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The Political Economy of Social Security under Differential Longevity and Voluntary Retirement[1]

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Abstract

This paper studies a model where the existence of a pension system is decided by majority voting. We assume that individuals have the same income but different longevity. Retirement is voluntary and the pension system is characterised by a payroll tax on earnings and a flat pension benefit. Individuals vote only on the tax level. We show that a pension system emerges when there is a majority of long-lived individuals and that voluntary retirement enables to lower the size of the transfers received by the long-lived. A rise in average longevity will also increase the size of the pension system.

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1 Introduction

All industrialized economies have seen an unprecedented rise in *average* longevity during the last two centuries. This positive trend should, however, not obscure the fact that significant *longevity differentials* continue to exist within a given cohort. Clearly, the rise in average longevity was not shared equally by everyone.

The existence of such longevity differentials raises difficult issues for policy-makers. Considering equity issues, one could then argue that redistributive Social Security systems should take into account these longevity differences. This is because PAYG pension systems operate redistribution between individuals with different income but, as those systems provide individuals with an annuity which *does not* depend on life expectancy, they also generate unexpected transfers between individuals with different life spans. For instance, Coronado *et al.* (2000), Liebman (2001) in the US case and Bommier *et al.* (2006) in the French case, have shown that even though these systems continue to be redistributive, part of income redistribution is neutralized by mortality differentials. Bommier *et al.* (2006) also estimated that in France, differential mortality offsets between one fourth and one half of aggregate redistribution and that replacement rates certainly over-estimate redistributive effects of pension systems. Consequently, longevity is one of the many dimensions, other than productivity, which should affect the design of pension schemes.

This paper studies the design of a pension system taking into account longevity differentials. While this issue has already been studied from a normative perspective (see for example, Bommier *et al.*, 2007), our paper, on the contrary, adopts a positive approach and investigates how differences in life spans influence the vote over the emergence and the size of a pension system.

To this purpose, we consider a generation of individuals characterized by a given distribution of life durations. Unlike in Browning (1975), all voters have the same age, but differ according to longevity. There is no other source of heterogeneity. We also assume that individuals perfectly know their longevity.[1]

[1] The reasons why individuals have different longevities might be difficult to identify. However, many studies show that there exists a correlation between longevity and some individual characteristics like wealth, education, gender or location. Individuals know to which group they

In the first periods of their life, they work, receive a fixed labour income, pay a proportional tax (in order to finance the pension system) and save. In the later periods of their life, they retire and receive a flat pension benefit as well as the returns of their savings. In the benchmark situation, we assume that the retirement age is uniformly set by the government. We relax this assumption later in the paper and assume that individuals can choose their own retirement age. The level of the tax rate is chosen by majority voting. We determine the voting equilibrium and provide the conditions under which a pension system emerges in a setting where individuals have different life spans.

Our main results concern the voting equilibrium and the implications of endogenous retirement. We first find that, if individuals have different survival chances, a pension system emerges only if the distribution is left-skewed.[2] Allowing for voluntary retirement reduces the size of the system and the size of the transfers made to long-lived individuals. Finally, we study the impact of an increase in average longevity and of higher disutility of work on the voting outcome.

This paper is part of a growing literature on the political economy of Social Security which studies the effects of longevity differentials on the design of a pension system. The two most closely related papers are De Donder and Hindriks (2002) and Borck (2007). The former studies a political economy model in which individuals differ both in their productivity and in their survival chances. They examine how various changes in the link between taxes and benefits (flat pension, benefits positively related to contributions and means-tested program) affect the political support for Social Security. They show that tightening the link between contributions and benefits does not always imply less distortions "when political economy considerations are thrown in". Borck (2007) also analyses the majority voting equilibrium when individuals have both different incomes and different survival chances, assuming a positive correlation between the two.[3] Depending

belong to and thus perfectly anticipate how long they will live. We also consider life duration as exogenous which rules out the possibility for individuals to modify their life span (through monetary or non monetary investments).

[2] In the following, we will use equivalently left-skewed for negatively-skewed and right-skewed for positively-skewed.

[3] In his paper, productivity is the only source of heterogeneity and survival enters in the

on the magnitude of this correlation and on the link between contributions and pension benefits, the preferred tax rate increases or decreases with the income and the coalition for high tax rates also changes. While these models and ours have in common that longevity is differentiated inside the population, they both differ from ours in that they all assume a uniform retirement age, excluding for the possibility of a link with life expectancy. On the opposite, our model assumes that retirement is a voluntary decision and shows that life expectancy certainly influences it. We also compare the redistributive consequences of having voluntary retirement with a framework where retirement would be uniform.

Up to now, political economy models have always assumed either differential productivity and endogenous retirement (like in Casamatta *et al.*, 2005, Casamatta *et al.*, 2006, and Lacomba and Lagos, 2006) or differential longevity and uniform retirement (see Borck, 2007 and De Donder and Hindriks, 2002), but none of them assume both heterogeneity in life durations and endogenous retirement. On the contrary and as we just discussed, the key point of our paper is to stress the importance of the retirement decision in a world where individuals face different life durations. To this respect, the present paper fills the gap.

Of course, assuming a voluntary retirement age is debatable and this may raise several objections regarding the modelling of our problem.[4] For instance, we could have assumed a uniform legal retirement age and studied the political outcome of a vote on both the tax rate level and the legal retirement age.[5] There are several reasons why we did not follow this road. First, raising the retirement age appears to be difficult to implement and may encounter strong political resistance. Second, multidimensional voting is always a complicated task, and a majority voting equilibrium may not always exist.[6] But, more importantly, we believe that a lot of elements influence the individual's decision to retire; among them, some individual characteristics, like health condition, wealth and longevity. The existence and the generosity of a Social Security sys-

model as a positive function of it.

[4]For a complete survey, see Cremer *et al.* (2008).

[5]For example, Lacomba and Lagos (2007) assume that individuals have different ages and make them vote on the legal retirement age.

[6]For a complete survey on bi-dimensional voting, see Persson and Tebellini (2002).

tem also play a crucial role and some parameters, like the level of contributions, of benefits and the statutory retirement age are certainly taken into account by the individual when retiring. As proven by Sheshinski (1978) and Crawford and Lilien (1981), only an actuarially fair pension system does not distort retirement decisions (with no borrowing constraint). Yet, the study of Gruber and Wise (1997) show that most PAYG pension systems are far from being actuarially fair and that the decision to retire depends on the incentives created by Social Security systems. They show that because the payroll tax is not age-dependant and because pension benefits do not adapt to an extended length of activity, the system creates "an implicit tax on continued activity" which distorts the retirement decision. Thus, in order to stick to this view, we decided to assume endogenous retirement and for simplicity, we retricted the vote to only one instrument, the tax rate. To this extent, our paper can also be related to this branch of political economy which studies the political support for the pension system under the assumption of voluntary retirement. In our model, individuals do not simply vote for the existence of a specific pension scheme but more, *for a level of implicit taxation.*

Our paper is structured as follows. The next section gives the main assumptions of the model and the benchmark equilibrium under uniform retirement. In Section 3, we relax this assumption and study individuals' decisions and the voting equilibrium. In Section 4, we simulate our theoretical model. Section 5 discusses the implications of assuming flexible retirement and the last section concludes.

2 The model

2.1 The economy

Consider a population of individuals differentiated according to their life span T and assume that individuals know their type with certainty.[7] The distribution of longevity has support $[T_{\min}, T_{\max}]$, density function $f(T)$ and cumulative distri-

[7] In order to make our framework more realistic, we could have equally assumed uncertainty on the life span. Under the assumption of risk neutral agents which rules out insurance motivations, we would have obtained identical conclusions.

bution function $F(T)$. $E(T)$ and T_m are respectively the mean and the median longevity. For the time being, no assumption is made on the distribution of life durations. We also assume that the population size is constant.[8]

Under no pure time preferences, the intertemporal utility function of a type T individual is:

$$U(c(t), z, T) = \int_0^z [u(c(t)) - r(t)]\, dt + \int_z^T u(c(t))\, dt \qquad (1)$$

where $c(t)$ is consumption at period t and z is the retirement age. The utility function $u(.)$ is increasing and strictly concave. $r(t)$ denotes the per period disutility of labor and $r'(t) > 0$. We denote $R(z) = \int_0^z r(t)\, dt$, the disutility of a working life of length z. In the following section, we briefly develop the benchmark case in which the retirement age is assumed to be uniformly fixed by government policy, $z = \bar{z}\ \forall T$; in Section 3, we relax this assumption.

We further assume that each individual is subject to a payroll tax rate $\tau \in [0, 1]$ in order to contribute to the pension system so that he earns a net income of $w(1-\tau)$ over z units of time.[9] There is no heterogeneity in instantaneous labour income w and it is constant across periods. He retires for a length of time $(T - z)$ and receives a per period pension benefit p. Assuming a zero interest rate and no liquidity constraint, the lifetime budget constraint of a type -T individual can be represented by:

$$\int_0^T c(t)\, dt \leq \int_0^z w(1-\tau)\, dt + (T - z)\, p \qquad (2)$$

We now turn to the specification of the pension benefit formula. In our framework, per period pension benefit, p is obtained by balancing the government budget. A feasible pension scheme must satisfy the government budget constraint:

$$\int_{T_{\min}}^{T_{\max}} w\tau z f(T)\, dT = \int_{T_{\min}}^{T_{\max}} (T - z)\, p f(T)\, dT$$

[8]It can be equivalently assumed that our population is constitued of several generations as in Browning (1975), with earlier generations transmiting their life expectancy to future generations.

[9]We implicitly assume that the tax rate chosen today remains the same over the following periods. At least, individuals believe that today's decision on the tax rate will remain until the end of their life.

so that

$$p=\frac{w\tau E\left(z\right)}{\left(E\left(T\right)-E\left(z\right)\right)} \tag{3}$$

with $E\left(z\right)\equiv\int_{T_{\min}}^{T_{\max}}zf\left(T\right)dT$, the mean retirement age. Then, per period benefit is flat and depends on the population characteristics and on the tax rate.

The tax rate τ is determined by majority voting. The sequence of decisions is the following one. The vote over the contribution rate takes place ex-ante.[10] Each individual determines his preference for the tax rate depending on his preferences for consumption, for work and on his length of life. He also anticipates the consequences of choosing a specific value of the tax rate over the level of pension benefit. Individuals then vote over the level of the tax rate and it is such that, at the voting equilibrium, a majority of individuals prefers this level.

The natural benchmark from which to begin our analysis is the uniform retirement age.

2.2 Majority voting equilibrium under uniform retirement age

As a first step in explaining the relation between longevity differentials, retirement and the emergence of a pension system, we consider an economy where the retirement age is fixed by the government and uniform, so that $\bar{z}=E\left(z\right)=z$ $\forall T$.[11]

Whenever the individual is not liquidity constrained, consumption is smoothed across time and equal to $c_T=\left[w\left(1-\tau\right)\bar{z}+\left(T-\bar{z}\right)\bar{p}\left(\tau\right)\right]/T$ where the per period pension benefit $\bar{p}\left(\tau\right)$ is equal to $w\tau\bar{z}/\left(E\left(T\right)-\bar{z}\right)$.[12] Replacing for c_T into (1) and differentiating it with respect to τ, we find that individuals with $T<E\left(T\right)$ prefer a zero tax rate ($\tau_T^*=0$) while for $T\geq E\left(T\right)$, the preferred tax rate is $\tau_T^*=1$.

[10] We assume that individuals vote for a pension system when they are born. Assuming on the contrary that individuals vote later in life, would not make any difference; we would only need to consider a life duration distribution starting at $T_{\min}\neq 0$, which does not modify our results.

[11] Note that it would be plausible to consider a scenario in which the individual would be asked to vote over the policy allocation $(\tau,\bar{z})$. Since the vote would be bi-dimensional, it would complicate the model so that we decided to leave it as future work.

[12] Note that we assume $\bar{z}\leq E\left(T\right)$ to ensure that, with budget balance, $\bar{p}\left(\tau\right)$ and τ are always positive.

To understand this, we define the net contribution to the pension system of a type -T individual as $NC_T = w\tau z - (T - z)\, p$. Replacing for $\bar{z}$ and $\bar{p}(\tau)$, the net contribution is simply

$$NC_T = w\tau\bar{z}\left(\frac{E(T) - T}{E(T) - \bar{z}}\right)$$

so that an individual with $T < E(T)$ always prefers a zero tax rate as he will be a net contributor while for $T \geq E(T)$, his preferred tax rate level is maximum (he is net recipient). Note also that the ex-post return of the pension system is increasing in longevity since it represents additional years of pension benefits, with no adjustment between total contributions paid and the individual's longevity.

We now study the voting equilibrium. Since the indirect utility function is single-peaked in the tax rate, the median voter theorem applies and the Condorcet winner corresponds to the preferred tax rate of the voter with life duration T_m. Our results are summarized below:

Proposition 1 *Assume a population of individuals with different longevity T. The pension system is characterized by pension parameters $(\tau, \bar{p}(\tau))$ and a uniform retirement age $\bar{z}$. If there are no liquidity constraints, the majority voting tax rate τ^{mv} is such that:*

(i) if $T_m < E(T)$, $\tau^{mv} = 0$.

(ii) if $T_m \geq E(T)$, $\tau^{mv} = 1$.

If the life duration distribution is negatively-skewed, $T_m \geq E(T)$, the majority voting tax rate is maximum.[13]

We now turn to the case where individuals can privately choose their retirement age.

[13] Note that this section is derived under the assumption of no liquidity constraints. If individuals had been liquidity constrained, the preference for the tax rate would be null for any individual with life duration below $\bar{z}$ and strictly positive and increasing in life duration for any individual above. In this latter case, increasing the tax rate creates a trade-off between on the one hand, an increase in the return obtained from the pension system and on the other hand, smaller pre retirement consumption. The voting outcome may then be modified and an interior majority voting solution may be possible depending on the characteristics of the population.

3 Majority voting equilibrium under voluntary retirement

In this section, we first derive individuals' optimal retirement age and consumption levels. We then study the consequences of longevity differentials over the individuals' tax rate preference and we find the majority voting equilibrium.

3.1 Individual decisions and comparative statics

The problem of an individual with longevity T amounts to choose consumption path $c_T(t)$ and retirement age z_T in order to maximize (1) subject to the budget constraint (2). Because of the separability of preferences and the no liquidity constraint assumption, the optimum is obtained when consumption is constant over time, $c_T(t) = c_T$ for every t and the objective function can be rewritten as:

$$U(z_T, T) = Tu\left(\frac{w(1-\tau)z_T + (T - z_T)p}{T}\right) - R(z_T) \tag{4}$$

where we replaced for the budget constraint. First order condition of this problem yields

$$\frac{r(z_T)}{u'(c_T)} = w(1-\tau) - p \tag{5}$$

In equation (5), we assume $p < w(1-\tau)$ which ensures that the individual always works a positive amount of time. Note also that the existence of a pension system implies earlier retirement for the individual as it decreases the price of leisure with respect to consumption, $w(1-\tau) - p < w$.

Using both the lifetime budget constraint and (5), we find optimal per period consumption $c_T^*(\tau, p)$ and retirement age $z_T^*(\tau, p)$ which depend on the level of pension parameters and on longevity. We show in Appendix A that consumption decreases with longevity while the retirement age increases with it.[14] We also find that the elasticity of retirement age with respect to life duration $\varepsilon_{z^*(\tau,p),T} = (dz_T^*(\tau,p)/dT) \times (T/z_T^*(\tau,p))$ is lower than one, i.e. if life duration increases, the retirement age increases less, so that the retirement period $(T - z_T^*(\tau,p))$

[14]Empirical evidence would show the opposite relation between life duration and retirement age. This difference between our theoretical result and empirical evidence can be explained by the fact that there exist other major components such as wealth (positively correlated with longevity), which we do not take into account in our model but certainly enters in the individual's decision to retire.

also increases. This comes from a trade-off arising at the optimum between both higher total disutility of work and the need for additional resources (in order for the individual to consume during these additional periods of life).[15] As a result, even if lifetime income increases, the level of per period consumption decreases so as to adjust between a longer life duration and a proportionally higher retirement period. These results are summarized in the following proposition.

Proposition 2 *Assume a population of individuals with different longevity T and a pension system defined by pension parameters (τ, p). At the optimum, for any two individuals with longevity T_1 and T_2 such that $T_1 < T_2$*

(i) $c^*_{T_1}(\tau, p) > c^*_{T_2}(\tau, p)$

(ii) $z^*_{T_1}(\tau, p) < z^*_{T_2}(\tau, p)$

(iii) $\varepsilon_{z^*_T(\tau,p),T} < 1$

Our findings are similar to the framework of Sheshinski (2005) with the difference that he assumes uncertain lifetime. Thus, individuals with higher life expectancies consume less per period and retire later.

3.2 The individual's preference for the tax rate

As a first step in determining the individual's preference for the tax rate, we replace for $z^*_T(\tau, p)$ into (3) so that per period pension benefit can be expressed as a function of τ only:

$$p(\tau) = w\tau \frac{E(z^*_T(\tau))}{\left[E(T) - E\left(z^*_T(\tau)\right)\right]} \tag{6}$$

where $z^*_T(\tau) \equiv z^*_T(\tau, p(\tau))$ and $E(z^*_T(\tau)) = \int_{T_{\min}}^{T_{\max}} z^*_T(\tau) f(T)\,dT$. We define the indirect utility function attained by type-T individuals for given τ as

$$\begin{aligned} V(\tau, T) &\equiv U(c^*_T(\tau), z^*_T(\tau), T) \\ &= Tu\left(\frac{w(1-\tau) z^*_T(\tau) + (T - z^*_T(\tau)) p(\tau)}{T}\right) - R(z^*_T(\tau)) \end{aligned} \tag{7}$$

Preferred tax rates of individuals are obtained by differentiating the above function with respect to τ (see Appendix B). Our results are summarized in the following proposition:

[15] Because consumption smoothing is optimal, a framework in which the individual has zero consumption for some periods cannot be optimal.

Proposition 3 *The individual's preferred tax rate is such that:*

(i) $\tau_T^* = 0$ *for any individual with longevity* $T \in [T_{\min}; T_{TR}(\tau)]$ *with*

$$T_{TR}(\tau) = \frac{E(T)}{E\left(z_T^*(\tau)\right)} z_{T_{TR}}^*(\tau) \tag{8}$$

(ii) $\tau_T^* \in]0,1]$ *and is implicitly determined by*

$$[T - z_T^*(\tau_T^*)] \frac{dp(\tau_T^*)}{d\tau} = w z_T^*(\tau_T^*) \tag{9}$$

for any individual with longevity $T \in]T_{TR}(\tau); T_{\max}]$. *This tax level* τ_T^* *is increasing in* T.

In the above proposition, $z_{T_{TR}}^*(\tau)$ corresponds to the optimal retirement age of an individual with life duration $T_{TR}(\tau)$. Note also that if the retirement age is uniform, $z_{T_{TR}}^*(\tau) = E(z_T^*(\tau)) = \bar{z}$ and $T_{TR}(\tau) = E(T)$ and we are back to Proposition 1.

Below $T_{TR}(\tau)$, individuals always prefer a zero tax rate while beyond this threshold, the preferred tax rate is strictly positive and is implicitly determined by (9). This equation states that at the preferred tax rate level τ_T^* of an individual with life duration $T \in]T_{TR}(\tau), T_{\max}]$, the marginal benefit of a tax rate increase (i.e. a marginal increase of per period pension benefit over the length of retirement) is equal to its marginal cost (i.e. a decrease in the net labour income).

Using these results, the population is then divided into two categories: the short-lived individuals (with $T \leq T_{TR}(\tau)$) who always prefer a zero tax rate and no pension system and the long-lived individuals (with $T > T_{TR}(\tau)$) who always prefer a positive tax rate. Again, this can be explained through a net-contribution argument. Under voluntary retirement, the net contribution is equal to

$$NC_T = w\tau \left[\frac{E(T) z_T^*(\tau) - E(z_T^*(\tau)) T}{E(T) - E\left(z_T^*(\tau)\right)}\right]$$

so that it is positive (resp. strictly negative) if $T \leqslant (resp. >) T_{TR}(\tau)$. Indeed, there are two means of transferring resources from young ages toward old ages: private savings with a zero return and the pension system. For individuals with life duration below the threshold $T_{TR}(\tau)$, the implicit return of the pension system is always negative (the net contribution is positive), so that they always prefer private savings which provide them with a zero return and no pension system. On the other hand, individuals above the threshold obtain a strictly positive return from the pension system and always prefer a strictly positive tax rate.[16]

We also find that the individual's preference for the tax rate is interior which is a direct consequence of the labour distortions created by the existence of a pension system when retirement is endogenous. Increasing τ creates a trade-off between on the one hand, higher net benefit from the pension system and on the other hand, a reduction in resources obtained from work (due both to an increase in the tax rate and to increased labor supply distortions). We also find that the preference for the tax rate increases with the individual's life duration; since the elasticity of the retirement age with life duration is lower than one, individuals with higher life duration obtain a higher net benefit from the pension system.[17]

Finally, our findings can be related to the empirical results obtained by Coronado *et al.* (2000), Liebman (2001) and Bommier *et al.* (2006) which state that even if replacement rates are higher for low-income workers, part of the redistribution made toward them is neutralized due to mortality differentials. In our setting, transfers from short-lived toward long-lived individuals explain the individual's preference for the existence and the size of the pension system. As we will see in the next section, it will also determine the political equilibrium.

3.3 Majority voting equilibrium

We now determine the majority voting tax rate and pension benefit. We first check that the median voter theorem applies. Preferences are defined over two-dimensional variables (τ, p) but with the government budget constraint, it is

[16] Note that removing the assumption of a zero interest rate would push the value of $T_{TR}(\tau)$ to the right as individuals would now support the system if the return of the pension system is greater than $(1 + \textit{interest rate})$, which would be achieved only for higher life durations.

[17] Under the existence of liquidity constraints, the threshold life duration would simply be pushed to the right. In this case, individuals always prefer lower tax rate so as to ensure positive consumption in the first period; thus, for a given life duration, the length of activity (which increases the net contribution) is higher so that only individuals with higher life durations are going to be net beneficiaries and prefer a strictly positive τ.

effectively unidimensional. We verify that the single crossing condition defined by Gans and Smart (1996) is satisfied in our framework.[18] Defining $MRS_{p,\tau}^{T}$ as the marginal rate of substitution between τ and p of a type-T individual, we obtain

$$MRS_{p,\tau}^{T} = -\frac{\partial V(\tau, p, T)/\partial \tau}{\partial V(\tau, p, T)/\partial p} = w\frac{z_T^*(\tau, p)}{T - z_T^*(\tau, p)} > 0$$

and find that it is monotonically decreasing in the individual's type since $\varepsilon_{z_T^*(\tau,p),T} < 1$.[19] Individuals' marginal rate of substitution between τ and p can be ranked independently of the pension allocation (τ, p). Then, the single crossing property holds and it guarantees that a political equilibrium exists under pure majority rule; the voter with median life duration T_m is decisive and the Condorcet winner is the preferred tax rate of this individual.

Relying on Proposition 3, the existence and the size of the pension system at the voting equilibrium depend crucially on the position of $T_{TR}(\tau)$ relative to T_m and thus on life duration distribution. This is summarized in the following proposition:

Proposition 4 *Assume a population characterized by a life duration distribution with support* $[T_{\min}, T_{\max}]$*, density function* $f(T)$ *and median life duration* T_m*. At the voting equilibrium,*

(i) if $T_m \leq T_{TR}(\tau)$*, the voting outcome is* $\tau^{mv} = 0$ *and no pension system emerges.*

(ii) if $T_m > T_{TR}(\tau)$*, the majority voting tax rate corresponds to the preferred tax rate of the median voter,* $\tau^{mv} = \tau_{T_m}^*$*. It belongs to the interval* $]0,1]$ *and satisfies*

$$\left[T_m - z_{T_m}^*(\tau^{mv})\right]\frac{dp(\tau^{mv})}{d\tau} = wz_{T_m}^*(\tau^{mv}) \tag{10}$$

The majority voting tax rate is increasing in T_m*.*

The crucial point is now to determine the position of $T_{TR}(\tau)$ with respect to T_m. To do so, we represent $z_T^*(\tau)$ as a concave function of T (since $\varepsilon_{z_T^*(\tau),T} < 1$),

[18] Individuals preferences may not be single peaked in the tax rate since $p(\tau)$ may not be strictly concave in τ. For this reason, we use the single crossing property.

[19] $V(\tau, p, T)$ is the intermediate utility function and is equal to $Tu((w(1-\tau)z_T^*(\tau,p) + (T - z_T^*(\tau,p))p)/T) - R(z_T^*(\tau,p))$ with $z_T^*(\tau,p)$ defined by (5).

for a given tax rate:

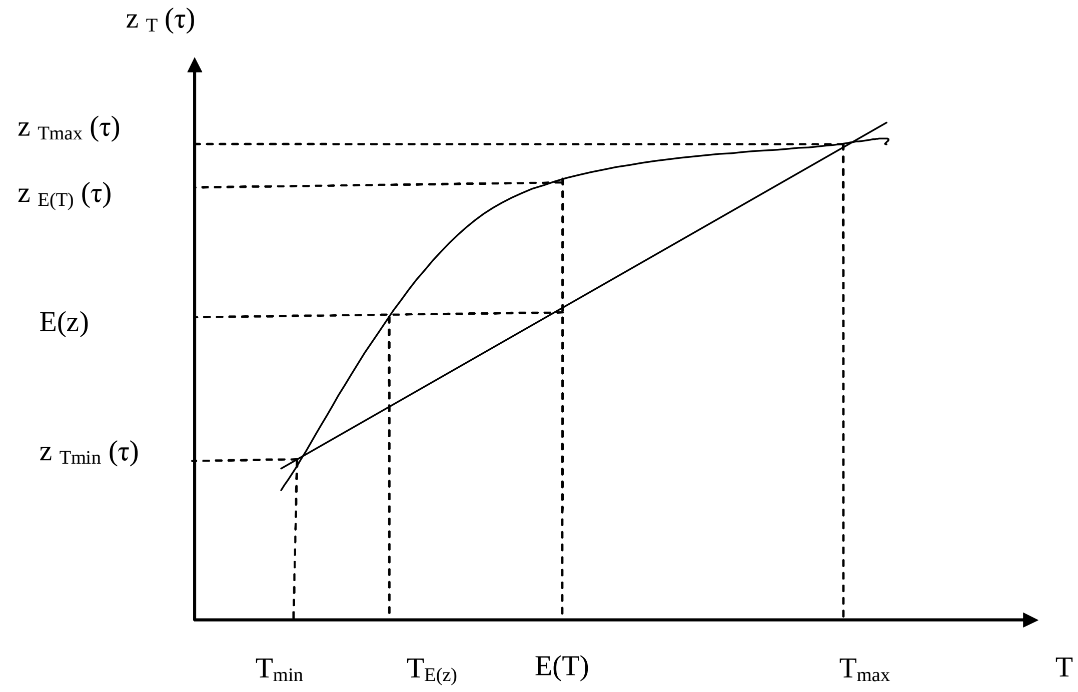

Figure 1: Distribution of retirement ages as a function of life durations

Using the above figure and the expression for $T_{TR}(\tau)$, it is possible to prove that $T_{TR}(\tau) \notin \left[T_{E(z)}, E(T)\right]$.[20] On the opposite, the position of T_m depends only on the skewness of the longevity distribution.

Let first consider the case where $T_{TR}(\tau)$ belongs to $\left[T_{\min}, T_{E(z)}\right[$. If the longevity distribution is centered or left-skewed ($T_m \geq E(T)$), a pension system always emerges. This might also be the case that a pension system emerges when the distribution is right-skewed; one only needs to have $T_{TR}(\tau) < T_m (< ET)$). Let now consider the case where $T_{TR}(\tau)$ belongs to $]E(T), T_{\max}]$. A pension system emerges only if $T_m > T_{TR}(\tau)$, which corresponds to a very left-skewed longevity distribution. When the conditions for the existence of a pension system are satisfied, the majority voting tax rate τ^{mv} satisfies (10) evaluated at T_m.

[20]Sketch of proof: we compare $T_{TR}(\tau)/z^*_{T_{TR}}(\tau)$ with $E(T)/E(z)$ depending on whether T_{TR} belongs to $\left[T_{\min}, T_{E(Z)}\right]$, $\left[T_{E(Z)}, E(T)\right]$ or $[E(T), T_{\max}]$ and find the corresponding $z^*_{T_{TR}}(\tau)$; we check whether $T_{TR}(\tau)/z^*_{T_{TR}}(\tau)$ and $E(T)/E(z)$ can be equal and find that it is never possible in the interval $\left[T_{E(Z)}, E(T)\right]$.

At this point of our theoretical analysis, we need to specify a form for the longevity distribution so as to find whether $T_{TR}(\tau)$ belongs to the intervals $[T_{\min}, T_{E(z)}[$ or $]E(T), T_{\max}]$; we determine it through simulations in the next section.

4 Numerical Example

Consider the following specifications for the various components of our model. We set $T_{\min} = 0$ and $T_{\max} = 100$ and normalize $T = T_{\max}x$ assuming that x follows a Beta distribution $B[a, b]$ with a, $b > 0$.[21] The density function of the longevity distribution has then the following form:

$$f(T) = \frac{1}{T_{\max}} f_x\left(\frac{T}{T_{\max}}\right)$$

where $f_x(.)$ is the density function of the xs. As we show in the following, making parameters a and b vary, enables to change the skewness of the density function in the same way for both the xs and the Ts. We assume that per period utility of consumption is $u(c) = \log c$ and that disutility of work is quadratic, $R(z) = \gamma z^2/2$. We set $w = \gamma = 1$.

Our results are presented below. The first table gives the value of $T_{TR}(\tau)$ (defined by (8)) as a function of the tax rate (in line) and for different shape of the life duration distribution (in column):[22]

[21] We obtain similar qualitative results for any other value of $T_{\min}$ and $T_{\max}$.

[22] We assume successively a centered distribution, with parameters $a = b = 2$; a right-skewed distribution, $a = 2$ and $b = 4$; and a left-skewed distribution, $a = 4$ and $b = 2$.

Table 1: Threshold levels as a function of the tax rate and of the shape of life duration distribution

Distribution of T			
	right	**centered**	**left**
$E(T)$	33.33	50	66.67
T_m	31.38	50	68.62
$T_{TR}(0.25)$	36.12	53.1	68.03
$T_{TR}(0.5)$	35.84	52.88	67.90
$T_{TR}(0.6)$	35.62	52.59	67.79
$T_{TR}(0.75)$	35.09	52.03	67.53
$T_{TR}(0.9)$	34.18	51.02	67.07
$T_{TR}(0.99)$	33.38	50.06	66.69

Regarding the values of $T_{TR}(\tau)$, we first find that, for any shape of the life duration distribution and any value of τ, $T_{TR}(\tau)$ *always belongs to the interval* $]E(T), T_{\max}]$. We also find that $T_{TR}(\tau)$ is decreasing in the tax rate. This can be explained as follows. An increase in τ reduces the length of work and increases the retirement period so that individuals at the bottom of the distribution (below $T_{TR}(\tau)$) who were net contributors to the system and preferred no pension system are now likely to be net recipients and prefer a strictly positive tax rate.

Our second set of results concern the voting equilibrium. As Table 1 demonstrates, when the distribution is centered or right-skewed, $T_m < T_{TR}(\tau)$ so that no pension system emerges. On the contrary, *when the distribution is left-skewed,* $T_{TR}(\tau) < T_m$ *so that the median-type individual always votes for a positive tax rate.* In this case, a pension system emerges with tax rate $\tau^{mv} = \tau^*_{T_m}$ defined by (10).

We now study the consequences on pension parameters of an increase in longevity. We represent it through a *rectangularisation of survival curves*, i.e. the proportion of older individuals increases while the maximum expected length of life remains almost constant.[23] In our framework, it is equivalent to an increase the skewness of the life duration distribution (through an increase in a) while maintaining $T_{\max}$ constant. The following table reports the voting outcome under this scenario:

[23] Demographers estimate that in the recent decades, the increase in longevity took on this character.

Table 2: Majority voting pension parameters

skewness	$\boldsymbol{\tau}^{mv} = \boldsymbol{\tau}^*_{T_m}$	$\mathbf{p}\left(\boldsymbol{\tau}^{mv}\right)$
$a = 3.5$	0.0346	0.0048
$a = 4.5$	0.0989	0.0125
$a = 5.5$	0.1143	0.0139
$a = 6.5$	0.1196	0.0142

We find that, in this case, both the voting equilibrium tax rate and pension benefit increase.[24] The increase in the tax rate is a direct consequence of the increase in median life duration (see Proposition 4). On the opposite, the effect on the pension benefit is less straightforward. First, we show in unreported simulations that the impact of increased longevity overcompensates the distortions on labor supply created by a higher tax rate, which results in a higher $E\left(z_T^*\left(\tau\right)\right)$. Thus in our example, the increase in total resources (both due to a higher tax rate and higher average length of activity) exceeds the increase in expenditures (due to higher average retirement period) so that per period benefit increases.

Finally, we study the effect of a marginal increase in the disutility of work.[25] The following table, where γ accounts for marginal disutility of work, shows that the majority voting tax rate $\boldsymbol{\tau}^{mv}$ increases with the disutility of work and $p\left(\boldsymbol{\tau}^{mv}\right)$ decreases with it:

Table 3: Majority voting pension parameters as functions of the labour disutility

γ	$\boldsymbol{\tau}^{mv} = \boldsymbol{\tau}\left(T_m\right)$	$\mathbf{p}(\boldsymbol{\tau}^{mv})$
1	0.0989	0.0125
2	0.1008	0.0087
3	0.1017	0.0071
5	0.1025	0.0054
7	0.1030	0.0046
10	0.1034	0.0038
15	0.1037	0.0031
20	0.1040	0.0027

For a given left-skewed longevity distribution, when the disutility of work increases, the length of activity of the median type individual decreases so that

[24] The level of the tax rate $\tau^*_{T^m}$ is obtained by crossing equations (6) and (10) for given values of T^m and $E\left(T\right)$.

[25] We assume a left-skewed distribution with $a = 4.5$ and $b = 2$.

the net benefit he obtains from the pension system increases; as a consequence, his support for the pension system is higher and the tax rate he votes for is also higher. Thus, $E(z_T^*(\tau))$ decreases, due to both an increase in the tax rate and in the disutility of work, and $[E(T) - E(z_T^*(\tau))]$ increases. Under our specifications, we find that the increase in the tax rate is not enough to compensate for lower length of activity and higher retirement period so that per period pension benefit decreases. This explains the above result.

5 Discussion

In this section, we discuss the implications of assuming voluntary retirement on the voting outcome. For this purpose, let first remember that a pension system emerges when $E(T) < T_m$ in the mandatory retirement case while with voluntary retirement, this is the case only when $T_{TR}(\tau) < T_m$ with $E(T) < T_{TR}(\tau)$. Thus, in the latter scenario, the condition for the emergence of a pension system is more stringent and a pension system is less likely to emerge. We also find that when a pension system exists, its size is lower with endogenous retirement as the tax rate solution is likely to be interior while under mandatory retirement, $\boldsymbol{\tau}^{mv} = 1$.

Moreover, when voluntary retirement is allowed, the net contribution partly adapts to longevity differentials. To see this, we compare how the net contribution varies with the individual's type for a given tax rate,

$$\frac{\partial NC_T}{\partial T} = [w\tau + p(\tau)] \frac{\partial z_T^*(\tau)}{\partial T} - p(\tau)$$

under voluntary retirement with $\partial NC_T / \partial T = -\bar{p}$, in the uniform retirement case. Then, the negative direct effect of life duration over the net contribution (i.e. additional $p(\tau)$) is now mitigated by the endogeneity of the retirement decision. Here, a higher life duration implies a higher length of activity, which tends to lower the impact of T over the net contribution through additional contributions and delayed pension benefits. According to this new feature, the distribution between net contributors / net beneficiaries is going to be modified. Under voluntary retirement, the number of net contributors increases, which is due to the fact that individuals with $T \in [E(T), T_{TR}(\tau)]$ who were net beneficiaries under mandatory retirement, become additional net contributors under voluntary retirement. Equivalently, the number of net beneficiaries decreases. Consequently, every individuals with $T \in [T_{\min}, E(T)[$ are now better-off under voluntary retirement as their own net contribution is reduced (the size of the system is smaller and the number of contributors is larger). Individuals with $T \in [E(T), T_{TR}(\tau)]$ are clearly worse-off under voluntary retirement. This is also the case for individuals with $T \in [T_{TR}(\tau), T_{\max}]$ as they obtain a lower net benefit under a voluntary retirement scheme than under a mandatory one.

6 Conclusion

This paper examines the determination of the size of a pension system through the political process, assuming that individuals have different longevities and that retirement is an individual decision. We find that, with longevity as the only source of population heterogeneity, a pension system emerges only when there is a majority of long-lived individuals (i.e. the distribution of longevities is such that the median type is above the average). We also discuss the implications of assuming either voluntary or uniform retirement. We show that voluntary retirement enables to decrease the net contribution of short-lived individuals (i.e. the ones at the bottom of the distribution). We also conclude that, even if individuals with high longevity (i.e. the ones at the top of the distribution) continue to be net beneficiaries from the system, allowing for voluntary retirement enables to lower the size of the transfers they receive. Finally, we show that a rise in average longevity is likely to increase the size of the pension system, both through a increase in the tax rate and in the pension benefit.

Until recently, the political economy literature has mainly focused on the impacts of income heterogeneity and of the population age structure on the existence of a pension system. For instance, it is by now well recognized that the size of the pension system increases with the proportion of the elderly and that intra-generational redistribution increases with the number of the poor young.[26]. However, it has largely neglected another important dimension, i.e. life duration which certainly influences the individual's support for a pension system. In

[26] For a complete survey on the political economy of Social Security, see Galasso and Profeta (2002).

this paper, we have shown that even if individuals have the same income, they may have different preferences for the pension scheme. This is because pension systems do not only transfer resources from the young to the old and from the rich to the poor but also redistribute resources from short-lived to long-lived individuals. Thus, by including longevity differentials, our paper questions the widespread belief that high-income workers always prefer a zero tax rate and no pension system. If we believe that individuals with higher income also have higher life expectancy, it is not clear whether this preference pattern continues to hold since they may also prefer a strictly positive tax rate, due to higher life duration. Their preferred tax rate level would depend on which dimension of heterogeneity, income or longevity, dominates.

Finally, we are concerned with the fact that our model relies on very simplistic assumptions such as a flat rate benefit, exogenous and certain lifetime, and identical productivity. First, the assumption of exogenous longevity is certainly very restrictive and one could argue that longevity can be influenced by, for example, health spending. An interesting extension would be to consider this possibility and to assume that the government provides individuals with health care benefits (in addition to pension benefits). Under some conditions, this could mitigate the redistribution from short-lived to long-lived individuals and increase the support for the Social Security system. Second, relaxing the assumption of certain lifetime, Social Security would be welfare enhancing for risk-averse individuals as it would work as an insurance against the risk of a long life. This constitutes a straightforward extension. As already discussed, introducing differences in productivity also constitutes a relevant and interesting extension. These are on our research agenda.

APPENDIX

A Proof of Proposition 2

Differentiation with respect to life expectancy T of the individual's budget constraint and of the first order condition gives

$$\begin{aligned}
\frac{r'\left(z_T^*(\tau,p)\right)}{u''\left(c_T^*(\tau,p)\right)}\frac{dz_T^*(\tau,p)}{dT} &= \left[w(1-\tau)-p\right]\frac{dc_T^*(\tau,p)}{dT} \\
c_T^*(\tau,p)+T\frac{dc_T^*(\tau,p)}{dT} &= \left[w(1-\tau)-p\right]\frac{dz_T^*(\tau,p)}{dT}+p
\end{aligned}$$

Substituting one equation into the other, we obtain:

$$\frac{dz_T^*(\tau,p)}{dT}=\frac{p-c_T^*(\tau,p)}{\frac{T}{w(1-\tau)-p}\frac{r'\left(z_T^*(\tau,p)\right)}{u''\left(c_T^*(\tau,p)\right)}-\left(w(1-\tau)-p\right)}$$

where both the numerator and the denominator are negative; then $dz_T^*(\tau,p)/dT > 0$ and

$$\frac{dc_T^*(\tau,p)}{dT}=\frac{r'\left(z_T^*(\tau,p)\right)}{\left[w(1-\tau)-p\right]u''\left(c_T^*(\tau,p)\right)}\frac{dz_T^*(\tau,p)}{dT}<0$$

for any values of $\tau\in[0,1]$. We now derive the elasticity $\varepsilon_{z_T^*(\tau,p),T}$ of the retirement age with respect to life duration:

$$\varepsilon_{z_T^*(\tau,p),T}=\frac{dz_T^*(\tau,p)}{dT}\frac{T}{z_T^*(\tau,p)}=\frac{-\left(w(1-\tau)-p\right)}{\frac{T}{w(1-\tau)-p}\frac{r'\left(z_T^*(\tau,p)\right)}{u''\left(c_T^*(\tau,p)\right)}-\left(w(1-\tau)-p\right)}<1$$

This proves Proposition 1.

B Proof of Proposition 3

Using optimality conditions (5), we differentiate (7) with respect to τ

$$\frac{\partial V(\tau,T)}{\partial\tau}=u'\left(c_T^*(\tau)\right)\left[-wz_T^*(\tau)+\left(T-z_T^*(\tau)\right)\frac{dp(\tau)}{d\tau}\right] \tag{11}$$

with $c_T^*(\tau)\equiv c_T^*(\tau,p(\tau))$. The variation of the pension benefit with respect to τ is given by

$$\frac{dp(\tau)}{d\tau}=w\left[\frac{E\left(z_T^*(\tau)\right)}{E(T)-E\left(z_T^*(\tau)\right)}+\frac{E(T)\,\tau}{\left[E(T)-E\left(z_T^*(\tau)\right)\right]^2}\frac{dE\left(z_T^*(\tau)\right)}{d\tau}\right] \tag{12}$$

and

$$\begin{aligned}\frac{dE\left(z_T^*\left(\tau\right)\right)}{d\tau} &= \frac{dE\left(z_T^*\left(\tau, p\left(\tau\right)\right)\right)}{d\tau} \\ &= \frac{\partial E\left(z_T^*\left(\tau, p\left(\tau\right)\right)\right)}{\partial\tau} + \frac{\partial E\left(z_T^*\left(\tau, p\left(\tau\right)\right)\right)}{\partial p\left(\tau\right)}\frac{dp\left(\tau\right)}{d\tau} < 0\end{aligned}$$

since $\partial E\left(z_T^*\left(\tau, p\left(\tau\right)\right)\right)/\partial\tau$ and $\partial E\left(z_T^*\left(\tau, p\left(\tau\right)\right)\right)/\partial p\left(\tau\right)$ are negative (as mentioned in Section 4.1, the length of activity is a normal good) and $dp\left(\tau\right)/d\tau > 0$ (the economy is on the increasing part of the Laffer curve). Replacing (12) into (11), we show that $\partial V\left(\tau, T\right)/\partial\tau < 0$ whenever the individual's life duration T is such that:

$$\frac{E\left(z_T^*\left(\tau\right)\right)\left(T - z_T^*\left(\tau\right)\right)}{E\left(T\right) - E\left(z_T^*\left(\tau\right)\right)} - z_T^*\left(\tau\right) \leq 0$$

$$\text{or equivalently } T \leq T_{TR}\left(\tau\right)$$

where $T_{TR}\left(\tau\right) = z_T^*\left(\tau\right)\left[E\left(T\right)/E\left(z_T^*\left(\tau\right)\right)\right]$. Then $\tau_T^* = 0$ for any individual with life span $T \leq T_{TR}\left(\tau\right)$. For any individual with $T \in \left]T_{TR}\left(\tau\right), T_{\max}\right]$, the solution τ_T^* is interior and defined by $\partial V\left(\tau, T\right)/\partial\tau \mid_{\tau=\tau_T^*} = 0$ or equivalently:

$$\left(T - z_T^*\left(\tau_T^*\right)\right)\frac{\partial p\left(\tau_T^*\right)}{\partial\tau} = wz_T^*\left(\tau_T^*\right)$$

We assume that the second order condition is satisfied to ensure that the individual's utility is effectively maximized in τ_T^*.

For any individual with life duration $T \in \left]T_{TR}; T_{\max}\right]$, we find how his preferred tax rate τ_T^* varies with life span T. By the implicit function theorem,

$$sign\left(\frac{d\tau_T^*}{dT}\right) = sign\left(\frac{\partial^2 V\left(\tau_T^*, T\right)}{\partial\tau\partial T}\right)$$

with

$$\frac{\partial V^2\left(\tau_T^*, T\right)}{\partial\tau\partial T} = u'\left(c_T^*\left(\tau_T^*\right)\right)\left\{\left(1 - \frac{dz_T^*\left(\tau_T^*\right)}{dT}\right)\frac{dp\left(\tau_T^*\right)}{d\tau} - w\frac{dz_T^*\left(\tau_T^*\right)}{dT}\right\}$$

Substituting for equation (9), one gets

$$\frac{\partial V^2\left(\tau_T^*, T\right)}{\partial\tau\partial T} = u'\left(c_T^*\left(\tau_T^*\right)\right)\frac{w}{T - z_T^*\left(\tau_T^*\right)}\left\{z_T^*\left(\tau_T^*\right) - T \times \frac{\partial z_T^*\left(\tau_T^*\right)}{\partial T}\right\}$$

Since $0 < \varepsilon_{z_T^*(\tau),T} < 1$, the term inside parenthesis is always positive and $d\tau_T^*/dT > 0$.

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