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Synergies and Investment Decisions

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Abstract

I examine optimal investment policies when there are synergies between two investment projects, in that joint operation reduces operating costs. These synergies create interactions between two investments projects, therefore two investments decisions can't be determined separately. These interactions suggest that decisions of conglomerate firms may be rational.

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1 Introduction

The purpose of this note is to use the real options approach to examine optimal investment policies when there are synergies between two investment projects, in that joint operation reduces operating costs. The real options approach provides a strong tool to investigate phenomena governed by uncertainty.[1] In the presence of synergy between two projects, a firm with both projects, i.e., a diversified firm, can raise its value by considering joint operation of the projects, which creates interactions between projects.

My main conclusions are that (i) investment policies for a project are affected by a synergy project even if business conditions of both projects are independent; that (ii) a firm expanding its synergy project (the first investment) can invest in another project (the second investment) in worse business conditions; that (iii) the first investment is promoted by the synergy effect only when the simultaneous investment is optimal, that is the first and the second investments are undertaken simultaneously.

The remainder of this paper proceeds as follows. The model is presented in Section 2. Section 3 investigates optimal investment policies for a firm facing two synergy projects. Section 4 discusses the possibilities of the simultaneous investments. Section 5 concludes.

2 The Model

Consider a competitive firm facing two projects A and B. I assume that there are synergies between projects A and B. That is, with joint operation, the firm reduces operating costs.

Letting p_t denote project A output price at time t, I assume that p_t follows a geometric Brownian motion:

$$dp_t = \alpha p_t dt + \sigma p_t dB_t^Q, \tag{1}$$

where B_t^Q is a standard Brownian motion under an equivalent martingale measure Q. The instantaneous expected percentage change α in p_t is assumed to be positive. Constant σ represents the instantaneous standard deviation

[1] Dixit and Pindyck (1994) provide nice treatments of this approach. In the recent developments, Grenadier (2002), Weeds (2002), Aguerrevere (2003) and Lambrecht (2004) incorprate game theoretical models into the real options approach.

of percentage change in p_t. In contrast, for simplicity, project B output price q is assumed to be constant.[2]

The firm operating either projects A or B (say, a single-segment firm) determines input so as to maximize profits:

$$\pi_A(p_t) \equiv \max_{y_A}[p_t y_A - \frac{1}{2}cy_A^2] \;\; and \;\; \pi_B(q) \equiv \max_{y_B}[qy_B - \frac{1}{2}cy_B^2],$$

where y_i represents output and $\frac{1}{2}cy_i^2$ represents cost function (i=A, B).

On the other hand, the firm operating both projects A and B (say, a diversified firm), owing to joint operation, determines output so as to maximize the joint profits,

$$\pi_{AB}(p_t; q) \equiv \max_{y_A, y_B}\left[p_t y_A + qy_B - \frac{1}{2}cy_A^2 + \lambda y_A y_B - \frac{1}{2}cy_B^2\right], \tag{2}$$

where $0 < \lambda < c$ captures the synergy effect resulting from joint operation.[3] Solving this equation gives joint profit:

$$\pi_{AB}(p_t; q) = \frac{c}{2\Delta}p_t^2 + \frac{\lambda}{\Delta}p_t q + \frac{c}{2\Delta}q^2,$$

where $\Delta \equiv c^2 - \lambda^2$.

3 Optimal investment policies for diversified firms

This section derives the optimal investment policies for a firm facing two synergy projects A and B, i.e., a diversified firm. There exist two investment strategies, which are classified by the firm's initial project. The first is strategy A →B, that is, the firm invests in project A at T_A and then in project B at T_B ($\geq T_A$). The second is strategy B→A, that is, the firm invests in project B at T_B and then in project A at T_A ($\geq T_B$).[4]

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[2]If both projects A and B's output prices follow stochastic processes, the main result of the paper does not change.

[3]The more general cost function with synergies is proposed by Eaton and Lemche (1991).

[4]The reason why the investment order is emphasized is that owing to synergy effects, the first investment alters the profitabilities of the second investment, thereby affecting the investment timing of the second. If there is no synergies, T_A and T_B are determined independently.

The firm selects a strategy so as to maximize its firm value, which depends on the business conditions of both projects. Therefore, letting $F_{A\to B}(T_A, T_B)$ and $F_{B\to A}(T_A, T_B)$ be the values of the strategies A→B and B→A resulting from the investment policy (T_A, T_B), respectively, the optimization problem facing a diversified firm is expressed as follows

$$\sup_{T_A, T_B}\Big[\max\Big\{F_{A\to B}(T_A, T_B), F_{B\to A}(T_A, T_B)\Big\}\Big].$$

In fact, this problem has three cases; (i) $T_A < T_B$ (say, sequential investments A→B); (ii) $T_A = T_B$ (say, simultaneous investments AB); (iii) $T_A > T_B$ (say, sequential investments B→A). Roughly speaking, sequential investments A→B (B→A) is optimal when q is low (high). Simultaneous investments AB is optimal when q is medium.

3.1 Optimal investment policies in strategy A→B

The first investment (in project A) with initial sunk costs I_A yields profit flows π_A until the second investment (in project B) has been undertaken. Once the firm invests in project B with initial sunk costs I_B, it obtains joint profits π_{AB}. Therefore the optimization problem facing the firm is given as

$$\begin{aligned} F_{A\to B} \equiv \sup_{T_A, T_B} E_t^Q\Big[\int_{T_A}^{T_B} e^{-r(\tau-t)}\pi_A(p_\tau)d\tau - I_A e^{-r(T_A-t)} \\ + \int_{T_B}^{\infty} e^{-r(\tau-t)}\pi_{AB}(p_\tau; q)d\tau - I_B e^{-r(T_B-t)}\Big] \quad s.t.\ \ T_B \geq T_A, \end{aligned} \tag{3}$$

where r is risk-free rate. Provided that $T_A(p^x) \equiv \inf\{\tau : p_\tau = p^x\}$ and $T_B(p^y) \equiv \inf\{\tau : p_\tau = p^y\}$, as shown in Appendix A, the optimization problem (3) is reduced to

$$\begin{aligned} &\sup_{p^x, p^y}\Big\{\Big(\frac{p}{p^x}\Big)^\beta [V_A(p^x) - I_A] + \Big(\frac{p}{p^y}\Big)^\beta [V_{AB}(p^y; q) - V_A(p^y) - I_B]\Big\} \\ &s.t.\quad p^y \ \geq \ p^x, \end{aligned} \tag{4}$$

where the constraint $T_B(p^y) \geq T_A(p^x)$ is substituted by $p^y \geq p^x$ because the lower the threshold is, the shorter the hitting time is.

Letting γ be Lagrange multiplier, the first order conditions with respect to p^x and p^y are given as

$$\left(\frac{p}{p^x}\right)^\beta \frac{\beta}{p^x}\left[\left(1-\frac{2}{\beta}\right)\frac{1}{2c}\frac{(p^x)^2}{\delta_A} - I_A\right] + \gamma = 0, \tag{5}$$

$$\begin{aligned} & \left(\frac{p}{p^y}\right)^\beta \frac{\beta}{p^y}\left[\left(1-\frac{2}{\beta}\right)\frac{\lambda^2}{2c\Delta}\frac{(p^y)^2}{\delta_A} + \left(1-\frac{1}{\beta}\right)\frac{\lambda}{\Delta}\frac{p^y q}{\delta} + \frac{c}{2\Delta}\frac{q^2}{r} - I_B\right] \\ -\gamma &= 0, \end{aligned} \tag{6}$$

respectively. The complementary-slackness condition is given as

$$\gamma(p^x - p^y) = 0.$$

There exist two cases: binding or not, which, depend on the constant parameter q.

In not binding case ($T_B > T_A$): sequential investments A→B, the complementary-slackness condition gives $p^y > p^x$ and $\gamma = 0$. Substituting these conditions into the first order conditions gives threshold levels p^x and p^y (say, threshold A→B):

$$\frac{1}{2c}\frac{(p^x)^2}{\delta_A} = \frac{\beta}{\beta-2} I_A, \tag{7}$$

$$\left(1-\frac{2}{\beta}\right)\frac{\lambda^2}{2c\Delta}\frac{(p^y)^2}{\delta_A} + \left(1-\frac{1}{\beta}\right)\frac{\lambda}{\Delta}\frac{p^y q}{\delta} + \frac{c}{2\Delta}\frac{q^2}{r} = I_B, \tag{8}$$

respectively. As shown in Appendix B, since $\mathrm{d}p^y/\mathrm{d}q < 0$ and p^x is constant, the condition satisfying the constraint $p^y > p^x$ is replaced by $\underline{q} > q$ where $\underline{q}$ is determined by $p^y = p^x$.

In the case $q < \underline{q}$, it is optimal to invest in project A when $p \geq p^x$ and then invest in project B when $p \geq p^y$. This looks like suggesting that synergy promotes the investment in only project B because equation (7) does not contain synergy parameter λ. Further discussions are presented in section 4.

In binding case ($T_A = T_B$) $q \geq \underline{q}$, the complementary-slackness condition gives $p^x = p^y \equiv p^s$. Eliminating γ from (5) and (6) gives p^s, say threshold AB

$$\left(1-\frac{2}{\beta}\right)\frac{c}{2\Delta}\frac{(p^s)^2}{\delta_A} + \left(1-\frac{1}{\beta}\right)\frac{\lambda}{\Delta}\frac{p^s q}{\delta} + \frac{c}{2\Delta}\frac{q^2}{r} = I_A + I_B. \tag{9}$$

In the case $q \geq \underline{q}$, it is optimal to invest in both projects A and B simultaneously when $p \geq p^s$. Section 4 discusses the implications of simultaneous investments and threshold AB.

3.2 Optimal investment policies in strategy B→A

In this strategy, the optimization problem facing the firm is given by

$$\begin{aligned} F_{B\to A} &= \sup_{T_A,T_B} E_t^Q \Big[\int_{T_B}^{T_A} e^{-r(\tau-t)} \pi_B(q) d\tau - I_B e^{-r(T_B-t)} \\ &+ \int_{T_A}^{\infty} e^{-r(\tau-t)} \pi_{AB}(p_\tau; q) d\tau - I_A e^{-r(T_A-t)} \Big] \quad s.t. \ \ T_A \geq T_B. \end{aligned} \tag{10}$$

Provided that $T_B(p^z) \equiv \inf\{\tau : p_\tau = p^z\}$ and $T_A(p^w) \equiv \inf\{\tau : p_\tau = p^w\}$, as the same way in Section 3.1, in the case $p^w > p^z$ thresholds p^w and p^z (say, threshold B→A) are given by

$$\frac{1}{2c}\frac{\bar{q}^2}{r} = I_B, \tag{11}$$

$$\left(1 - \frac{2}{\beta}\right)\frac{c}{2\Delta}\frac{(p^w)^2}{\delta_A} + \left(1 - \frac{1}{\beta}\right)\frac{\lambda}{\Delta}\frac{p^w q}{\delta} + \frac{\lambda^2}{2c\Delta}\frac{q^2}{r} = I_A. \tag{12}$$

Equation (11) is a simple NPV rule and furthermore implies that when $q > \bar{q}$, the firm should invest in project B regardless of p. Therefore the condition $p^w > p^z$ is equivalent to $q > \bar{q}$. It is optimal for the firm operating in project B to invest in project A when $p \geq p^w$.

In binding case $q \leq \bar{q}$, the complementary condition gives $p^w = p^z \equiv p^s$, which is reduced to equation (9).

4 Possibilities of simultaneous investments

This section provides complete investment policies comparing two investment strategies and discusses the possibilities of simultaneous investments. Section 3.1 provides that $F_{A\to B} > F_{AB} \ \ if \ \ q < \underline{q}$, and $F_{A\to B} > F_{AB} \ \ if \ \ q < \underline{q}$, where F_{AB} is the firm value resulting from the simultaneous investment strategy p^s. On the other hand, Section 3.2 provides that $F_{B\to A} < F_{AB} \ \ if \ \ q < \bar{q}$, and $F_{B\to A} \geq F_{AB} \ \ if \ \ q \geq \bar{q}$.

Therefore in the case $q < \underline{q}$, the sequential investments A→B are the best. In the case $\underline{q} \leq q < \bar{q}$, the simultaneous investments AB are the best. In the case of $q \geq \bar{q}$, the sequential investments B→A are the best. As a result, three lemmas are provided as follows.

Lemma 1
In the case $q < \underline{q}$, optimal investment policies for the diversified firm are to invest in project A when $p \geq p^x$ and to invest in B when $p \geq p^y$.

When business conditions of project B are bad, the diversified firm should invest in project A first and wait the chance of further expansions of project A to undertake the project B profitably. See the threshold $A \rightarrow B$ in Figure 1.

Lemma 2
In the case $\underline{q} \leq q < \bar{q}$, optimal investment policies for the diversified firm are to invest in both projects A and B simultaneously when $p \geq p^s$.

When the business conditions of project B are not so bad, the diversified firm should wait the chance to invest in both projects A and B simultaneously. See the threshold $B \rightarrow A$ and the region B(1) in Figure 1.

Lemma 3
In the case $q > \bar{q}$, optimal investment policy for the diversified firm is to invest in project B immidiately and in project A when $p \geq p^w$.

In the case the business conditions of project B are good, the diversified firm should invest in project B immediately and wait the chance that the investment in project A can be profitably undertaken. See the threshold AB and the region AB(1) in Figure 1.

Lemma 1 and 2 imply that synergy effect promotes not the first but the second investments. Lemma 2 implies that synergy effect promotes the first investment only when the simultaneous investments are optimal. The reason is that the synergy effect reveals only when the firm operates in both projects.

5 Conclusion

This note investigates the effects of synergy on optimal investment policies. Boom of the one business segment allows a firm to invest in the project of the other segment facing the worse business condition. Moderate business conditions of the both segments induce intensive investments: simultaneous investment. These findings may explain the investment policies of diversified firms.

Appendix A

This appendix derives equation (4) and associated first order conditions (5) and (6). Equation (3) is reduced to

$$\begin{aligned}
&\sup_{T_A,T_B}\Bigg\{E_t^Q\left[e^{-r(T_A-t)}\int_0^\infty e^{-r\tau^*}\pi_A(p_{\tau^*})d\tau^* - I_A e^{-r(T_A-t)}\right]\\
+\ \ & E_t^Q\left[e^{-r(T_B-t)}\int_0^\infty e^{-r\tau^{**}}(\pi_{AB}(p_{\tau^{**}};q)-\pi_A(p_{\tau^{**}}))d\tau^{**} - I_B e^{-r(T_B-t)}\right]\Bigg\}\\
& s.t.\ T_B \geq T_A,
\end{aligned}$$

where $\tau^* \equiv \tau - T_A$ and $\tau^{**} \equiv \tau - T_B$. From the definitions $T_A(p^x)$ and $T_B(p^y)$, this is reduced to

$$\begin{aligned}
&\sup_{p^x,p^y}\Bigg\{E_t^Q[e^{-r(T_A(p^x)-t)}]E_t^Q\left[\int_0^\infty e^{-r\tau^*}\pi_A(p_{\tau^*})d\tau^* - I_A\right]\\
+\ \ & E_t^Q[e^{-r(T_B(p^y)-t)}]E_t^Q\left[\int_0^\infty e^{-r\tau^{**}}(\pi_{AB}(p_{\tau^{**}};q)-\pi_A(p_{\tau^{**}}))d\tau^{**} - I_B\right]\Bigg\}\\
& s.t.\ p^y \geq p^x,
\end{aligned}$$

As shown in Harrison (1985), the expected present values of 1 dollar delivered at the uncertain future dates $T_A(p^x)$ and $T_B(p^y)$ are given by

$$E_t^Q[e^{-r(T_A(p^x)-t)}] = \left(\frac{p}{p^x}\right)^\beta \quad and \quad E_t^Q[e^{-r(T_B(p^y)-t)}] = \left(\frac{p}{p^y}\right)^\beta,$$

respectively, where β is positive root of the quadratic equation:

$$Q(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1)+\alpha\beta - r = 0.$$

The expected current values of future profit flows π_A and π_{AB} are given by

$$E_t^Q\left[\int_0^\infty e^{-r\tau^*}\pi_A(p_{\tau^*})d\tau^*\right] = \frac{1}{2c}\frac{(p^x)^2}{\delta_A} \equiv V_A(p^x),$$

$$E_t^Q\left[\int_0^\infty e^{-r\tau^{**}}\pi_{AB}(p_{\tau^{**}};q)d\tau^{**}\right]=\frac{c}{2\Delta}\frac{(p^y)^2}{\delta_A}+\frac{\lambda}{\Delta}\frac{p^yq}{\delta}+\frac{c}{2\Delta}\frac{q^2}{r}\equiv V_{AB}(p^y;q),$$

respectively, where $\delta_A \equiv r-2\alpha-\sigma^2$ which is assumed to be positive for the convergence of V_A and $\delta \equiv r-\alpha$. Using above results yields (4)

Appendix B

This appendix shows that there exist $\underline{q}$ such that $p^y > p^x$ is equivalent to $q < \underline{q}$. Differentiating equation (8) with respect to q gives the following relation:

$$\mathrm{d}p^y/\mathrm{d}q = -\frac{cq/r+(1-1/\beta)\lambda p^y/\delta}{(1-2/\beta)\lambda^2 p^y/c\delta_A+(1-1/\beta)\lambda q/\delta}.$$

Since $\beta > 2$, the sign of $\mathrm{d}p_y/\mathrm{d}q$ is negative. See Figure 1. From (7), p^x is constant and independent of q. On the other hand, p^y is a decreasing function in q. Therefore there exists $\underline{q}$ such that $p^y > p^x$ is equivalent to $\underline{q} > q$ where $\underline{q}$ is determined by $p^x = p^y$.

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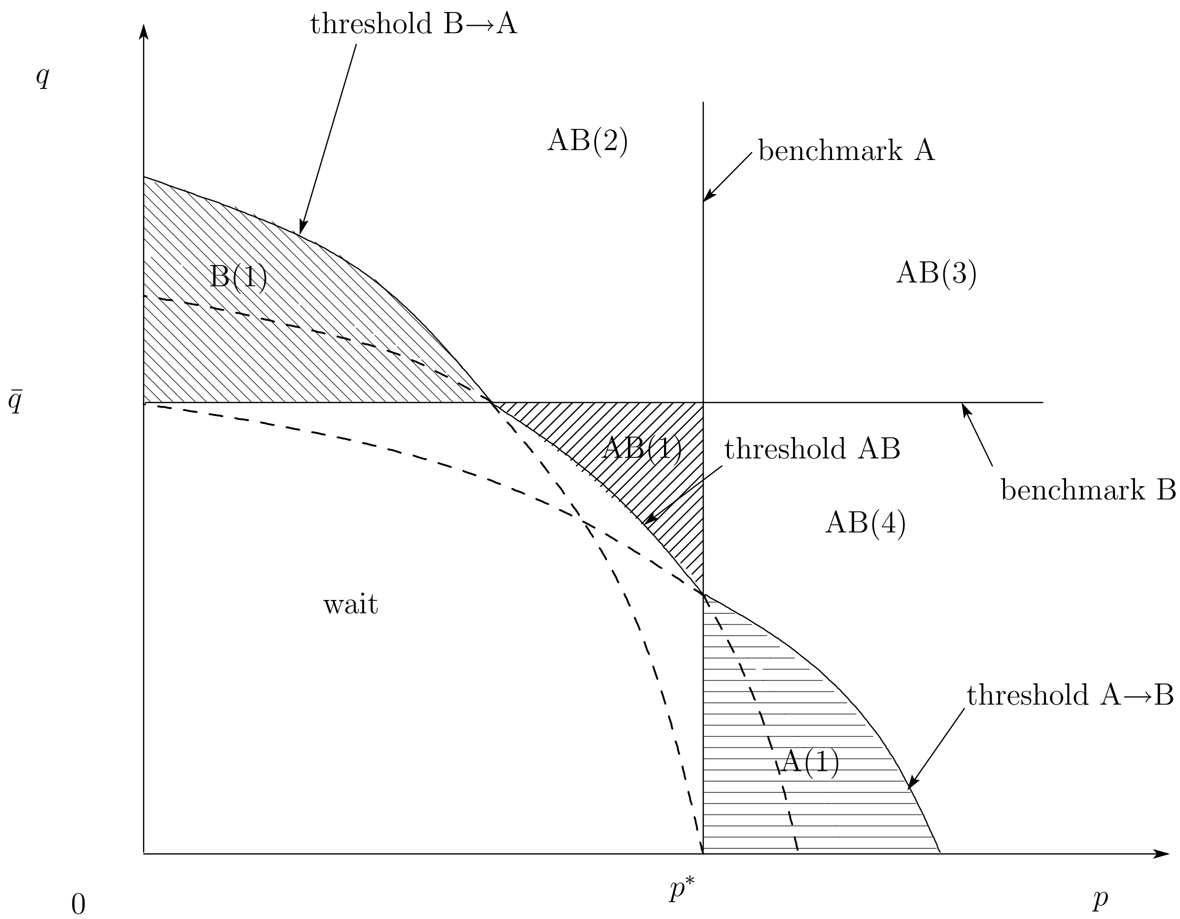

Figure 1: Optimal policy for the first investment