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# Monopolistic Marginal Cost Pricing 

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#### Abstract

It is usually argued that the monopolistic pricing distortion arises because "a monopoly can raise its price above marginal cost without losing all its clients" (Tirole, 1988). We discuss a simple well-behaved example in which: i) monopoly price gets as close as desired to marginal cost, and ii) nevertheless it is associated to a significant deadweight welfare loss.


## 1 Monopolistic Marginal Cost Pricing: An Example

Monopolistic pricing above marginal cost is the paradigmatic example of the distortions created by the presence of market power: see e.g. Tirole (1988: chapter 1). Yet, it is well known that if the firm can "price discriminate" perfectly it leads to no "dead-weight" welfare loss for society (Tirole, 1988: chapter 3). In this note we investigate the somehow polar case, that as far as we know has attracted no attention, in which (almost) no price distortion arises and yet society may suffer a significant welfare loss.

Consider the standard pricing problem for a monopolist (see Armstrong and Vickers, 2015 for the more general case of multiproduct Ramsey pricing), assuming that: i) consumers have quasi-linear preferences, which implies that demand can be generated by a single representative consumer with (differentiable) utility function $U(q)=u(q)+y$, where $y$ is the composite commodity and $u$ is strictly increasing and strictly concave with respect to the quantity $\left.q(u(0)=0) ;^{1} i i\right)$ the (differentiable) cost function $c(q)$ is strictly increasing and strictly convex $(c(0)=0)$. As is well known, an internal profit-maximizing solution must satisfy the first-order condition:

$$
\begin{equation*}
p(q)=c^{\prime}(q)+s^{\prime}(q) \tag{1}
\end{equation*}
$$

where $p(q)=u^{\prime}(q)$ is the inverse demand function and $s(q)=u(q)-u^{\prime}(q) q$ is consumer surplus, so that $s^{\prime}(q)=-u^{\prime \prime}(q) q>0$ for $q>0$ (profits are given by $\pi(q)=r(q)-c(q)$, where $r(q)=p(q) q$ is revenue, and the Marshallian social welfare is $W(q)=u(q)-c(q))$.

The way condition (1) is written immediately enlightens the fact that the distance between monopolistic pricing and first-best marginal cost pricing need not to be large. Of course, this is well known through the "inverse elasticity rule" which can be expressed by re-writing the previous condition as:

$$
\frac{p(q)-c^{\prime}(q)}{p(q)}=\frac{1}{\varepsilon(q)}
$$

where

$$
\varepsilon(q)=-\frac{p(q)}{p^{\prime}(q) q}=\frac{u^{\prime}(q)}{s^{\prime}(q)}
$$

is demand elasticity. Thus almost marginal cost pricing follows if $\varepsilon(q) \approx \infty$. However, demand elasticity is a somewhat more involved function with respect to $s^{\prime}(q)$, and it is easier to focus on the latter.

The purpose of this note is to illustrate the possibility of a welfare-costly monopolistic (almost) marginal-cost pricing by the following simple example. Suppose that:

$$
\begin{equation*}
u(q)=q+\frac{q^{\rho}}{\rho} \tag{2}
\end{equation*}
$$

[^0](see Bertoletti et al., 2007 for this functional form) and
\[

$$
\begin{equation*}
c(q)=q+\beta \frac{q^{2}}{2} \tag{3}
\end{equation*}
$$

\]

where $1>\rho>0$ and $\beta>0$. It is easy to see that (2)-(3) respect all previous assumptions and, in addition, imply that marginal revenue everywhere decreases while marginal cost increases from below unbounded, thus ensuring that (1) characterizes the unique solution. In particular, inverse demand, marginal revenue $\left(r^{\prime}(q)=p(q)+p^{\prime}(q) q\right)$ and marginal cost are given by

$$
p(q)=1+q^{\rho-1}, \quad r^{\prime}(q)=1+\rho q^{\rho-1}, \quad c^{\prime}(q)=1+\beta q
$$

while consumer surplus is such that

$$
s^{\prime}(q)=(1-\rho) q^{\rho-1}, \quad s^{\prime \prime}(q)<0
$$

and

$$
\lim _{q \rightarrow \infty} s^{\prime}(q)=0
$$

One immediately obtains from (1) that the monopoly quantity, price and profit as functions of the "supply-side" parameter $\beta$ are given by:

$$
\begin{gathered}
q(\beta)=\left(\frac{\rho}{\beta}\right)^{\frac{1}{2-\rho}}>0 \\
p(\beta)=1+\left(\frac{\rho}{\beta}\right)^{\frac{\rho-1}{2-\rho}}>1 \\
\pi(\beta)=\frac{2-\rho}{2}\left(\frac{\rho}{\beta}\right)^{\frac{\rho}{2-\rho}}>0
\end{gathered}
$$

It follows that $q^{\prime}(\beta), \pi^{\prime}(\beta)<0<p^{\prime}(\beta)$ and that as $\beta$ goes to zero the monopolistic price decreases and approaches monotonically and continuously the marginal cost. This well-behaved example in which $p(\beta)$ can be as close as one wishes to $c^{\prime}(q(\beta))$ is illustrated in Figure 1.

Notice that the (first-best) Pareto efficient quantity is given by $q^{e}(\beta)=$ $\beta^{\frac{1}{\rho-2}}$, and thus that the monopoly dead-weight loss $\left(W L=\int_{q(\beta)}^{q^{e}(\beta)}\left[p(x)-c^{\prime}(x)\right] d x\right)$

$$
W L(\beta)=\frac{(2-\rho) \beta^{\frac{\rho}{\rho-2}}}{2 \rho} \Psi(\rho)>0
$$

(where $\Psi=1-\frac{2-\rho^{2}}{2-\rho} \rho^{\frac{\rho}{2-\rho}}$ ) is such that $W L^{\prime}(\beta)<0$ : accordingly, a decrease of $\beta$ reduces the price distortion but in fact increases the dead-weight loss (this is due to the fact that it raises the distance $\left.q^{e}(\beta)-q(\beta)\right)$. However, also notice that

$$
\begin{aligned}
W^{e}(\beta) & =u\left(q^{e}(\beta)\right)-c\left(q^{e}(\beta)\right) \\
& =\frac{(2-\rho) \beta^{\frac{\rho}{\rho-2}}}{2 \rho}
\end{aligned}
$$



Figure 1
so that both the relative dead-weight loss $W L / W^{e}=\Psi \in\left(0, \frac{1}{10}\right)$ and the fraction $\pi / W^{e}=\rho^{\frac{2}{2-\rho}} \in(0,1)$ of the potential welfare captured by the monopolist are constant with respect to $\beta$ (see Tirole, 1988: p. 67). ${ }^{2}$

[^1]
## References

Armstrong, M. and J. Vickers (2015) "Multiproduct Pricing Made Simple", mimeo, University of Oxford.
Bertoletti, P., Fumagalli, E. and C. Poletti (2008) "Price-Increasing Monopolistic Competition? The Case of IES Preferences" IEFE working paper number 15, Bocconi University, Milan.
Tirole, J. (1988) The Theory of Industrial Organization, MIT Press: Cambridge (MA).


[^0]:    ${ }^{1}$ It is assumed in the following that $y>0$.

[^1]:    ${ }^{2}$ Figure 1 suggests that, at the cost of weakening the concavity of $u$, it would be possible to construct examples in which the inverse demand function becomes locally constant, so possibly inducing the monopolist to adopt a fully-fledged policy of marginal cost pricing. However, in those cases there would be no welfare loss.

