E C O N O M I C S   B U L L E T I N

How large is the social cost of an asset bubble?

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Abstract

Using the Grossman and Yanagawa(1993) model, we investigate how large the social cost of an asset bubble is. We show that if the utilities of all generations are evaluated almost equally, the extent of its social cost is equivalent to imposing on all generations a wage income tax whose rate is the ratio of the quantity of bubble asset to the total savings in the balanced growth equilibrium.

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1. Introduction

What effects does the existence of an asset bubble have on the welfare aspect of the economy? An answer to this question was given by Tirole (1985). Using the neoclassical growth model with overlapping generations, he showed that an asset bubble can exist in equilibrium only when an economy without bubble is dynamically inefficient, and that the introduction of an asset bubble into such an economy can be Pareto improving because the bubble eliminates over-accumulation of capital. Accordingly, the existence of a bubble in equilibrium is desirable from the standpoint of welfare in the neoclassical growth model[1]. However, this conclusion no longer holds in the endogenous growth model. By introducing an asset bubble into the standard endogenous growth models, Grossman and Yanagawa (1993) demonstrated that the introduction improves the welfare of the initial old generation but harms the welfare of future generations, and that the former positive welfare effect is exceeded by the latter negative welfare effect. Therefore, in the framework of endogenous growth theory the net welfare effect of an asset bubble on the whole economy is negative. However, Grossman and Yanagawa (1993) do not examine how large the net welfare cost is[2].

The purpose of this paper is to investigate the extent of the social cost of an asset bubble. Concretely, we compare the social welfare level of an economy with an asset bubble with that of an economy without a bubble asset but a wage income tax imposed on all generations, and calculate the social cost of the bubble in terms of the wage income tax rate. We show that if the utilities of all generations are evaluated almost equally, the extent of the social cost of the bubble is equivalent to imposing the tax whose rate is the ratio of the quantity of bubble asset ($=B_t$) to the total savings ($=S_t$) in the balanced growth equilibrium, namely B_t / S_t.

2. Model and result

The model we consider is basically identical to that of Grossman and

[1] The same conclusion holds in the case of the exchange economy of Samuelson (1958) and Wallace (1980).

[2] Futagami and Shibata (1999) also study the welfare effects of the asset bubble in an endogenous growth model. They consider the case where the supply of the bubble asset is variable, and investigate which generation has the welfare gain (or loss) when the growth rate of the supply of bubble is changed. However, they too do not argue the

Yanagawa (1993). Each generation lives for two periods, and we assume the population of each generation is one for simplicity. There are two methods of savings: physical capital and bubble asset, which is the intrinsically worthless paper asset. The representative household of generation t solves the following problem:

(1) Max $u_t = \log c_t^y + \beta \log c_{t+1}^o \quad (0 < \beta < 1)$

$$\text{s.t.} \quad c_t^y + [s_t + p_t m_t] = w_t \ , \quad c_{t+1}^o = (1 + r_{t+1}) s_t + p_{t+1} m_t$$

where β , c_t^y , c_{t+1}^o , w_t , r_{t+1} , s_t , p_t , m_t denote, respectively, subjective discount factor, young period consumption, old period consumption, labor income, return rate of physical capital, quantity of savings in physical capital, price of one unit of bubble asset in terms of goods, and quantity of bubble asset purchased. Considering that $1 + r_{t+1} = p_{t+1} / p_t$ must hold in equilibrium, we can derive the following optimal plans for consumption and savings.

(2) $c_t^y = \dfrac{1}{1+\beta} w_t , \quad c_{t+1}^o = (1 + r_{t+1}) \dfrac{\beta}{1+\beta} w_t , \quad S_t = \dfrac{\beta}{1+\beta} w_t$

where S_t is the quantity of the total savings, namely $S_t = s_t + p_t m_t$.

On the production side of the economy there are many identical firms behaving competitively, and the production function of the representative firm j is given by

(3) $y_t^j = F[k_t^j , A(K_t) l_t^j]$,

where y_t^j , k_t^j K_t and l_t^j represent output of firm j , physical capital employed by firm j , aggregate capital stock, and labor force employed by firm j , respectively. It is assumed that the production function exhibits constant returns to scale, that the functional form of labor productivity is $A(K_t) = aK_t$, and that the total labor force supplied inelastically by households is one. Under these assumptions perfect competition yields

(4) $r_t = F_1(1, a) = \bar{r} \quad , \quad w_t = aF_2(1, a) K_t$

where $F_1 \equiv \partial F / \partial K_t$ and $F_2 \equiv \partial F / \partial A_t L_t$.

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extent of the social cost of bubble.

The market equilibrium conditions of physical capital and bubble asset are given respectively by

(5) $K_{t+1} = s_t$, $m_t = 1$

where we assume the aggregate nominal supply of bubble asset is one and is fixed over time. Let us define $1+\kappa_t \equiv K_{t+1}/K_t$, $B_t \equiv p_t m_t$ and $b_t \equiv B_t / K_t$, where $1+\kappa_t, B_t$ and b_t are gross economic growth rate, total real stock of bubble asset and real bubble-capital ratio. From (2), (4) and (5) the following dynamic equations of this economy can be obtained.

(6) $$b_{t+1} = \frac{(1+F_1)b_t}{\frac{\beta}{1+\beta}aF_2 - b_t} \quad \text{and} \quad 1+\kappa_t = \frac{\beta}{1+\beta}aF_2 - b_t$$

We can see from (6) that the growth rate of the economy with bubble is lower than that without bubble, namely the potential growth rate is $\frac{\beta}{1+\beta}aF_2$. In the balanced growth equilibrium, (6) can be rewritten as

(7) $$1+\kappa^b = 1+F_1, \quad b = \frac{\beta}{1+\beta}aF_2 - (1+F_1)$$

where $1+\kappa^b$ is the gross growth rate of the steady state with bubble. So, the positive bubble can exist in equilibrium only if the potential growth rate is higher than the interest rate:

(8) $$\frac{\beta}{1+\beta}aF_2 > 1+F_1$$

Hereafter we assume this condition is satisfied.

From (2), (4) and (7) the consumption levels of generations –1 and t in the balanced growth equilibrium are

(9) $$c_0^o = [(1+F_1)+b]K_0$$

(10) $$c_t^y = \frac{1}{1+\beta}aF_2(1+\kappa^b)^t K_0, \quad c_{t+1}^o = (1+F_1)\frac{\beta}{1+\beta}aF_2(1+\kappa^b)^t K_0$$

As we can see from (9) and (10), the existence of bubble improves the utility of generation –1 because the bubble asset can be sold to the next generation, but harms the utilities of future generations because it depresses the economic growth rate. So, how large is the social cost of bubble? To examine this, we assume the existence of the following liner social welfare function:

(11) $$W(u_{-1},u_0,u_1,\cdots)=\sum_{i=-1}^{\infty}\psi^i u_i \quad (0<\psi<1)$$

where u_t is given by (1) and u_{-1} is defined as $\beta\log(c_0^o)$. From (9), (10) and (11) the social welfare level of the steady state with bubble W^b can be calculated as

(12) $$W^b=\psi^{-1}\beta\log[\{(1+F_1)+b\}K_0]+\frac{\psi}{(1-\psi)^2}(1+\beta)\log(1+\kappa^b)+\frac{\overline{C}}{1-\psi}$$

$$(\overline{C}\equiv\log\left[\frac{1}{1+\beta}aF_2K_0\right]+\beta\log\left[(1+F_1)\frac{\beta}{1+\beta}aF_2K_0\right])$$

Next, we consider an economy where there is no bubble asset and the wage income tax whose rate is τ is imposed on all generations, and examine how large the social cost of the bubble is in terms of the wage income tax rate. In this case the optimization problem of the representative household of generation t is:

$$\text{Max} \quad u_t=\log c_t^y+\beta\log c_{t+1}^o$$

$$\text{s.t.} \quad c_t^y+s_t=(1-\tau)w_t\,, \quad c_{t+1}^o=(1+r_{t+1})s_t$$

Thus the optimal plans for consumption and savings are

(13) $$c_t^y=\frac{1}{1+\beta}(1-\tau)w_t\,,\ c_{t+1}^o=(1+r_{t+1})\frac{\beta}{1+\beta}(1-\tau)w_t\,, \quad S_t=\frac{\beta}{1+\beta}(1-\tau)w_t$$

The profit maximization conditions of the representative firm and the market equilibrium condition of physical capital are the same as before and are given by (4) and (5). So the economic growth rate and the consumption plans of generations –1 and t on the balanced growth equilibrium are respectively

(14) $$1+\kappa^\tau=(1-\tau)\frac{\beta}{1+\beta}aF_2$$

(15) $$c_0^o=(1+F_1)K_0$$

(16) $$c_t^y=(1-\tau)\frac{1}{1+\beta}aF_2(1+\kappa^\tau)^tK_0\,, \quad c_{t+1}^o=(1-\tau)(1+F_1)\frac{\beta}{1+\beta}aF_2(1+\kappa^\tau)^tK_0$$

where $1+\kappa^\tau$ denotes the gross economic growth rate of this case. Using (11), (15) and (16) we can calculate the social welfare level of this case as

(17) $$W^{\tau} = \frac{1+\beta}{1-\psi}\log(1-\tau) + \psi^{-1}\beta\log[(1+F_1)K_0]$$
$$+\frac{\psi}{(1-\psi)^2}(1+\beta)\log(1+\kappa^{\tau}) + \frac{\overline{C}}{1-\psi}$$

From (12) and (17) the tax rate which equates W^b with W^{τ} can be derived as

(18) $$\tau = 1 - (x_2 / x_1)^X$$
$$(X \equiv \frac{\beta}{1+\beta}\frac{(1-\psi)^2}{\psi} - \psi, \quad x_1 \equiv 1+F_1, \quad x_2 \equiv \frac{\beta}{1+\beta}aF_2)$$

Because $x_2 / x_1 > 0$ holds by (8), the tax rate τ which equates W^b with W^{τ} satisfies the following relationship according to the sign of X.

(19) $\tau > 0$ if $X < 0$, $\tau = 0$ if $X = 0$, $\tau < 0$ if $X > 0$

Relationship (19) means that the asset bubble harms (improves) the social welfare if $X < 0$ ($X > 0$) holds. For example, suppose $\psi = \beta$, which means that the discount factor of the next generation's utility in the social welfare function is equal to the individual subjective discount factor in the utility function. In such a case the value of X satisfies $X = \frac{1-3\beta}{1+\beta}$, so the asset bubble harms social welfare if $\beta > 1/3$ holds.

The most realistic case is that of ψ 1, where the utilities of all generations are evaluated almost equally. In this case the value of X satisfies X -1, so the existence of the asset bubble necessarily harms the social welfare. Using (2), (4) and (7) we can calculate the extent of the social cost in terms of the wage income tax rate as

(20) $$\tau \quad 1 - (x_2 / x_1)^{-1} = \frac{b}{\frac{\beta}{1+\beta}aF_2} = \frac{B_t}{S_t}$$

This means that if the utilities of all generations are evaluated almost equally, the extent of the social cost of the bubble is equivalent to imposing the wage income tax whose rate is the ratio of the quantity of bubble asset B_t to the total savings S_t. Therefore, in the Grossman and Yanagawa-type endogenous growth model, the larger the size of the bubble in the balanced growth equilibrium is, the more serious the social cost of the bubble.

How should we interpret this result more concretely? Suppose an economy where the prices of all existing stocks are continuously above their fundamental values by 5%, which roughly corresponds to the case where

B_t / S_t in our model is 5%. In such an economy the larger amount of national savings is allocated to the purchase of existing bubbly stocks and therefore the amount of national savings allocated to the new investment projects (namely, newly issued stocks) is inevitably lowered. The results obtained here show that in such a case the social cost is equivalent to imposing on all generations a 5% wage income tax.

3. Final Remarks

In this paper we showed that if the utilities of all generations are evaluated almost equally the social cost of the stationary bubble in the Grossman and Yanagawa (1993) model is equivalent to imposing on all generations a wage income tax whose rate is equal to B_t / S_t. Note that this conclusion holds true only in the Grossman and Yanagawa model and different conclusions can be obtained if different formulations of the model are supposed.

The reason for the social cost of the bubble arising in the Grossman and Yanagawa model is that the bubble crowds out investment for physical capital and lowers the growth rate in the balanced growth path. Recently, some authors have proposed models where the bubble can promote economic growth through stimulating investment activities. For example, Olivier (2000) developed a model where the bubble on the market value of R&D firms encourages R&D activities and raises the growth rate of the economy measured by the speed of the increase in the variety of consumption goods. Ventura (2003) constructs a simple model where entrepreneurs can create bubbly firms and shareholders buy them for the purpose of saving, and shows that the creation of bubbly firms can stimulate capital accumulation when the financial market is imperfect, in the sense that shareholders must pay the monitoring cost because of the informational asymmetry. In such models the existence of the bubble can improve the welfare of the economy and brings about social benefit. Examining how large the social benefit of the bubble is in such models is a question for future research.

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