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A note on market power in bilateral oligopoly

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Abstract

In this note we consider a simple bilateral oligopoly model of an exchange economy. We characterize the Cournot-Nash equilibrium and we explore the effectiveness of market power. We provide some measures of relative market power. We show it depends on the relative size of the market and on the ratio of price elasticities of supply. Finally, we study free entry. We show it does not always lead to the competitive equilibrium market outcome.

This note is a piece of a more general research devoted to strategic interactions and general equilibrium. I am grateful to an anonymous referee for her/his helpful suggestions.
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1. INTRODUCTION

In the standard Cournot oligopoly model firms behave strategically since they perceive how their individual supply influences the market price. The market power of any firm is measured with the Lerner index, which is strictly positive in an oligopoly equilibrium. The market distortions can be eliminated notably by enlarging the economy in such a way the number of firms increases without limit, in which case the Lerner index becomes zero. Then, the aggregate supply, not the individual supply, and the market price coincide with their competitive equilibrium values (Frank (1965), Seade (1980), and Amir and Lambson (2000)). In this note, we wonder whether these features will hold when strategic interactions occur in interrelated markets. Three issues are dealt with. First, we characterize the general oligopoly equilibrium in terms of first-order conditions. Second, we provide some measures of market power when traders exchange more than one commodity. Third, we study free entry by replicating the economy.

We investigate these issues by building a bilateral oligopoly model in the spirit of the models developed by Gabszewicz and Michel (1997), Bloch and Ghosal (1997), and Bloch and Ferrer (2001). Our simple model constitutes an illustration and a two-commodity version of the Dubey and Shubik (1978), Sahi and Yao (1989) and Amir et al. (1990) models. In the bilateral oligopoly models, there are two divisible commodities and two types of traders, with a finite number of traders for each type. Each type has corner endowments but wants to consume both goods. There is market price mechanism which relies both sides of the market. This mechanism captures strategic interactions within each side and between both sides of the market (Dickson and Hartley (2011), Dickson (2013)). The Cournot-Nash equilibrium (CNE thereafter) is the market outcome. The existence of a CNE is beyond the scope of this note (see notably Cordella and Gabszewicz (1998), Busetto and Codognato (2006)). We rather propose to explore some properties of the bilateral oligopoly model by putting forward the market power at stake.

We first characterize the equilibrium conditions of the CNE with general assumptions regarding preferences. We notably consider heterogeneity among traders within each side and between both sides of the market. Then, we turn to the study of market power. To this end, we define two concepts, namely the *relative* Lerner indexes and the *relative* Herfindahl index. The first measures the market power between two traders who belong to the same side or to opposite side of the market. The second measures concentration in a two-sided market. These measures enable to capture some features of market power in a two-commodity economy. We also show that the relative indexes may coincide. Finally, we consider free entry. We replicate the economy without assuming that each side of the market grows symmetrically. Therefore, the convergence toward the competitive equilibrium depends on the way the economy is replicated. An example shows that the sequence of CNE price and strategies, can coincide in the limit with the competitive equilibrium price and individual strategies.

The paper is organized as follows. In section 2, we describe the model. The CNE is studied in Section 3. Section 4 defines two indexes of market power. Section 5 is devoted to free entry. In section 6 we provide an example. In Section 7 we conclude.

2. THE MODEL

Consider an economy with two divisible homogeneous commodities labeled 1 and 2. Let p_1 and p_2 be the corresponding unit prices. The economy also includes a finite set T of $n_1 + n_2$ traders which is partitioned into two subsets T_1 and T_2, with $T_1 \cap T_2 = \{\varnothing\}$. We assume $T_1 = \{1, ..., n_1\}$ where each trader is indexed by i, and $T_2 = \{1, ..., n_2\}$, where each trader is indexed by j. There are fixed initial endowments which satisfies the following assumption.

Assumption 1. $\mathbf{w}^i = (w_1^i, 0)$, with $w_1^i > 0$, for each $i \in T_1$, and $\mathbf{w}^j = (0, w_2^j)$, with $w_2^j > 0$ for each $j \in T_2$.

The preferences of each trader $i \in T_1$ (resp. $j \in T_2$) are described by a utility function $U^i : \mathbb{R}_+^2 \to \mathbb{R}$ (resp. $U^j : \mathbb{R}_+^2 \to \mathbb{R}$), satisfying the following assumptions.

Assumption 2. For each $i \in T_1$ (resp. $j \in T_2$), the utility function U^i (resp. U^j) is twice-continuously differentiable, strictly monotonic and quasi-concave.

To this exchange economy we associate a strategic market game $\mathbf{\Gamma}$.

Assumption 3. For each $i \in T_1$, $B^i = \left\{b_1^i \in \mathbb{R}_+ : 0 \leqslant b_1^i \leqslant w_1^i\right\}$ and for all $j \in T_2$, $B^j = \left\{b_2^j \in \mathbb{R}_+ : 0 \leqslant b_2^j \leqslant w_2^j\right\}$.

The quantity b_1^i denotes the pure strategy of any trader $i \in T_1$ and represents the amount of commodity 1 she sells on the market. Equivalently, b_2^j is the pure strategy of trader $j \in T_2$. There is a trading post which specifies the relative price at which exchange occurs. Given a price system $\mathbf{p} = (p_1, p_2)$ and a strategy profile $\mathbf{b} = (b_1^1, ..., b_1^{n_1}; b_2^1, ..., b_2^{n_2})$, with $\mathbf{b} \in \prod_i^i B^i \times \prod_j B^j$, the market clearing price $\frac{p_1}{p_2}(\mathbf{b})$ obtains as:

$$\frac{p_1}{p_2}(\mathbf{b}) = \frac{\sum_{j=1}^{n_2} b_2^j}{\sum_{i=1}^{n_1} b_1^i}. \tag{1}$$

So, the resulting allocation of commodities are:

$$\begin{aligned} (x_1^i, x_2^i) &= \left(w_1^i - b_1^i, \frac{\sum_{j=1}^{n_2} b_2^j}{\sum_{k=1}^{n_1} b_1^k} b_1^i\right), \; i \in T_1 \\ (x_1^j, x_2^j) &= \left(\frac{\sum_{i=1}^{n_1} b_1^i}{\sum_{k=1}^{n_2} b_2^k} b_2^j, w_2^j - b_2^j\right), \; j \in T_2. \end{aligned} \tag{2}$$

The corresponding utility levels may be written as payoffs:

$$\begin{aligned} V^i(\mathbf{b}) &= U^i\left(w_1^i - b_1^i, \frac{\sum_{j=1}^{n_2} b_2^j}{\sum_{k=1}^{n_1} b_1^k} b_1^i\right), \; i \in T_1 \\ V^j(\mathbf{b}) &= U^j\left(\frac{\sum_{i=1}^{n_1} b_1^i}{\sum_{k=1}^{n_2} b_2^k} b_2^j, w_2^j - b_2^j\right), \; j \in T_2. \end{aligned} \tag{3}$$

3. COURNOT-NASH EQUILIBRIUM

Definition 1. A pair $(\tilde{\mathbf{b}}, \tilde{\mathbf{x}})$, where $\tilde{\mathbf{b}} = (\tilde{b}_1^1, ..., \tilde{b}_1^{n_1}; \tilde{b}_2^1, ..., \tilde{b}_2^{n_2})$ is a strategy profile, with $\tilde{\mathbf{b}} \in \prod_i B^i \times \prod_j B^j$, and $\tilde{\mathbf{x}}$ is an allocation such that $\tilde{\mathbf{x}}^i = \mathbf{x}^i(\tilde{b}_1^i, p(\tilde{\mathbf{b}}))$, for $i \in T_1$, and $\tilde{\mathbf{x}}^j(t) = \mathbf{x}^j(\tilde{b}_2^j, p(\tilde{\mathbf{b}}))$, for $j \in T_2$, constitutes a Cournot-Nash equilibrium of $\mathbf{\Gamma}$, with respect to a price system $\mathbf{p}(\tilde{\mathbf{b}})$, if:
a. $U^i(\mathbf{x}^i(\tilde{b}_1^i, \mathbf{p}(\tilde{\mathbf{b}}))) \geqslant U^i(\mathbf{x}^i(b_1^i, \mathbf{p}(b_1^i, \tilde{\mathbf{b}}^{-i}))), \forall b_1^i \in B^i, i \in T_1$
b. $U^j(\mathbf{x}^j(\tilde{b}_2^j, \mathbf{p}(\tilde{\mathbf{b}}))) \geqslant U^j(\mathbf{x}^j(b_2^j, \mathbf{p}(b_2^j, \tilde{\mathbf{b}}^{-j}))), \forall b_2^j \in B^j, j \in T_2$.

Proposition 1. *If $(\tilde{\mathbf{b}}, \tilde{\mathbf{x}})$ is a Cournot-Nash equilibrium of the game $\mathbf{\Gamma}$, then:*

$$\frac{p_1}{p_2}(\tilde{\mathbf{b}})\left(1 - s_1^i\right) = MRS_{1/2}^i(\tilde{\mathbf{x}}^i),\ i \in T_1 \tag{C1}$$

$$\frac{p_2}{p_1}(\tilde{\mathbf{b}})(1 - s_2^j) = MRS_{2/1}^j(\tilde{\mathbf{x}}^j),\ j \in T_2 \tag{C2}$$

where MRS and s represent the marginal rate of substitution and the market share.

Proof. The traders' programs are solutions to:

$$\max_{b_1^i \in [0, w_1^i]} U^i\left(w_1^i - b_1^i, \frac{p_1}{p_2}(\mathbf{b})\, b_1^i\right),\ i \in T_1 \tag{4}$$

$$\max_{b_2^j \in [0, w_2^j]} U^j\left(\frac{p_2}{p_1}(\mathbf{b})\, b_2^j, w_2^j - b_2^j\right),\ j \in T_2. \tag{5}$$

Differentiating (4) with respect to b_1^i, and (5) with respect to b_2^j, leads respectively to the first-order conditions:

$$\frac{p_1}{p_2}(\tilde{\mathbf{b}})\left(1 + \frac{s^i}{\epsilon_1}\right) = \frac{\frac{\partial U^i}{\partial x_1^i}}{\frac{\partial U^i}{\partial x_2^i}}\left(w_1^i - \tilde{b}_1^i, \left(\frac{\tilde{p}_1}{p_2}\right)\tilde{b}_1^i\right),\ i \in T_1 \tag{6}$$

$$\frac{p_2}{p_1}(\tilde{\mathbf{b}})\left(1 + \frac{s^j}{\epsilon_2}\right) = \frac{\frac{\partial U^j}{\partial x_2^j}}{\frac{\partial U^j}{\partial x_1^j}}\left(\left(\frac{\tilde{p}_2}{p_1}\right)\tilde{b}_2^j, w_2^j - \tilde{b}_2^j\right),\ j \in T_2 \tag{7}$$

where $\bar{\mathbf{b}}_1 \equiv \sum_i b_1^i$ and $\bar{\mathbf{b}}_2 \equiv \sum_j b_2^j$, with $\frac{\partial \bar{\mathbf{b}}_1}{\partial b_1^i} = 1$ and $\frac{\partial \bar{\mathbf{b}}_2}{\partial b_2^j} = 1$, and where $\epsilon_1 := \frac{d \log \bar{\mathbf{b}}_1}{d \log(p_1/p_2)} = -1$ and $\epsilon_2 := \frac{d \log \bar{\mathbf{b}}_2}{d \log(p_2/p_1)} = -1$ are the price elasticity of supply of good 1 and good 2 respectively. ∎

Therefore, (C1) and (C2) state that, in equilibrium, the marginal revenue in real terms equals the marginal rate of substitution of any trader. The strategic behavior of each trader consists in contracting her supply to manipulate the rate of exchange. The markup on MRS increases with the market share of any trader. These conditions share some similarities with those obtained in partial equilibrium models. The optimal conditions say that the markup on the rate of tradeoff between the two goods (a real opportunity cost) depends positively on the trader's market share, but also on the price elasticity of supply. But they differ since they are linked to a market price mechanism which explain how relative price is formed. Here no specific assumption is made regarding the market demand function since the price is determined through the aggregate strategic supplies of traders.

Remark 1. If $(\tilde{\mathbf{b}}, \tilde{\mathbf{x}})$ is a symmetric CNE, then $\frac{p_1}{p_2}(\tilde{\mathbf{b}})\left(1 - \frac{1}{n_1}\right) = MRS_{1/2}(\tilde{\mathbf{x}}^i)$, $i \in T_1$ and $\frac{p_2}{p_1}(\tilde{\mathbf{b}})\left(1 - \frac{1}{n_2}\right) = MRS_{2/1}(\tilde{\mathbf{x}}^j)$, $j \in T_2$.

4. MARKET POWER

Let L_1^i be the Lerner index of trader $i \in T_1$, with $L_1^i \equiv \frac{\frac{p_1}{p_2} - MRS_{1/2}^i}{\frac{p_1}{p_2}}$.

Corollary 1. *If $(\tilde{\mathbf{b}}, \tilde{\mathbf{x}})$ is a CNE of the game $\mathbf{\Gamma}$, then:*

$$L_1^i = s_1^i,\ i \in T_1 \tag{C1'}$$

$$L_2^j = s_2^j,\ j \in T_2. \tag{C2'}$$

Conditions (C1)' and (C2)' provide an expression for the market power caused by imperfectly competitive behavior. In a symmetric equilibrium (the same utility function and endowments within each sector), one has: $L_1^i = \frac{1}{n_1}$, $i \in T_1$, and $L_2^j = \frac{1}{n_2}$, $j \in T_2$.

Definition 2. Let L_1^i, $i \in T_1$ and L_2^j, $j \in T_2$. The relative Lerner index $l_{i,i'}$ of trader $i \in T_1$ with respect to trader $i' \in T_1$ is given by $l_{i,i'} \equiv \frac{L_1^i}{L_1^{i'}}$, i, $i' \in T_1$. Equivalently, the relative Lerner index of trader $i \in T_1$ with respect to trader $j \in T_2$ is given by $l_{i,j} \equiv \frac{L_1^i}{L_2^j}$, $i \in T_1$, $j \in T_2$.

Proposition 2. *Assume both commodities are substitutes. In a CNE, the relative market power of any trader decreases (increases) with the strategy of any trader who belongs to the same (other) side of the market (when the price elasticity of individual supply exceeds unity).*

Proof. From Definition 2, i.e., $l_{i,i'} \equiv \frac{L_1^i}{L_1^{i'}}$, $i, i' \in T_1$, one gets $\frac{\partial l_{i,i'}}{\partial \tilde{b}_1^{i'}} = -\frac{\tilde{b}_1^i}{(\tilde{b}_1^{i'})^2} < 0$. In addition, $\frac{\partial l_{i,j}}{\partial \tilde{b}_2^j} = \frac{\tilde{b}_1^i}{(\tilde{b}_2^j)^2}\left(\frac{\partial\left(\frac{\tilde{p}_1}{p_2}\right)}{\partial \tilde{b}_2^j}\tilde{b}_2^j - \left(\frac{\tilde{p}_1}{p_2}\right)\right)$. Therefore, $\frac{\partial l_{i,j}}{\partial \tilde{b}_2^j} = \left(\xi^j - 1\right)\frac{1}{\tilde{b}_2^j} l_{i,j}$, where $\xi^j \equiv \frac{\partial\left(\frac{\tilde{p}_1}{p_2}\right)}{\partial \tilde{b}_2^j}\tilde{b}_2^j\left(\frac{\tilde{p}_2}{p_1}\right)$, $j \in T_2$, with $\frac{\partial\left(\frac{\tilde{p}_1}{p_2}\right)}{\partial \tilde{b}_2^j} > 0$ by assumption. ∎

The intersectoral effect puts forward that the increase in price must be sufficiently important for the relative market power to increase. It requires that strategies between both sides of the market must be complements, which stems from assuming the substituability of commodities (see Bloch and Ferrer (2001)).

Definition 3. Let H_1 and H_2 be the Herfindahl indexes within each side of the market. The relative concentration index $h_{1,2}$ between the two sides of the market is given by $h_{1,2} \equiv \frac{H_1}{H_2}$, $h_{1,2} > 0$.

Proposition 3. *In a symmetric CNE: $l_{i,j} = h_{1,2}$, $i \in T_1$, $j \in T_2$.*

Proof. In a symmetric CNE: $L_1^i = \frac{1}{n_1}$, $i \in T_1$ and $L_2^j = \frac{1}{n_2}$, $j \in T_2$. We deduce $l_{i,j} = \frac{n_2}{n_1}$, $i \in T_1$ or $l_{ji} = \frac{n_1}{n_2}$, $j \in T_2$. In addition, we have $H_1 \equiv \sum_i (s_1^i)^2$ and $H_2 \equiv \sum_j (s_2^j)^2$. So, we deduce $\sum_i s_1^i L_1^i = \frac{1}{n_1}$ and $\sum_j s_2^j L_2^j = \frac{1}{n_2}$. Then $h_{1,2} = \frac{n_2}{n_1}$. ∎

The measures of market power and market concentration are equivalent whenever each side of the market embodies symmetric behavior and the market price formation given by (1) provides unitary price elasticities of supply. The relative Herfindahl index puts forward that the degree of market concentration is critically linked to the relative size of the market. When the two sectors are concentrated in the same way, then $h_{1,2} = 1$, so $H_1 = H_2$. But, since the degree of competition is relative, this means that when one side of the market is thicker, it can benefit to the traders on the other side of the market.

5. FREE ENTRY

We perform a replication of the basic economy. Let r_1 and r_2 be two integers, with $r_1 \geqslant 1$ and $r_2 \geqslant 1$. We let the possibility that the market may not be enlarged in the same way. The new game $\Gamma(r_1, r_2)$ now embodies $r_1 n_1 + r_2 n_2$ traders, with r_1 traders $i \in T_1$, each being indexed by ik_1 for $k_1 = 1, ..., r_1$, and r_2 traders $j \in T_2$, each being indexed by jk_2 for $k_2 = 1, ..., r_2$. Assumptions 1-4 still hold. Therefore, a CNE for the replicated game $\Gamma(r_1, r_2)$ is now given by the $(r_1 n_1 + r_2 n_2)$-tuple of strategies $(\tilde{b}_1^{11}, \tilde{b}_1^{12}, ..., \tilde{b}_1^{n_1 r_1}; \tilde{b}_2^{11}, \tilde{b}_2^{12} ..., \tilde{b}_2^{n_2 r_2})$.

PROPOSITION 4. *If $r_1 \to \infty$ and $r_2 \to \infty$, then the symmetric Cournot-Nash equilibrium of the replicated game coincides with the competitive equilibrium of the market game.*

Proof. The market price is now given by:

$$\frac{p_1}{p_2}\left(\mathbf{b}(r_1, r_2)\right) = \frac{\sum_{k_2=1}^{r_2} \sum_{j=1}^{n_2} b_2^{jk_2}}{\sum_{k_1=1}^{r_1} \sum_{i=1}^{n_1} b_1^{ik_1}}. \tag{8}$$

Using the same procedure as for the Proof of Proposition 1, one obtains the following first-order conditions in a symmetric CNE:

$$L_1^{ik_1} = \frac{1}{r_1 n_1}, \text{ where } L_1^{ik_1} \equiv \frac{\frac{p_1}{p_2} - MRS_{1/2}^{ik_1}\left(\tilde{\mathbf{x}}^{ik_1}\right)}{\frac{p_1}{p_2}}, \quad k_1 = 1, ..., r_1, \; i \in T_1 \tag{9}$$

$$L_2^{jk_2} = \frac{1}{r_2 n_2}, \text{ where } L_1^{ik_1} \equiv \frac{\frac{p_2}{p_1} - MRS_{2/1}^{jk_2}(\tilde{\mathbf{x}}^{jk})}{\frac{p_2}{p_1}}, \; k_2 = 1, ..., r_2, \; j \in T_2. \tag{10}$$

First, let $r_1 \to \infty$ and $r_2 \to \infty$. Then from (13) and (14), we get $\lim_{r_1 \to \infty} L_1^{ik_1} = 0$, $k_1 = 1, ..., r_1$, $i \in T_1$, and $\lim_{r_2 \to \infty} L_2^{jk_2} = 0$, $k_2 = 1, ..., r_2$, $j \in T_2$. Therefore, from (19) and (20), one gets $\frac{p_1}{p_2} = MRS_{1/2}^{ik_1}$, $k_1 = 1, ..., r_1$, $i \in T_1$, and $\frac{p_2}{p_1}\left(1 - s_2^{jk_2}\right) = MRS_{2/1}^{jk_2}$, $k_2 = 1, ..., r_2$, $j \in T_2$. ∎

Proposition 5 shows that when the game is replicated an infinite number of times, the market outcome (price and aggregate supply) becomes competitive. Whilst this result is not new (see notably Amir and Bloch (2009), Dickson (2013)), the way we replicate the economy differs from the literature since we allow for considering "asymmetric" replication. This motivates the next corollary.

COROLLARY 2. *Let $r_1 \to \infty$ or $r_2 \to \infty$. Then the symmetric CNE coincides with the Cournot-Walras equilibrium of the market game.*

Proof. Immediate from (9) and (10). ∎

In case the economy is enlarged asymmetrically, the traders belonging to the first side of the market behave as oligopolists while the traders on the opposite side behave as price takers (see the Cobb-Douglas examples in Gabszewicz (2002)). Our result holds without assuming specific utility functions.

6. AN EXAMPLE

In this example Assumption 1 is $\mathbf{w}^i = \left(\frac{1}{n_1}, 0\right)$, $i \in T_1$ and $\mathbf{w}^j = \left(0, \frac{1}{n_2}\right)$, $j \in T_2$. Assumption 2 is $U^i(x_1^i, x_2^i) = \sqrt{x_1^i} + \sqrt{x_2^i}$, $i \in T_1$ and $U^j(x_1^j, x_2^j) = x_1^j . x_2^j$, $j \in T_2$. Assumption 3 is $0 \leqslant b_1^i \leq \frac{1}{n_1}$, $i \in T_1$, and $0 \leqslant b_2^j \leqslant \frac{1}{n_2}$, $j \in T_2$. The competitive equilibrium is given by $\left(\frac{p_1}{p_2}\right)^* = 1$ and $((x_1^i)^*, (x_2^i)^*) = \left(\frac{1}{2n_1}, \frac{1}{2n_1}\right)$, $i \in T_1$ and $((x_1^j)^*, (x_2^j)^*) = \left(\frac{1}{2n_2}, \frac{1}{2n_2}\right)$, $j \in T_2$.

The equilibrium strategies profile obtains as the solution to:

$$\max_{b_1^i \in [0, \frac{1}{n_1}]} \sqrt{\frac{1}{n_1} - b_1^i} + \sqrt{\frac{\sum_{j=1}^{n_2} b_2^j}{b_1^i + \bar{\mathbf{b}}_1^{-i}} b_1^i}, \; i \in T_1 \tag{11}$$

$$\max_{b_2^j \in [0, \frac{1}{n_2}]} \frac{\sum_{i=1}^{n_1} b_1^i}{b_2^j + \bar{\mathbf{b}}_2^{-j}} b_2^j . \left(\frac{1}{n_2} - b_2^j\right), \; j \in T_2. \tag{12}$$

The equilibrium strategies for $i \in T_1$ and $j \in T_2$, and the market price are:

$$\tilde{b}_1^i = \frac{1}{2n_1}\left(\frac{n_1 - 1}{n_1}\right)^2 \frac{n_2 - 1}{2n_2 - 1}\left(\sqrt{1 + 4\left(\frac{n_1}{n_1 - 1}\right)^2 \frac{2n_2 - 1}{n_2 - 1}} - 1\right) \tag{13}$$

$$\tilde{b}_2^j = \frac{1}{n_2}\frac{n_2 - 1}{2n_2 - 1}. \tag{14}$$

$$\left(\frac{\tilde{p}_1}{p_2}\right) = \frac{1}{2}\frac{n_2 - 1}{2n_2 - 1}\left(\sqrt{1 + 4\left(\frac{n_1}{n_1 - 1}\right)^2 \frac{2n_2 - 1}{n_2 - 1}} + 1\right). \tag{15}$$

When $n_1 \to \infty$ and $n_2 \to \infty$, $\lim \left(\frac{\tilde{p}_1}{p_2}\right) = 1$, but $\lim \tilde{b}_1^{ik_1}(r_1, r_2) \neq (b_1^i)^*$, $k_1 = 1, ..., r_1$, $i \in T_1$, and $\lim \tilde{b}_2^{jk_2}(r_1, r_2) \neq (b_2^j)^*$, $k_2 = 1, ..., r_2$, $j \in T_2$. The replication yields the following equilibrium strategies and market price:

$$\tilde{b}_1^{ik_1} = \frac{\frac{r_2}{r_1}\left(\frac{r_1 n_1 - 1}{r_1 n_1}\right)^2 \frac{r_2 n_2 - 1}{2r_2 n_2 - 1}}{2n_1}\left(\sqrt{1 + \frac{4r_1}{r_2}\left(\frac{r_1 n_1}{r_1 n_1 - 1}\right)^2 \frac{2r_2 n_2 - 1}{r_2 n_2 - 1}} - 1\right) \tag{16}$$

$$\tilde{b}_2^{jk_2} = \frac{1}{n_2}\frac{r_2 n_2 - 1}{2r_2 n_2 - 1}. \tag{17}$$

$$\left(\frac{\tilde{p}_1}{p_2}\right) = \frac{1}{2}\frac{r_2 n_2 - 1}{2r_2 n_2 - 1}\left(\sqrt{1 + \frac{4r_1}{r_2}\left(\frac{r_1 n_1}{r_1 n_1 - 1}\right)^2 \frac{2r_2 n_2 - 1}{r_2 n_2 - 1}} + 1\right). \tag{18}$$

When $r_1 \to \infty$ and $r_2 \to \infty$, $\lim \left(\frac{\tilde{p}_1}{p_2}\right) = 1$ and $\lim \tilde{b}_1^{ik_1}(r_1, r_2) = \frac{1}{2n_1} = (b_1^i)^*$, $k_1 = 1, ..., r_1$, $i \in T_1$, and $\lim \tilde{b}_2^{jk_2}(r_1, r_2) = \frac{1}{2n_2} = (b_2^j)^*$, $k_2 = 1, ..., r_2$, $j \in T_2$. This result is obviously no longer true when the number of traders increases without limit on only one side of the market.

7. CONCLUSION

We propose a simple model with general assumptions regarding preferences and with heterogeneity within and between both sides of the market. First, the market power is relative. Second, when the market is enlarged, the competitive equilibrium is not always reached.

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