \widehat{e}_t \end{bmatrix} = \mathbf{\Pi}_2 \begin{bmatrix} \widehat{c}_t \\ \widehat{k}_t \end{bmatrix}.$$

The dynamic equations (A6) through (A8) give

$$\mathbf{J}_1 \begin{bmatrix} E_t\widehat{c}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} + \mathbf{J}_2 \begin{bmatrix} E_t\widehat{y}_{t+1} \\ E_t\widehat{x}_{t+1} \\ E_t\widehat{l}_{t+1} \\ E_t\widehat{u}_{t+1} \\ E_t\widehat{\delta}_{t+1} \\ E_t\widehat{e}_{t+1} \end{bmatrix} = \mathbf{J}_3 \begin{bmatrix} \widehat{c}_t \\ \widehat{k}_t \end{bmatrix} + \mathbf{J}_4 \begin{bmatrix} \widehat{y}_t \\ \widehat{x}_t \\ \widehat{l}_t \\ \widehat{u}_t \\ \widehat{\delta}_t \\ \widehat{e}_t \end{bmatrix}.$$

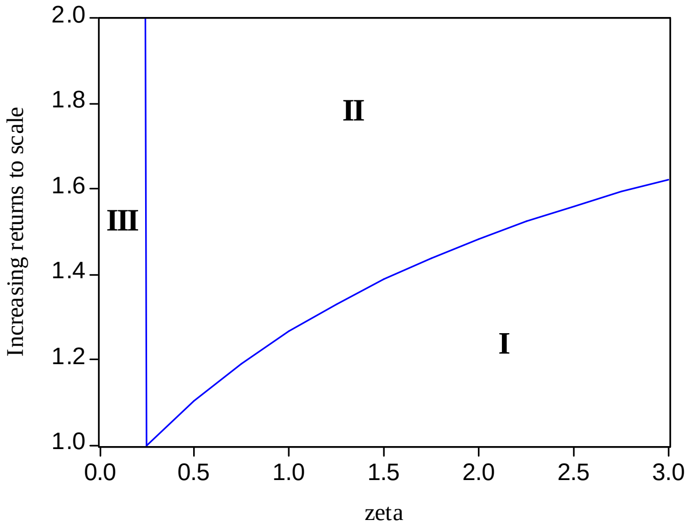

Figure 1: Sunspot zones $\eta = 1/2$

The model reduces to

$$\mathbf{J}_1 \begin{bmatrix} E_t \widehat{c}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} + \mathbf{J}_2 \mathbf{\Pi}_1^{-1} \mathbf{\Pi}_2 \begin{bmatrix} E_t \widehat{c}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{J}_3 \begin{bmatrix} \widehat{c}_t \\ \widehat{k}_t \end{bmatrix} + \mathbf{J}_4 \mathbf{\Pi}_1^{-1} \mathbf{\Pi}_2 \begin{bmatrix} \widehat{c}_t \\ \widehat{k}_t \end{bmatrix}$$

or

$$\begin{bmatrix} E_t \widehat{c}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \left(\mathbf{J}_1 + \mathbf{J}_2 \mathbf{\Pi}_1^{-1} \mathbf{\Pi}_2\right)^{-1} \left(\mathbf{J}_3 + \mathbf{J}_4 \mathbf{\Pi}_1^{-1} \mathbf{\Pi}_2\right) \begin{bmatrix} \widehat{c}_t \\ \widehat{k}_t \end{bmatrix} \equiv \mathbf{M} \begin{bmatrix} \widehat{c}_t \\ \widehat{k}_t \end{bmatrix}.$$

This is equation (6) in the main text.

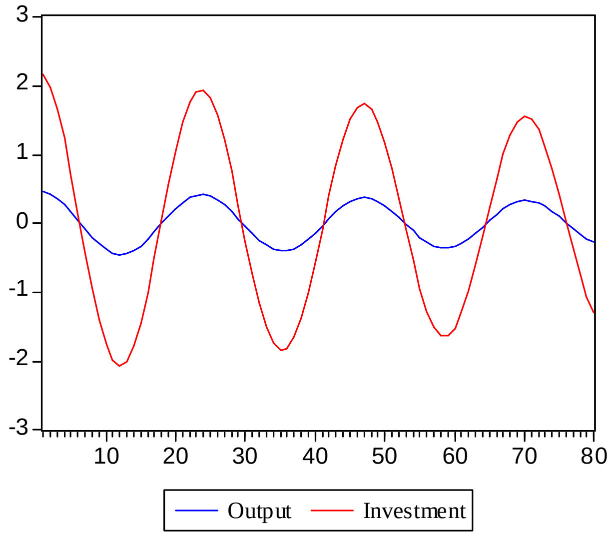

Figure 2: Response of output and investment to one time sunspots shock

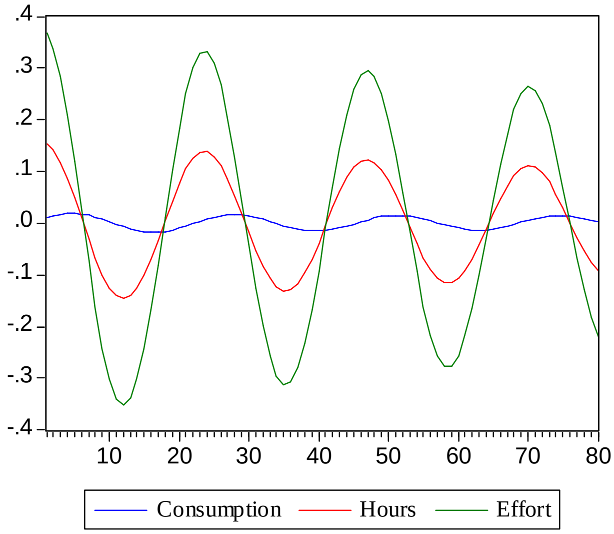

Figure 3: Response of consumption, hours and effort to one time sunspots shock