E C O N O M I C S   B U L L E T I N

Menu costs, (s,S) rule, imperfect information and the neutrality of money

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Abstract

A dynamic macroeconomic model of monopolistic competition and imperfect information with menu costs and (s,S) pricing rule is proposed, in the lines of Caballero and Engel [1991]. The model can be seen as an imperfect competition version of Lucas [1973] with menu costs. The presence of informational imperfection destroys the neutrality result of Caplin and Spulber [1987], and the effect of a monetary shock on output is shown to be an increasing function of the degree of strategic complementarity between firms.

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1 Introduction

This paper proposes a dynamic model of monopolistic competition with menu costs, as introduced by Caplin and Spulber [1987] and Caballero and Engel [1991], and extended more recently by Dotsey, King and Wolman [1999]. Such a model cannot by itself exhibit a durable and systematic non-neutrality of money once firm prices have reached their stationary distribution. We propose here an enrichment of the informational structure of the model by introducing imperfect information and rational expectations of aggregate variables. Firms cannot fully separate real local and nominal aggregate shocks in their pricing behavior. The result of the model[1] is that imperfect information implies a non-neutrality of money, even at the stationary price distribution. A necessary condition for this result is the relaxation of the Caplin and Spulber [1987] assumption of no strategic complementarity.

2 The Model

The model is derived from Caplin and Spulber [1987] and Caballero and Engel [1991][2].

2.1 Description of the Model

There is a continuum of firms of measure one, each indexed by i, $i \in [0,1]$. All variables are logs and the model is composed of

1. An aggregate demand equation

$$y(t) = m(t) - p(t) \tag{1}$$

where $y(t)$ is the output, $p(t) = \int_0^1 p_i(t)di$ the aggregate price index[3] and $m(t)$ the money supply.

2. An optimal pricing policy without menu costs

$$p_i^\star(t) = \alpha(m(t) - p(t)) + p(t) + \varepsilon_i(t) \tag{2}$$

where $\varepsilon_i(t)$ are real local shocks *iid*, $\varepsilon_i(t) \rightsquigarrow \mathcal{N}(0, \sigma_l^2)$ $\forall i$ and $\forall t$. We do not impose the constraint $\alpha = 1$ (as in Caplin and Spulber [1987]) and the firms problem has two state variables; $m(t)$ and $p(t)$.

3. A stochastic money process

$$m(t) = m(t-1) + g + \varepsilon_m(t) \tag{3}$$

where $g > 0$ and $\varepsilon_m(t)$ is a macroeconomic monetary shock, $\varepsilon_m(t) \rightsquigarrow \mathcal{N}(0, \sigma_m^2)$. Money follows a random walk with positive drift g. We assume that g/σ_m is large enough to

[1]This model can be seen as an imperfect competition version of Lucas [1973] with menu costs.

[2]The log-linear equations are derived from maximizing behavior of firms and households and from the solving of the general equilibrium.

[3]In an explicit Dixit and Stiglitz [1977] *CES* framework, this price index must be seen as an approximation of the effective *CES* price index.

avoid downward movements of $m(t)$, as this condition guarantees that a (s, S) pricing rule is a good approximation of the optimal pricing rule[4].

4. A menu cost β incurred each time a firm resets its price. This menu cost leads the firm to adopt a (s, S) pricing rule[5].

2.2 The informational structure

Let $z_i(t)$ be the difference between effective nominal price and the optimal price without menu cost.

$$z_i(t) = p_i(t) - p_i^\star(t) \tag{4}$$

Firms control $z_i(t)$ by following a (s, S) rule in perfect information models. For instance, in the Caplin and Spulber [1987] model with perfect information, one have $\alpha = 1$ et $\sigma_l = 0$, and therefore

$$z_i(t) = -gt - \varepsilon_m(t) + z_i(0)$$

As we abandon those assumptions in this paper, the informational structure of the model must be made explicit. In the Lucas [1973] model, as firms are not price-setters on their island, they observe past values of the model variables as well as the current price on their island. An over simplified version of that model[6] is given by the two following equations:

$$\begin{aligned} y_i(t) &= \nu \left(p_i(t) - E\left[p(t) \mid_i\right]\right) && (5) \\ m_t + \varepsilon_i(t) &= p_i(t) + y_i(t) && (6) \end{aligned}$$

Equation (5) is the supply function on island i and (6) is the demand function on that island. As firms know the ε_i and have a *correct* prior distribution on the general price level p_t, the model can be solved using an undetermined coefficient method.

We adopt here the same inobservability assumption of real (local) and nominal (aggregate) components of the shock on the level of demand $(\alpha\varepsilon_m(t) + \varepsilon_i(t))$. The sequence of the model is then the following. Each firm knows the past of the model. At period t, *after* local and monetary shocks but *before* eventually resetting its price, each firm observes its demand level *for* $(t-1)$ *prices*, and can then compute the demand scale parameter $(\alpha\varepsilon_m(t) + \varepsilon_i(t))$. Nevertheless, $\varepsilon_m(t)$ and $\varepsilon_i(t)$ are not observed separately.

Knowing the money process (equation (3)) and the optimal pricing policy without menu cost (equation 2)), one gets:

$$z_i(t) = p_i(t) - \alpha(m(t-1) + g) - (1-\alpha)p(t) - (\alpha\varepsilon_m(t) + \varepsilon_i(t)) \tag{7}$$

To decide whether or not it must reset its price, firm i forms a rational expectation of $p_i^\star(t)$ – and therefore a rational expectation of the aggregate price level – conditionally to

[4] See for instance Dixit [1992] on that point. More correctly, we shall assume that $g/\sigma_{p^\star}$ is large enough, where $\sigma_{p^\star}$ is the standard-error of the firm target price innovation. In our model, it is obvious to check that the first condition implies the second for the stationary distribution of prices.

[5] See the seminal paper of Sheshinski and Weiss [1977] and Bertola and Caballero [1991] for a complete description of (s, S) pricing policies.

[6] With the secular component of output and the coefficient of its cyclical component past value set to zero.

the information available on island i (the current value of the composite shock and the past values of all variables) and resets its price each time the expectation of $z_i(t)$ hits the upper barrier of $[s, S]$[7]. Let

$$\widehat{z}_i(t) = E\left[z_i(t) \mid_i\right],$$

equation (4) becomes:

$$\widehat{z}_i(t) = p_i(t) - E\left[p_i^\star(t) \mid_i\right] \tag{8}$$

with

$$E\left[p_i^\star(t) \mid_i\right] = \alpha(m(t-1)+g) + (1-\alpha)E\left[p(t) \mid_i\right] + (\alpha\varepsilon_m(t) + \varepsilon_i(t)) \tag{9}$$

Each firm knows $m(t-1)$ and the composite shock, and may reset $p_i(t)$ to maintain $\widehat{z}_i(t)$ in the interval $[s, S]$, forming a rational expectation of $p_i^\star(t)$ and $p(t)$.

Finally, we assume that information is perfect at date $t = 0$, that $m(0) = 0$, which implies

$$\widehat{z}_i(0) = z_i(0) = p_i(0) ,$$

and that the $z_i(0)$ are uniformly distributed[8] over the interval $[s, S]$.

Under these hypotheses and letting $\sigma = S - s$, the process of $\widehat{z}_i(t)$ is given by:

$$\begin{aligned} \widehat{z}_i(t) &= S - [S + \alpha(m(t-1)+g) + (1-\alpha)E\left[p(t) \mid_i\right] \\ &+ (\alpha\varepsilon_m(t) + \varepsilon_i(t)) - z_i(0)] \bmod(\sigma) \end{aligned} \tag{10}$$

where $x \bmod(y)$ is the rest of the Euclidian division of x by y.

Solving the model requires at this stage the computation of the rational expectation $E\left[p(t) \mid_i\right]$.

2.3 Computation of $E\left[p(t) \mid_i\right]$

Summing equation (8) over i[9] , one gets:

$$\begin{aligned} p(t) &= \alpha m(t) + (1-\alpha)\int_0^1 E\left[p(t) \mid_i\right] di \\ &+ \int_0^1 \widehat{z}_i(t) di \end{aligned} \tag{11}$$

and using (10)

$$\begin{aligned} p(t) &= \alpha m(t) + (1-\alpha)\int_0^1 E\left[p(t) \mid_i\right] di + S \\ &- \int_0^1 [S + \alpha(m(t-1)+g) + (1-\alpha)E\left[p(t) \mid_i\right] \\ &\quad +(\alpha\varepsilon_m(t) + \varepsilon_i(t)) - z_i(0)] \bmod(\sigma)\, di \end{aligned} \tag{12}$$

[7]These barriers are not the same than in the perfect information model.

[8]It is shown in Caballero and Engel [1991] that the model converges under particular conditions on the shocks processes to a uniform distribution of the $z_i(t)$ if this hypothesis is removed.

[9]We assumed a large number of firms in the economy, which implies $\int_0^1 \varepsilon_i(t) di = 0$.

We use the undeterminate coefficients method to get this expectation. Firm i forms the expectation:

$$E\left[p(t)\mid_i\right] = A(m(t-1)+g) + B(\alpha\varepsilon_m(t)+\varepsilon_i(t)) + C \tag{13}$$

where we distinct arbitrarily $m(t-1)+g$ from the constant to facilitate the economic interpretation of the results.

Substituting in (12) $E\left[p(t)\mid_i\right]$ by its value in (13), one gets:

$$\begin{aligned} p(t) &= (\alpha+(1-\alpha)A)(m(t-1)+g) + (1+(1-\alpha)B)\alpha\varepsilon_m(t) + (1-\alpha)C \\ &+ S - \int_0^1 [x_i(t) - z_i(0)] \bmod(\sigma) di \end{aligned} \tag{14}$$

with

$$\begin{aligned} x_i(t) &= (\alpha+(1-\alpha)A)(m(t-1)+g) \\ &+ (1+(1-\alpha)B)(\alpha\varepsilon_m(t)+\varepsilon_i(t)) + (1-\alpha)C + S \end{aligned}$$

To solve this equation, we use the following result: if $z(0)$ is a stochastic variable uniformly distributed over $[s, S]$ and independent from the stochastic variable $x(t)$, then $[z(0)+x(t)] \bmod(\sigma)$ is uniformly distributed over $[0,\sigma]$, with $\sigma = S - s$ (see Caballero and Engel [1991] for a proof).

Then, equation (14) becomes

$$\begin{aligned} p(t) &= (\alpha+(1-\alpha)A)(m(t-1)+g) + (1+(1-\alpha)B)\alpha\varepsilon_m(t) \\ &+ (1-\alpha)C + \frac{S+s}{2} \end{aligned} \tag{15}$$

and taking rational expectation of (15)

$$\begin{aligned} E\left[p(t)\mid_i\right] &= (\alpha+(1-\alpha)A)(m(t-1)+g) \\ &+ (1+(1-\alpha)B)\alpha E\left[\varepsilon_m(t)\mid_i\right] + (1-\alpha)C + \frac{S+s}{2} \end{aligned} \tag{16}$$

As ε_m et ε_i are independent, one can compute the rational expectation of $\varepsilon_m(t)$ conditionally to the information set of firm i at time t –*i.e.* $\alpha\varepsilon_m(t)+\varepsilon_i(t)$ –:

$$E\left[\varepsilon_m(t)\mid_i\right] = \frac{1}{\alpha}\,\frac{\alpha^2\sigma_m^2}{\alpha^2\sigma_m^2+\sigma_l^2}(\alpha\varepsilon_m(t)+\varepsilon_i(t)) = \gamma\varepsilon_m(t) + \frac{\gamma}{\alpha}\varepsilon_i(t) \tag{17}$$

with

$$\gamma = \frac{\alpha^2\sigma_m^2}{\alpha^2\sigma_m^2+\sigma_l^2}$$

Using (17), equation (16) becomes:

$$\begin{aligned} E\left[p(t)\mid_i\right] &= (\alpha+(1-\alpha)A)(m(t-1)+g) \\ &+ (1+(1-\alpha)B)\gamma(\alpha\varepsilon_m(t)+\varepsilon_i(t)) + (1-\alpha)C + \frac{S+s}{2} \end{aligned} \tag{18}$$

and identifying coefficients of equations (13) and (18), one gets

$$\begin{aligned} A &= 1 \\ B &= \frac{\gamma}{1-(1-\alpha)\gamma} \\ C &= \frac{S+s}{2\alpha} \end{aligned}$$

The rational expectation of the aggregate price index is thus given by

$$\begin{aligned} E\left[p(t) \mid_i\right] &= (m(t-1)+g)+\frac{S+s}{2\alpha} \\ &+ \frac{\gamma}{1-(1-\alpha)\gamma}(\alpha\varepsilon_m(t)+\varepsilon_i(t)) \end{aligned} \tag{19}$$

3 Output dynamics and the role of strategic complementarity

Substituting in (15) A, B and C by their values, we are now able to compute the aggregate price index:

$$p(t) = m(t) - \Theta\varepsilon_m(t) + \frac{S+s}{2\alpha} \tag{20}$$

with

$$\Theta = (1-\alpha) - \frac{\alpha(1-\alpha)\gamma}{1-(1-\alpha)\gamma}$$

and the process of output is then given by:

$$y(t) = m(t) - p(t) = -\frac{S+s}{2\alpha} + \Theta\varepsilon_m(t) \tag{21}$$

One can already notice that with perfect information (*e.g.* $\sigma_l^2 = 0$), the parameter Θ cancels out as $\gamma = 1$, and the variance of output is zero ($y(t) = y(0) = -\frac{S+s}{2\alpha}\ \forall t$), which is the result of Caplin and Spulber [1987], but without the restriction $\alpha = 1$. For the particular value of α where substitution and real balance effects counterbalance in the optimal pricing policy ($\alpha = 1$), we get exactly the result of Caplin and Spulber [1987].

Equation (21) shows that output does not depend on anticipated money growth rate g, which is the neutrality result of Caplin and Spulber. Nevertheless, the anticipation error induced by the informational structure creates a positive relation between unanticipated money and output.

It is worth noticing that the effect of a monetary shock is related to the degree of strategic complementarity[10] in the model. Following Caballero and Engle [1993], one gets from equations (2) and (4):

$$\widehat{z}_i(t) = E\left[p_i(t) - m(t) \mid_i\right] - \omega E\left[\int_0^1 z_u(t)du \mid_i\right]$$

[10] As defined by Cooper and John [1988]

where $\omega = \frac{1-\alpha}{\alpha}$ is the degree of strategic complementarity ($\omega < 0$ would correspond to strategic substituability). If firms are on the average *under* their target value $\left(E\left[\int_0^1 z_u(t)du \mid_i\right] < 0\right)$, $\widehat{z}_i(t)$ is high and firm i has an incentive to reduce $\widehat{z}_i(t)$ $-i.e.$ to get closer to the rest of the firms.

In our model, the parameter Θ is a function of the degree of strategic complementarity , $\Theta = \Theta(\omega)$, and one can easily check that:

$$\begin{cases} \Theta(\omega) > 0 \quad \forall \omega > 0 \\ \Theta'(\omega) > 0 \\ \Theta(0) = 0 \end{cases}$$

Therefore, the response of output to a monetary shock is an increasing function of the degree of strategic complementarity.

The Caplin and Spulber result is obtained in the absence of strategic complementarity ($\omega = 0$), even if information is imperfect. This result has an intuitive interpretation: as the optimal price of firm i does not depend on the general price level, idiosyncratic uncertainty, even if imperfectly observed, has no effect on aggregate behavior. One can verify that the same result can be derived in the Lucas model. From equation (5) and (6), one gets in that model:

$$p_i(t) = \frac{1}{1+\nu}m_t + \frac{\nu}{1+\nu}E\left[p(t) \mid_i\right] + \frac{1}{1+\nu}\varepsilon_i(t) \tag{22}$$

Equation (22) is the equivalent of equation (2) in our model, and the equivalent of our α parameter is $\frac{1}{1+\nu}$ in the Lucas model. Therefore, $\alpha = 1$ corresponds to $\nu = 0$, which means that supply is totally inelastic on island i. In that case, aggregate output is also constant in the Lucas model, despite imperfect information.

4 Conclusion

When we relax the assumptions of perfect information and no strategic complementarity, the strong neutrality result of Caplin and Spulber does not hold any more, even at the steady state. The unanticipated part of the money (which stems from imperfect information) implies a positive reaction of output, and this reaction is an increasing function of the degree of strategic complementarity. As Nishimura [1996] shows the interest on a imperfect information and imperfect competition approach in slightly different models, we demonstrated here that money matter when menu costs, imperfect information and strategic complementarity are assumed.

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