

Volume 38, Issue 1

Further Results on Preference Uncertainty and Monetary Conservatism

Keiichi Morimoto

Department of Economics, Meisei University

Abstract

This study re-examines the optimal delegation problem of monetary policy under preference uncertainty of the central banker. Liberal central bankers are desirable when uncertainty is strong, which is emphasized when the slope of the Phillips curve is flatter, as some empirical works report. However, appointing conservative central bankers is optimal with standard parameter values when monetary policies are conducted by committees, as in most actual economies.

This study is supported by a grant from the Japan Society for the Promotion of Science (JSPS) through a Grant-in-Aid for Young Scientists (B) (No. 16K17122). I am grateful to anonymous referees for helpful comments. I thank the participants of the MUETEI workshop at Meisei University. Of course, I am responsible for any remaining errors.

Citation: Keiichi Morimoto, (2018) "Further Results on Preference Uncertainty and Monetary Conservatism", *Economics Bulletin*, Volume 38, Issue 1, pages 583-592

Contact: Keiichi Morimoto - morimoto.keiichi@gmail.com

Submitted: December 17, 2017. **Published:** March 23, 2018.

1 Introduction

This study re-examines the optimal delegation problem of monetary policy under preference uncertainty of the central banker. Liberal central bankers are desirable when uncertainty is strong, which is emphasized when the slope of the Phillips curve is flatter, as some recent empirical works report.¹ However, appointing conservative central bankers is optimal with standard parameter values when monetary policies are conducted by committees, as in most of the actual economies.

As Bernanke (2004) suggests, new central bankers' preferences on inflation and output variability are, in general, unobservable and uncertain for the public. Tillmann (2008) shows that central bankers who are too conservative are more harmful than are those who are too liberal in a New Keynesian model with a discretionary monetary policy. This leads to the conjecture that pursuing simplistic delegations of Rogoff's (1985) conservative central bankers is problematic in the presence of preference uncertainty. Sorge (2013) tackles this issue by introducing Knightian uncertainty on central bankers' preferences, and shows that delegating monetary policy to liberal central bankers is optimal in a New Keynesian model with standard parameter values.

In this study, I introduce preference uncertainty of a simple form (other than Knightian uncertainty) into a New Keynesian model and derive a closed-form solution for the optimal delegation problem. This gives an empirically interesting parameter condition for the optimality of liberal central bankers explicitly. In addition, the condition becomes more likely to hold as the Phillips curve becomes flatter, which has been observed in modern developed economies. However, as in most actual countries over the last two decades, delegating monetary policy to committees mitigates the loss from uncertainty through preference aggregation. I show that central bank conservatism is again optimal in an extended model with monetary policy by committees of at least two persons. It is a simple finding, but can be interpreted as a theoretical justification for constituting monetary policy committees, which is an important issue in the recent debate on monetary policy design.

The role of central bank preference uncertainty is studied by the seminal works of Beetsma and Jensen (1998) and Muscatelli (1998). These papers examine the optimal design of inflation target regimes and linear inflation contracts for inflation bias in the models with the conventional Lucas supply curves.² In other words, they focus on whether monetary policy should be delegated to the central bank with some designed objective function through the specific contracts subject to (ex-ante) preference uncertainty. In view of the mechanism, these papers capture the trade-off between mitigating (not stabilization bias but) inflation bias and the inefficient variability due to preference uncertainty. They show the efficiency of imposing the penalties to excessive inflation under the delegation regimes, and the optimal quadratic contract of Beetsma and Jensen (1998) can be interpreted as the optimality of weight conservative central bankers. Thus, basically, these works differ from this paper in terms of the modeling, the underlying mechanism, and policy implications.

Sorge (2013) calculates the optimal weight on output gap of the central bank's loss function in a New Keynesian model where the central bank preference has Knightian

¹In the context of an optimal delegation of monetary policy, liberal (conservative) central bankers are those who put larger (smaller) weight on output stabilization relative to inflation than representative households.

²Beetsma and Jensen (1998) also analyze the optimal quadratic inflation contracts. For optimal inflation targets and linear inflation contracts, see Svensson (1997) and Walsh (1995).

uncertainty. I adopt a similar approach to Sorge (2013), but consider a given distribution of the central bank preference as common knowledge. While Sorge (2013)'s setting requires the robust control method and brings the rich properties of the solution such as discontinuity in parameter, the simple setting of this paper allows us to obtain a more tractable analytical solution with continuity. It leads to the clear and rich discussion on the relation between the optimal delegation and the slope of Phillips curve, comparing it with the implied result of Sorge (2013). It also helps to extend the model to the case of monetary policy committees. These are significant points in terms of the empirical evidence and the actual institution design. Hefeker and Zimmer (2015) also analyze the optimal design of monetary policy committees under uncertainty in the policy objectives in the New Keynesian model, but show the substantially different results. I provide further comparison with these in Section 3.

2 Model

I introduce preference uncertainty of central bankers into a standard New Keynesian model with discretionary monetary policy. The timing of the decisions is as follows:

- First, before the beginning of the initial period 0, the government appoints a central banker whose preference is characterized by an expected relative weight on output gap variance, $\hat{\lambda}$. The government chooses $\hat{\lambda}$ to minimize social loss under preference uncertainty, incorporating the following.
- Next, Nature draws the central banker's preference λ from a distribution with mean $\hat{\lambda}$. Then, each agent in the economy observes λ .
- Given the value of λ as common knowledge among all agents, the economic activity described in the standard New Keynesian model starts.

Setup

I analyze the model by backward induction. Given a value of λ , the model is identical to the popular New Keynesian model with discretionary monetary policy. I review it briefly below.

The private sector of the economy is characterized by the following New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + e_t, \quad (1)$$

where π_t and x_t denote the inflation rate and the output gap in period t . The parameter $\beta \in (0, 1)$ is the discount factor and $\kappa > 0$ is the slope of the Phillips curve. The cost shock e_t follows an AR(1) process: $e_{t+1} = \rho e_t + \varepsilon_{t+1}$, where $\rho \in (0, 1)$ is the degree of serial correlation and ε_{t+1} identically and independently follows the standard normal distribution, $N(0, 1)$.

The central bank conducts discretionary monetary policy using the period loss function $\pi_t^2 + \lambda x_t^2$, subject to the New Keynesian Phillips curve, (1).³ Then, the equilibrium dynamic path is given by

$$\pi_t = \frac{\lambda}{\kappa^2 + (1 - \beta\rho)\lambda} e_t, \quad (2)$$

$$x_t = -\frac{\kappa}{\kappa^2 + (1 - \beta\rho)\lambda} e_t. \quad (3)$$

To evaluate the long-run performance of a monetary policy regime, I adopt a standard social loss function

$$L^s \equiv V_\pi + \lambda^s V_x, \quad (4)$$

where λ^s is the relative weight on output gap variability derived from the representative households' utility function, and V_π and V_x are the asymptotic variances of the inflation rate and the output gap, respectively.⁴ I obtain these immediately from (2) and (3):

$$V_\pi = \frac{1}{1 - \rho^2} \left[\frac{\lambda}{\kappa^2 + (1 - \beta\rho)\lambda} \right]^2, \quad (5)$$

$$V_x = \frac{1}{1 - \rho^2} \left[\frac{\kappa}{\kappa^2 + (1 - \beta\rho)\lambda} \right]^2. \quad (6)$$

Optimal Delegation with Certain Preferences

Next, I solve the optimal delegation problem of monetary policy. To qualify the role of the preference uncertainty of central bankers, I first describe the case of certainty.⁵ In this case, the government chooses λ directly because $\lambda = \hat{\lambda}$. By (4), (5), and (6), the social loss given λ is

$$L^s = \frac{1}{1 - \rho^2} \left[\left[\frac{\lambda}{\kappa^2 + (1 - \beta\rho)\lambda} \right]^2 + \lambda^s \left[\frac{\kappa}{\kappa^2 + (1 - \beta\rho)\lambda} \right]^2 \right].$$

The optimal weight on the output gap, denoted by λ^* , solves the first-order condition with respect to λ : $\frac{\partial L^s}{\partial \lambda} \Big|_{\lambda=\lambda^*} = 0$. Now, we have

$$\lambda^* = (1 - \beta\rho)\lambda^s. \quad (7)$$

³Under a discretionary policy regime, in each period, the central bank minimizes her period loss function with expected inflation rate $\mathbb{E}_t \pi_{t+1}$ given. For details of the notion of discretionary policy and a derivation of equilibrium dynamics, see Walsh (2010).

⁴Note that $\lim_{\beta \uparrow 1} (1 - \beta) \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda^s x_t^2) = V_\pi + \lambda^s V_x$. That is, the social loss function is an approximation of the discounted sum of the social period loss function.

⁵For detail, see Clarida et al. (1999).

Equation (7) shows that in the presence of the serial correlation of cost shocks (i.e., $\rho > 0$), it is optimal to delegate monetary policy to a person who is concern more about inflation stabilization than society. That is, conservative central bankers are desirable. Intuitively, inflation expectations are stabilized by an inflation-averse policymaker responding aggressively to future cost shocks, which reduces the welfare loss due to stabilization bias under discretion. This is the basic rationale for the optimality of central conservatism in the literature.

Optimal Delegation under Preference Uncertainty

Next, we examine the case of uncertain preferences. Suppose that the government cannot observe the true preference, represented by λ , of a central banker before implementing the monetary policy. However, the government knows that λ is uniformly distributed in the interval $[\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$ when it delegates monetary policy to a person with expected policy weight $\hat{\lambda}$.

Under such preference uncertainty, the government incorporates this and minimizes the expected social loss by choosing $\hat{\lambda}$ before the beginning of period 0. The government's optimization problem is to minimize $\mathbb{E}_\lambda[L^s|\hat{\lambda}]$. We can obtain the optimal solution of this problem, as follows:⁶

$$\hat{\lambda}^* = \frac{-\kappa^2 + (1 - \beta\rho)^2\lambda^s + \sqrt{[\kappa^2 + (1 - \beta\rho)^2\lambda^s]^2 + 4(1 - \beta\rho)^2\varepsilon^2}}{2(1 - \beta\rho)}. \quad (8)$$

The optimal solution in (8) is increasing in the degree of uncertainty, ε . Since $\hat{\lambda}^* = \lambda^*$ when $\varepsilon = 0$, $\hat{\lambda}^* > \lambda^*$ for any $\varepsilon > 0$. That is, under preference uncertainty, it is desirable to select a policymaker who seems more output-minded.⁷ This basic result directly reflects Tillmann's (2008) conclusion that excessive conservatism is more harmful than is too much liberalism. The harm of excessive conservatism stems from the targeting rule under discretionary policy:

$$\frac{x_t}{\pi_t} = -\frac{\kappa}{\lambda}.$$

When the central bank sets an unreasonably small λ , the marginal transformation rate between the output gap and the inflation rate becomes very large in this targeting rule. It enlarges the variance of the output gap inefficiently and, thus, reduces social welfare significantly.

The optimal weight $\hat{\lambda}^*$ balances the benefit of conservatism, which is explained in the case of certainty, and the cost of emphasizing the above risk under uncertainty. Then,

⁶Appendix provides the calculations.

⁷Sorge (2013) obtains the qualitatively same result as this under some parameter condition. Since Sorge (2013) incorporates Knightian Uncertainty about the central bank's preference, the optimal delegation problem is solved by the minimax method. The minimax solution is complicated and discontinuous with respect to the underlying parameter. For example, there is a case of the knife-edge in which both of appointing conservative and liberal central bankers are the optima. Thus, the simplicity and continuity of the solution is a fine feature of the model in this paper. In Section 3, I refer to a case where Sorge (2013)'s results are directly comparable to this paper's ones.

which central banker should the government appoint, conservative or liberal? Inequality $\hat{\lambda}^* \geq \lambda^s$ is the condition for optimality of liberalism. By (8), it is equivalent to

$$\varepsilon \geq \sqrt{\left(\frac{\kappa^2}{1 - \beta\rho} + \lambda^s\right)\beta\rho\lambda^s} \equiv \bar{\varepsilon}. \quad (9)$$

Unsurprisingly, (9) is likely to hold when preference uncertainty is strong.

The threshold $\bar{\varepsilon}$ depends on ρ and κ , which play important roles in monetary policy analysis. By (9), $\bar{\varepsilon}$ is increasing in both of them. Since the benefit of conservatism is large when the serial correlation of cost push shock is strong (see (7)), appointing a conservative central banker is optimal as long as preference uncertainty is not so strong. Thus, $\bar{\varepsilon}$ is increasing in ρ . When the New Keynesian Phillips curve is steep, the central bank needs to stabilize inflation hard because the inflation rate is sensitive to a change in the output gap. Therefore, the inefficiency from preference uncertainty is relatively small and the range of ε in (9) is also small.

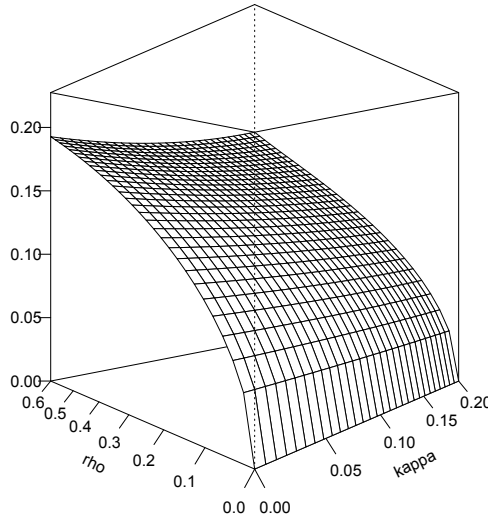


Figure 1: Sensitivity analysis of $\bar{\varepsilon}$ with respect to ρ and κ

I provide the result of a sensitivity analysis of $\bar{\varepsilon}$ in Figure 1. It visualizes $\bar{\varepsilon}$ as the function of (ρ, κ) for a sufficiently wide domain, using the standard parameter values of the simple New Keynesian models as shown in Table 1.⁸ For the benchmark parameter

⁸I adopt this parameter set from Walsh (2010)'s calibration. Here, I fix (λ^s, β) as in Table 1 and let (ρ, κ) be free.

Table 1: Baseline Parameter Value

λ^s	β	ρ	κ	ε
0.25	0.99	0.3	0.05	0.2

value in Table 1, $\bar{\varepsilon} \approx 0.1337$ and thus, the optimality of the liberal central banker is of true. ⁹ Since the benchmark value of (ρ, κ) is $(0.3, 0.05)$, Figure 1 shows that the result holds for a wide parameter set with plausible magnitude.

3 Discussion

Slope of Phillips Curves

The slope of the Phillips curve is one of the most important parameters because it captures the strength of the trade-off between inflation and output control. By (8), we can obtain $\frac{\partial \hat{\lambda}^*}{\partial \kappa} < 0$. Note that this is not true when preference uncertainty does not exist: see (7). This is because adjusting the policy weight λ using delegations brings only the stabilization effect on inflation expectations through the forward-looking behavior described by the New Keynesian Phillips curve. In contrast, when preference uncertainty exists, control errors of delegations can weaken output stabilization unreasonably and it generates the inefficient inflation volatility through the inflation-output trade-off. Such a risk is emphasized when the Phillips curve is steep. Thus, central bankers who are more conservative are desirable under the strong trade-off of monetary policy and preference uncertainty. ¹⁰ However, the empirical works on monetary economics suggest the opposite implication. As in Ball et al. (1988), Nishizaki and Watanabe (2000), and Roberts (2006), it is often reported that the Phillips curves have become flatter. The evidence on the slope of the Phillips curve is an argument supporting liberal central bankers according to the result of this model.

In the literature, a part of the main result of Sorge (2013) implies that when the Phillips curve is sufficiently flat, there exists a level of the degree of uncertainty which ensures the optimality of liberal central bankers for any degree of persistence in cost push shocks, ρ . In the model of this paper, one can find that the existence of such a degree of preference uncertainty depends on ρ because $\lim_{\kappa \rightarrow 0} \bar{\varepsilon} = \sqrt{\beta \rho} \lambda^s$ by (9). However, since plausible values of ρ are much smaller than 1 as the benchmark value, $\lim_{\kappa \rightarrow 0} \bar{\varepsilon} = \sqrt{\beta \rho} \lambda^s$ means that the range of ε in which appointing liberal central bankers is optimal is significantly large for given social preference, λ^s . While Sorge (2013)'s model with Knightian uncertainty has the discontinuous solution with respect to underlying parameter, the solution of the present model has continuity. Therefore, one can simply extend the above quantitative argument in this model for κ not close to 0: See Figure 1.

⁹Sorge (2013) uses the same parameter values. Because Sorge (2013) and the present study adopt different settings of uncertainty, I set the degree of uncertainty, ε , to a reasonable magnitude.

¹⁰In fact, $\frac{\partial}{\partial \varepsilon} \left(\frac{\partial \hat{\lambda}^*}{\partial \kappa} \right) < 0$ holds.

Monetary Policy by Committee

For the last two decades, over 100 central banks have formed monetary policy committees. Thus, delegating decision-making to legal committees is presently the usual style of monetary policy. This institutional change will affect the optimal delegation of monetary policy in the presence of preference uncertainty. Therefore, I extend the model in such a way that monetary policy is conducted by a committee.

Suppose that the government appoints N central bankers with expected preference $\hat{\lambda}$, and these members make up a monetary policy committee. Let λ_j be each member j 's preference, which is identically and independently drawn from the uniform distribution among $[\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$. To keep the model simple, assume that the decision-making process in the committee meeting is described by utilitarian bargaining. At each period t , given a set of publicly observable preferences $(\lambda_1, \lambda_2, \dots, \lambda_N)$, the committee implements a monetary policy to minimize the sum of all committee members' period loss functions:

$$\sum_{j=1}^N (\pi_t^2 + \lambda_j x_t^2),$$

subject to the New Keynesian Phillips curve

$$\pi_t + \kappa x_t = f_t, \quad (10)$$

where $f_t \equiv \beta \mathbb{E}_t \pi_{t+1} - e_t$ is a given state variable for the central bank because the policy regime is discretionary. The optimality condition is

$$\kappa \pi_t + \lambda x_t = 0, \quad (11)$$

where $\lambda \equiv \frac{1}{N} \sum_{j=1}^N \lambda_j$ is the policy weight that the committee chooses endogenously through bargaining. The equilibrium path is determined by (10) and (11) and is of the same form as (2) and (3).

As before, incorporating the above equilibrium dynamics, the government selects $\hat{\lambda}$ to minimize the expected value of the social loss function (4). In this extended model, the distribution of λ is complicated and we cannot solve the government's problem analytically. Therefore, I analyze the optimal weight $\hat{\lambda}^*$ numerically with parameter set of Table 1.

Figure 2 illustrates the relationship between the committee size N and the optimal weight $\hat{\lambda}^*$. When a single person implements a monetary policy ($N = 1$), the optimal weight on the output gap is larger than the society's weight. That is, liberal central bankers are optimal. In contrast, appointing conservative central bankers improves social welfare when the government appoints committees ($N \geq 2$). This is because preference aggregation in committee decision-making mitigates the detrimental effect of uncertainty and, thus, it is dominated by the benefit of reducing the stabilization bias by conservatism. Of course, actual decision-making processes in committees are complicated and probably have important aspects I abstract away here. However, the above result is significant because preference aggregation is the most basic and universal feature of collective decision-making. Furthermore, absorbing the risk of ex-post inefficient delegations is one of rationales for constituting monetary policy committees.

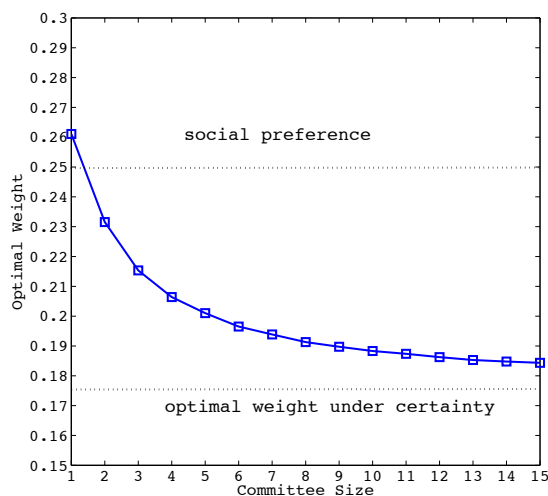


Figure 2: The relationship between committee size and optimal weight

Hefeker and Zimmer (2015) also address the optimal delegation in monetary policy committees under preference uncertainty in the New Keynesian framework. The most important difference from this paper is that uncertainty exists in not the policy weight but the target of output gap of each committee member. To mitigate the inefficient inflation variability from such preference shocks, it is optimal to appoint conservative committee members even when cost push shocks are serially uncorrelated. Thus, since the inefficiency from uncertainty shrinks as the committee size become larger, the optimal degree of conservatism is also decreasing (i.e., the optimal relative weight on output gap increases in this paper's setting) in the committee size. Remember that, in this paper's model, the benefit of liberalism due to preference uncertainty becomes smaller as the committee size increases. The difference in the source of the inefficiency brings the opposite result to this paper.

4 Conclusion

Preference uncertainty of central bankers is realistic and important for the optimal delegation problem of monetary policy. Although there is a benefit to output-minded monetary policy in the presence of preference uncertainty, it is not a decisive factor for the comparison between conservative and liberal central bankers alone. One of the channels through which uncertainty affects optimal delegation is the monetary policy trade-off, which is measured by the slope of the Phillips curve. However, decision-making by committees in actual central banks mitigates the harm of preference uncertainty significantly. A simple solution is to appoint potentially inflation-minded persons to committees.

References

- Ball, G., Mankiw, G. and D. Romer (1988), “The new Keynesian economics and the output-inflation tradeoff,” *Brookings Papers on Economic Activity* 1, 1–65.
- Beetsma, R. and H. Jensen (1998), “Inflation Targets and Contracts with Uncertain Central Banker Preferences,” *Journal of Money, Credit and Banking* 30, pp.384–403.
- Bernanke, B.S. (2004), “Remarks by Governor Ben S. Bernanke,” Speech given at the conference on Reflections on Monetary Policy 25 Years after October 1979, Federal Reserve Bank of St. Louis, October 8, 2004.
- Clarida, R., Galí, J. and M. Gertler (1999), “The Science of Monetary Policy: A New Keynesian Perspective,” *Journal of Economic Literature* 37, pp. 1661–1707.
- Hefeker, C. and B. Zimmer (2015), “Optimal Conservatism and Collective Monetary Policymaking under Uncertainty,” *Open Economies Review* 26, pp.259–278.
- Muscattelli, A. (1998), “Optimal Inflation Contracts and Inflation Targets with Uncertain Central Bank Preferences: Accountability through Independence?,” *Economic Journal* 108, pp.529–542.
- Nishizaki, K. and T. Watanabe (2000), “Output-Inflation Trade-off at Near-Zero Inflation Rates,” *Journal of the Japanese and International Economies* 14, 304–326.
- Roberts, J.M. (2006), “Monetary Policy and Inflation Dynamics,” *International Journal of Central Banking* 2, pp.193–230.
- Rogoff, K. (1985), “The Optimal Degree of Commitment to an Intermediate Monetary Target,” *Quarterly Journal of Economics* 100, pp. 1169–1189.
- Sorge, M. (2013), “Robust Delegation with Uncertain Monetary Policy Preferences,” *Economic Modelling* 30, pp. 73–78.
- Svensson, L. (1997), “Optimal Inflation Targets, ‘Conservative’ Central Banks, and Linear Inflation Contracts,” *American Economic Review* 87, pp. 1139–1153.
- Tillmann, P. (2008), “The Central Banker Revisited: Too Conservative is More Costly than too Liberal,” *European Journal of Political Economy* 28, pp. 737–741.
- Walsh, C. (1995), “Optimal Contracts for Central Bankers,” *American Economic Review* 85, pp. 150–167.
- Walsh, C. (2010), *Monetary Theory and Policy*, the 3rd ed., MIT Press.

Appendix

Here, I provide a brief derivation of the optimal weight, $\hat{\lambda}^*$. The expected social loss is

$$\mathbb{E}_\lambda[L^s|\hat{\lambda}] = \int_{\hat{\lambda}-\varepsilon}^{\hat{\lambda}+\varepsilon} (V_\pi + \lambda^s V_x) \frac{1}{2\varepsilon} d\lambda. \quad (12)$$

Let $t = \kappa^2 + (1 - \beta\rho)\lambda$. Then, $d\lambda = (1 - \beta\rho)^{-1}dt$ and the integral range becomes $[\kappa^2 + (1 - \beta\rho)(\hat{\lambda} - \varepsilon), \kappa^2 + (1 - \beta\rho)(\hat{\lambda} + \varepsilon)]$. Since $V_\pi = (1 - \rho^2)^{-1}[\frac{\lambda}{\kappa^2 + (1 - \beta\rho)\lambda}]^2$ and $V_x = (1 - \rho^2)^{-1}[\frac{\kappa}{\kappa^2 + (1 - \beta\rho)\lambda}]^2$, by this change of variables, we can obtain the expectation of inflation and output-gap variabilities with respect to λ :

$$\int_{\hat{\lambda}-\varepsilon}^{\hat{\lambda}+\varepsilon} V_\pi \frac{1}{2\varepsilon} d\lambda = (1 - \rho^2)^{-1}(1 - \beta\rho)^{-2} \left[1 - \kappa^2 \varepsilon^{-1} (1 - \beta\rho)^{-1} \ln \left(\frac{\kappa^2 + (1 - \beta\rho)(\hat{\lambda} + \varepsilon)}{\kappa^2 + (1 - \beta\rho)(\hat{\lambda} - \varepsilon)} \right) + \frac{\kappa^4}{[\kappa^2 + (1 - \beta\rho)(\hat{\lambda} + \varepsilon)][\kappa^2 + (1 - \beta\rho)(\hat{\lambda} - \varepsilon)]} \right], \quad (13)$$

$$\int_{\hat{\lambda}-\varepsilon}^{\hat{\lambda}+\varepsilon} V_x \frac{1}{2\varepsilon} d\lambda = \frac{\kappa^2}{[\kappa^2 + (1 - \beta\rho)(\hat{\lambda} + \varepsilon)][\kappa^2 + (1 - \beta\rho)(\hat{\lambda} - \varepsilon)]}. \quad (14)$$

Substituting (13) and (14) into (12), and taking the first-order condition, $\frac{\partial \mathbb{E}_\lambda[L^s|\hat{\lambda}]}{\partial \lambda} = 0$, we obtain the following quadratic equation of $\hat{\lambda}$, using long but straightforward calculations:

$$(1 - \beta\rho)\hat{\lambda}^2 + [\kappa^2 - (1 - \beta\rho)^2 \lambda^s] \hat{\lambda} - (1 - \beta\rho)(\kappa^2 \lambda^s + \varepsilon^2) = 0.$$

Choosing the positive root of the above equation, we obtain the optimal weight on the output-gap under preference uncertainty:

$$\hat{\lambda}^* = \frac{-\kappa^2 + (1 - \beta\rho)^2 \lambda^s + \sqrt{[\kappa^2 + (1 - \beta\rho)^2 \lambda^s]^2 + 4(1 - \beta\rho)^2 \varepsilon^2}}{2(1 - \beta\rho)}.$$