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Interpreting the concept of representational inequality to reckon between-group inequality for different types of data

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Abstract

Inter-group disparity is usually reckoned by using some conventional summary measures, which take into account the distributional characteristics of the underlying cardinally measurable attributes across groups. However, for categorical data these conventional summary measures cannot be applied in a meaningful way to assess between-group inequality. This note reintroduces the concept of 'representational inequality' (RI) presented by Reddy and Jayadev (2011a and 2011b) and develops some measures of such inequality, which are shown to produce meaningful results if applied to either cardinal or ordinal data. The empirical illustration of the developed measures of between-group inequality based on the concept of RI is then provided using some data set from India.

Representational Inequality
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1. Introduction

It is rather commonplace to observe that populations in different societies are not homogeneous in terms of social, geographical and personal factors, such as caste, race, ethnicity, religion, sector, gender, region of birth, and so on. While each individual is seen as the site of multiple identities, at the societal level the population is usually partitioned into mutually exclusive groups in accordance with each of the indicators of identities. The groups usually differ in terms of their well-being achievements. Relative disparities in achievements are often the concerns of the policy makers since sharpening disparities have the potential for creating political conflicts. The inequality in well-being achievement and/or deprivation across well-defined groups is usually referred to as 'horizontal' inequality. Besides being considered intrinsically bad from ethical standpoint, horizontal inequality has also been seen as having negative consequences on social coherence and peace (Stewart et al., 2005; Subramanian, 2009). An important step towards addressing such inequality is to develop a meaningful measure to assess the extent of inequality in a society.

There are several methods to measure horizontal inequality. It can be assessed directly by applying some summary measures commonly used to measure interpersonal inequality, such as group Gini coefficient, group coefficient of variation, etc. Inter-group inequality could alternatively be reckoned as a component of total or interpersonal inequality by using the subgroup decomposable measures.[1] All the measures reckon between-group inequality in terms of a scalar value representation of the distributional characteristics of the attribute across well-defined groups. One limitation of these measures is that their aggregative nature does not allow us to form any idea about group specific 'relative advantages and disadvantages'. Besides, these conventional summary measures are not well-suited for categorical data comprising ordered or permutable classes (Allison and Foster, 2004; Naga and Yalchin, 2008; Zheng, 2006).

The concept of 'representational inequality' (RI) introduced by Reddy and Jayadev (2011a) shows a conceptually rich way of assessing inter-group inequality, which differs from the conventional measures of between-group inequality. One advantage of this concept is that it enables us to focus on group-specific 'relative advantages and disadvantages', since the overall group inequality is measured as an aggregate of such advantages and disadvantages. 'Representational inequality' actually measures between group inequality in terms of the extent of the unequal sharing of the attribute by the members of the distinct groups in the society (Reddy and Jayadev, 2011a and 2011b; Subramanian, 2001 and 2011).

For assessing between-group inequality based on the concept of RI, one has to compare the proportional representations of the groups (measured in terms of the population proportions) at different categories of the attribute. It is hypothesized that if the groups are represented unequally at different categories, then between-group inequality exists in the society. Therefore, in order to apply this concept for cardinal data some categories or brackets on the distribution of the attribute should be pre-defined and unequal representation would then imply the inequality between the population shares of the groups and shares of the attribute by the respective groups. Hence, the cardinal nature of the data is suppressed when between-group inequality is assessed by the concept of RI in cardinal setting. For categorical data comprising order or permutable classes the concept of RI assesses horizontal inequality in terms of the disproportionate representations of the groups at different categories of the attribute. Thus, an important aspect

1. An additive subgroup decomposable measure reckons total inequality as the sum of between-group and within-group components.

of this concept is that it can be applied to measure between-group inequality for both cardinal and categorical data.

This paper first reviews some existing measures of RI suitable for cardinal data, presented in Jayaraj and Subramanian (2006; henceforth JS), then makes an attempt to develop an alternative measure of RI, which takes into account the problem of comparability due to the sensitivity of those measures to changes in population composition. Furthermore, the paper suggests a measure of RI suitable for categorical data, which is applicable for any number of identity and well-being categories. A second objective of this study is to apply these modified measures of RI to Indian data to evaluate group disparities in cardinal (consumer expenditure) and ordinal (educational status) dimensions of well-being in rural and urban areas of India.

The method of construction of a comparable and normalized index suitable for cardinal data is described in the next section. In section three we discuss the method to formulate some measures of RI compatible for the categorical data and applicable for any number of categories of the variable and identity groups. Irrespective of the nature of the data the RI measures developed in this study are based on some unique measurement approach, *i.e.*, these measures are based on the absolute deviations of the groups' realizations from a reference value. In case of cardinal data, the reference value is the population mean of the variable, and in case of categorical data the reference value is the expected or normative representations of the identity groups at different categories of the variable. Section four presents some empirical illustrations of the developed measures of 'representational inequality'.

2. Representational inequality with cardinally measurable data

JS have presented three society-wide indices of horizontal inequality in the cardinal setting, which are based on the concept of 'representational inequality'. These society-wide indices assess between-group inequality in terms of the disproportionate sharing of the attribute (for instance, income) by the members of distinct groups. The society-wide indices are the weighted sum of the group-specific indices of 'relative advantages and disadvantages', where weights are the population shares of the groups. This engenders the sensitivity of these indices to the population composition of the groups. Consequently, the difference in the degrees of horizontal inequality assessed by each of the society-wide indices in different settings might largely be due to the differences in the population composition of the groups rather than the differences between income shares of the groups, or differences in groups' mean incomes. So, the index values do not allow us to make meaningful comparisons between societies (Elbers et al., 2008). Therefore, we propose some modification of one of the society-wide indices of RI presented in JS by introducing a new term (maximum possible RI) in its denominator. This modification actually normalizes the measure by the relative sizes of the groups. The maximum possible RI is computed by considering the distributions across groups as non-overlapping with the original distribution of the attribute.[2] This modified measure represents the percentage contribution of the observed RI to its maximum possible value, and rises with the rise in observed RI.

Let the distribution of a cardinally measurable (income or expenditure) attribute be represented by the vector $\boldsymbol{x} = (x_1, x_2, \dots, x_N)$, where N is the population size and x_i is the income of the *i*th person. If the population is partitioned into K well-defined groups ($K \geq 2$), then μ, μ_j and n_j are the population mean of the attribute, mean of the attribute of the *j*th group and the population

2. In Appendix II, we discuss the importance of non-overlapping distribution of groups in conceptualizing the reference situation of maximal RI and describe the method of estimation of the 'maximum possible group inequality'.

size of the jth group. So, the population and income shares of the jth group are λ_j and θ_j, where $\lambda_j = (n_j/n)$ and $\theta_j = (n_j/n) \times (\mu_j/\mu)$ for j = 1, 2,…,K.

We take the first society-wide index of between-group inequality suggested in JS, and modify it to make it suitable for meaningful comparisons. The form of the first society-wide index is:

$$D = \sum_{j=1}^{K} \lambda_j \left|(\lambda_j - \theta_j)/\lambda_j\right| = \frac{1}{\mu} \sum_{j=1}^{K} (n_j/n) \left|\mu_j - \mu\right| \qquad (1)$$

This society-wide index is the population share weighted value of the absolute deviation of the groups' means from the population mean. It assesses RI on the basis of the deviation of the population shares of the groups from the income shares. The society-wide index (1) is constructed from the following group-specific index of relative advantage (or disadvantage):

$$\delta_j = (\lambda_j - \theta_j)/\lambda_j \qquad (2)$$

After modifying the index (1) introducing the maximum possible between-group inequality (measured by RI) in the denominator, the form of (1) becomes:

$$I_{RI}^{C} = \frac{\text{Observed between group inequality by } RI}{\text{Maximum possible between group inequality by } RI} = \frac{\sum_{j=1}^{K} \lambda_j \left|(\lambda_j - \theta_j)/\lambda_j\right|}{\sum_{j=1}^{K} \lambda_j \left|(\lambda_j - \theta_j^*)/\lambda_j\right|} \qquad (3)$$

Where $\theta_j^* = (n_j/n) \times (\mu_j^*/\mu)$ and μ_j^* is the mean outcome of the jth group, when the groups are completely segregated and non-overlapping and are ordered according to the original group means.[3] Incorporating θ_j^* in the denominator of (3), we can estimate the contribution of observed between-group inequality to the maximum possible between-group inequality.

The index (1) is the aggregated form of the group-specific advantages and disadvantages. A group is advantaged (disadvantaged) if population share of the group (λ_j) is smaller (or greater) than the proportion of possessing the attribute (θ_j). This classification is possible if the groups are unequally represented at different brackets of the attribute. If groups are equally represented at all categories of the attribute, then $\mu_j = \mu$, $\lambda_j = \theta_j$, and the value of δ_j for all groups is equal to 0, and the groups are neither advantaged nor disadvantaged. Any deviation from this situation would imply 'representational inequality'.

The index (3) is a measure of horizontal inequality based on the concept of RI. It takes the value 0 (its minimum value) when there is no disproportionate sharing of the attribute by the members of the groups and it takes the value 1 when horizontal inequality reaches its maximum. The value of this index I_{RI}^{C} is invariant with the permutations of the group identities which are associated with the individual attributes. Its value is also invariant with the replication of each individual by a given proportion, replication of every individual in different groups by different rates and replication of the groups. Thus, this index satisfies the axiomatic properties, such as, *within group anonymity, group identity anonymity, scale invariance, total population size invariance, population composition invariance, group replication invariance*.[4] In addition, the value of the index is invariant with the transfer of income from rich to poor persons within a group and its value declines with a transfer of income from a persons of the rich group to a person of the poor group. Hence it also satisfies the *transfer principles*. One important difference between the indices (1) and (3) is that the former does not satisfy the '*population composition invariance*' principle, while the latter satisfies this property, since it is normalized by the population composition of the groups. The index (3) enables us to compare representational inequality across settings.

3. The difference between 'complete segregation' and 'clustering' is explained in Appendix II.
4. The desirable axiomatic properties of RI measures in cardinal setting are given in Appendix I.

3. Representational inequality with categorical data

For cardinal data the summary statistics, such as mean, variance and different moments form the basis of the inequality measures. However, for categorical data these summary statistics are not available. To deal with this problem associated with the measurement of inequality for categorical data a new body of literature on inequality has been developed, which suggests inequality indices on the basis of the probability or cumulative probability distributions of the variable instead of the assigned values to the categories of a variable. In the case of assessing between-group inequality for categorical data same problems will arise due to the non-availability of the summary statistics. Since the categorical variable has some ordered or permutable classes, the populations of the identity groups could be assigned to the different classes of the attribute. Finally, the between-group inequality measures should be constructed on the basis of the probabilities or cumulative probabilities of the identity groups in different classes of the attribute. These measures are actually the scalar value representation of the difference in the concentrations of the groups at different categories of the attribute. The concept of representational inequality is a variant of between-group inequality, which reckons such inequality for categorical data by taking into account the difference in the proportional representations of the identity groups at different level of the attribute.

Thus, in case of categorical data the population is classified by two broad attributes, one is the well-being indicator and the other is identity category. This classified population defined by these two attributes can be represented by a matrix measuring different classes of the attributes along the rows and columns of the matrix. If we assign the percentages of classified population in the cells of the matrix corresponding to different identity and well-being categories, then this matrix is known as 'representational matrix' (RM), represented by: $[R] = [\gamma_{ij}]$. The cell element of RM (γ_{ij}) represents the proportion of population belonging to the *i*th well-being category (where *i* = 1, 2, ..., *m*) and *j*th identity category (where *j* = 1, 2, …., *K*). If there are *K* identity categories in the society and *m* categories of the well-being indicator, then the order of RM is $m \times K$.

The proportion of population constituted by the *i*th well-being category is $C_i = \sum_{j=1}^{K} \gamma_{ij}$ and the proportion of the population constituted by *j*th identity category is $R_j = \sum_{i=1}^{m} \gamma_{ij}$. Another form of the representational matrix is 'normative representational matrix' (NRM), represented by $[R^d] = [\gamma_{ij}^*]$. The cell elements of NRM are derived from RM by: $\gamma_{ij}^* = R_j \times C_i$; $\forall i = 1, 2, \ldots, m$ and $\forall j = 1, 2, \ldots, K$. So, the cell elements of NRM are the desirable or normative percentages of population of different identity groups at different levels of the well-being indicator. If there are *m* well-being categories and *K* number of identity groups in the society, then the representational and normative representational matrices have the following forms:

$$[R] = \begin{matrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1K} & \boldsymbol{C_1} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2K} & \boldsymbol{C_2} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \gamma_{m1} & \gamma_{m2} & \cdots & \gamma_{mK} & \boldsymbol{C_m} \\ \boldsymbol{R_1} & \boldsymbol{R_2} & \cdots & \boldsymbol{R_K} & \end{matrix} \qquad [R^d] = \begin{matrix} C_1R_1 & C_1R_2 & \cdots & C_1R_K \\ C_2R_1 & C_2R_2 & \cdots & C_2R_K \\ \vdots & \vdots & \cdots & \vdots \\ C_mR_1 & C_mR_2 & \cdots & C_mR_K \end{matrix}$$

Let $\gamma_{j|i}$ be the term representing the probability of being in the identity group conditional on belonging to the *i*th well-being class, and $\gamma_{j|i} = \gamma_{ij}/C_i$. Therefore, without any representational inequality, then $\gamma_{ij} = \gamma_{j|i} \times C_i = \gamma_{ij}^* = R_j \times C_i$. Thus, representational equality implies $\gamma_{j|i} = R_j$. Let η_{ij} represents the proportional deviation of the actual value of representation from its normative value, *i.e.*, $(\gamma_{ij} - \gamma_{ij}^*)/\gamma_{ij}^*$. If an identity group is either over or underrepresented at *j*th well-being category, then the value of η_{ij} will be greater than or less than zero.

Subramanian (2001) has explained the deviation from normal representation in terms of asymmetric representation considering only two identity categories and two well-being categories (*i.e.*, both RM and NRM are 2×2 ordered matrices). The society-wide index of asymmetric representation is computed from the group-specific asymmetric representations and its construction becomes difficult for $K > 2$ and $m > 2$. To solve this problem we develop some group-specific and society-wide indices of RI invoking the fundamental concept in the construction of the index of discrimination presented in Subramanian (2001). The group-specific misrepresentation η_j is computed by adding the absolute values of misrepresentations of each group in different categories of the well-being indicator. The absolute values are taken since the values of group-specific misrepresentations in different categories might cancel out each other. Assigning equal weight $(1/m)$ to all categories of the well-being indicator, the group-specific misrepresentation η_j could be estimated from η_{ij} by:

$$\eta_j = \frac{1}{m}\sum_{i=1}^{m}|\eta_{ij}| = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{\gamma_{ij}-\gamma_{ij}^*}{\gamma_{ij}^*}\right| = \frac{1}{m}\frac{1}{R_j}\sum_{i=1}^{m}|\gamma_{j|i} - R_j| \qquad (4)$$

The aggregate misrepresentation of a group is computed by adding the absolute values of misrepresentation of the particular group at different classes of the attribute, which enables us to account the actual total misrepresentation of the group. The measures of RI could not account the impact of the exchange of places between two classes of the categorical data. Thus the aggregate misrepresentation of each group is independent of the labels of the classes of the attribute. Any overrepresentation of a group in one (top) category does not compensate its underrepresentation in another (bottom) category.

The society-wide misrepresentation could now be computed by:

$$I_{RI}^{O1} = \sum_{j=1}^{K}\lambda_i\,\eta_i^{\epsilon};\ \text{for } \epsilon > 0 \qquad (5)$$

Where λ_i is the population share of the *i*th group and ϵ is the power term, which takes any positive value. Assigning equal weight $(1/K)$ to all groups, (5) would be:

$$I_{RI}^{O2} = (1/K)\sum_{j=1}^{K}\eta_i^{\epsilon};\ \text{for } \epsilon > 0 \qquad (6)$$

If groups are normally represented at all levels of well-being indicator, then the value of $\eta_j = 0$ and $I_{RI}^{O1} = I_{RI}^{O2} = 0$, *i.e.*, the index takes the minimum value zero, when there is no RI and the indices take positive value for representational inequality. The values of the indices I_{RI}^{O1} and I_{RI}^{O2} are invariant with the permutations of the attribute assigned to different individuals within a group and by the permutation of the group identities associated with the individual attribute. So, these indices satisfy *within-group anonymity* and *group-identity-anonymity* principles. Replication of every individual by a given proportion and replication of the groups do not influence the values of these indices. Thus, both of these indices satisfy the total *population size invariance* and *group replication invariance principles*. Since the index (5) is the sum of the group-specific indices of misrepresentation weighted by population shares, it is not invariant with the replication of every individual in different groups by different rates, *i.e.*, it does not satisfy the *population composition invariance* principle. On the contrary (6) satisfies this principle. In addition, the indices (5) and (6) are independent of the labels of the classes of the attribute. So, their values are invariant with the permutations of the classes of the attribute (*i.e.*, indices satisfy the 'symmetry principle'). After all, the values of these indices decline with *progressive bilateral transfer*.[5]

5. See the axiomatic properties of RI measures suitable for categorical data in Appendix I(B).

The indices (5) and (6) are closely linked to the chi-square statistic of the chi-square test for the association between attributes. In the chi-square test the cell values of the contingency table are interpreted quite differently, where the association between attributes is inferred by the sum total of the proportional deviations of the square values of the actual frequencies from the expected frequencies (i.e., chi-square statistic). It is hypothesized in the chi-square test that the attributes are associated if and only if the estimated value of the chi-square statistic is less than or equal to its tabulated value. However, the method of assessing horizontal inequality in terms of RI for categorical data takes one step further by reformulating the idea of the chi-square test for association between attributes, where a scalar index is developed on the basis of the proportional deviations of the actual population proportions from the desired or normative population proportions. The index (6) enables us to compare the estimated values of between-group inequalities in different settings; and the index (5) enables us to compare the between-group inequalities of different settings if population compositions of the groups in those settings are identical, which is not possible in the case of chi-square test.

4. Empirical illustration: RI of Indian castes across expenditure and educational dimensions

We use data in this study from the household consumer expenditure surveys (66th Round) conducted by the National Sample Survey Office (NSSO) of India. The data are collected from rural and urban sectors separately by two stage random sampling. In the Indian context population is usually classified into four broad social groups – Scheduled Tribes (ST), Scheduled Castes (SC), Other Backward Castes (OBC) and other castes. There are significant differences between these social groups in terms of their well-being achievements.

To assess RI in cardinal and ordinal settings we take household per capita consumer expenditure and educational achievements, respectively, as cardinal and ordinal variables. Since the information on income at the household levels is unavailable in India, monthly per capita expenditure (mpce) of the household is taken to be an indicator of living standard. In the NSS samples educational attainment is recorded by the level of education completed, which is an ordinal variable. To reckon RI in the ordinal setting among different social groups, we consider five educational levels – (i) illiterate and literate without formal schooling, (ii) below primary level and primary completed, (iii) above primary and secondary completed, (iv) above secondary and higher secondary completed, and (v) above higher secondary. In this present study we restrict the sample to the individuals aged 25 years and above both in rural and urban areas, to include only those persons in the analysis, who have already completed their education.

Comparing the mean 'mpce' and population shares of the groups in rural and urban areas (reported in Panel A and B of Table 1) it is possible to comment on the inter-group inequality in expenditure in rural and urban areas. However, to compare the between-group inequalities for alternative groupings in a setting and to compare the between group inequality for a particular grouping across settings, one needs a scalar index. Comparing the expenditure shares and population shares of different groups it is generally observed that the members of 'other castes' group's expenditure share is greater than their population share in both rural and urban areas. The computed values of the group-specific measures of relative advantages and disadvantages (*i.e.*, δ_j reported in Table 1) are positive for ST, SC and OBC groups, and negative for the 'other castes' group in both areas. This reveals that the OBC, SC, and ST groups are relatively disadvantaged groups and the 'other castes' group is relatively advantaged group in terms of sharing expenditure. The computed values of the society-wide indices D^R and D^U (for rural and urban areas) are 0.1 and 0.165 in rural and urban areas respectively, which reveal the existing RI among the social groups in the distribution of expenditure in India. In rural India it is less than in the urban areas.

Table – 1: Group-specific relative advantage and disadvantage, and society-wide index of RI

	Rural (A)					Urban (B)				
	OC	OBC	SC	ST	Overall	OC	OBC	SC	ST	Overall
Mean 'mpce' of different caste groups	1221.8	1001.69	889.86	944.5	1033.33	1898.5	1379.47	1191.89	1551.84	1580.29
Population Share of group j	0.275	0.384	0.174	0.165	---	0.418	0.367	0.136	0.079	---
Share of expenditure of group	0.325	0.372	0.149	0.151	---	0.492	0.322	0.103	0.078	---
$\delta_j^1 = (\lambda_j - \theta_j)/\lambda_j$	-0.182	0.031	0.143	0.085	---	-0.2	0.129	0.248	0.064	---
Value of μ_j^*	1767.91	909.64	458.97	647.99	---	2558.35	905.6	498.66	1323.05	---
Value of θ_j^*	0.47	0.338	0.077	0.103	---	0.663	0.212	0.043	0.066	---
Computed values of the society-wide Indices	$D^R = 0.1$; $D^{*R} = 0.11$; $I_{RI}^{C(R)} = 0.252$					$D^U = 0.169$; $D^{*U} = 0.152$; $I_{RI}^{C(U)} = 0.318$				

Table – 2: Percentage distributions of population of different social group according to educational achievement

Educational categories	Rural (A)				Urban (B)			
	OC	OBC	SC	ST	OC	OBC	SC	ST
1. Illiterate and literate without formal schooling	25.82	33.07	38.73	31.71	16.79	23.97	31.12	17.71
2. Below primary and primary completed	30.48	31.91	33.4	34.31	23.87	28.55	31.52	29.5
3. Above primary and secondary completed	29.08	25.26	20.80	25.28	28.34	28.52	24.46	33.46
4. Above secondary and higher secondary completed	7.85	5.50	4.05	5.06	11.83	8.63	6.59	9.72
5. Above higher secondary	6.77	4.25	3.03	3.64	19.17	10.33	6.31	9.61
Total	100	100	100	100	100	100	100	100

Permuting the population shares of the groups (considering the extreme case of permutation, *i.e.*, assigning the largest population share to the group originally having smallest population share, then assigning second largest population share to the group originally having second lowest population share and so on) the computed values of the society-wide indices D^R and D^U become 0.11 and 0.152 (represented by D^{*R} and D^{*U} in Table 1). This implies that a change in population composition of the groups has some influence on (1). Therefore, the index (1) is not comparable. However, we can compare RI existing in rural and urban areas by using the index (3). The estimated values of the indices $I_{RI}^{C(R)}$ and $I_{RI}^{C(U)}$ (for rural and urban areas) are 0.25 and 0.318, implying that the observed between-group inequality in the distribution of expenditure is almost one-fourth of the maximum possible between-group inequality in rural India, and the observed between-group inequality is approximately one-third of the maximum possible between-group inequality in urban India, with given current distributions of 'mpce', number of social groups, the population proportions of the social groups and the ranking of the groups in terms of the mean 'mpce' in both rural and urban areas.

Analyzing the percentage distributions of population of different social-groups (reported in Panel A and Panel B of Table 2) at different levels of education in rural and urban areas, the existing between-group inequality in education can be explained. It is observed that in rural areas there is a declining trend of population proportion of all social groups from lower to higher categories of education. However, the rate of decline is relatively higher for the ST, SC and OBC groups than the 'other caste' group. This declining trend of population proportion persists in urban areas for all social groups except the 'other castes'. Through this evaluative analysis, it is difficult to conceptualize the distinction between the inter-group disparities in educational achievement in rural and urban areas. Therefore, some scalar measures should be invoked for comparing existing between-group inequality in two or more settings.

Tables 3 and 4 are the RM and NRM for rural and urban areas (Panels A and B). These matrices are 5×4 ordered matrices, since there are five categories of education and four social groups. Tables 5(A) and 5(B) report the computed values of η_{ij} (proportional deviation of the actual representation from its normative value) in the rural and urban areas. It is observed that the OBC group is normally represented at fourth educational category in rural areas and ST group is normally represented at fourth educational category in the urban areas. In other categories of education in both rural and urban areas OBC and ST groups are either over or underrepresented. Likewise the 'other castes' and SC groups are either over or underrepresented at different categories of education in both areas. Therefore, there exists representational inequality in education in both rural and urban areas of India. Moreover, it is also observed that at the top categories the 'other castes' group is overrepresented and other social groups are underrepresented in both areas. Opposite in the case of the representations of these social groups at the bottom categories of education. Hence, the educational achievement of the members of the 'other castes' group is greater than the educational achievements of the members of all other groups.

The computed values of the society-wide indices (5) and (6) for different values of ϵ (given in the last row of Tables 5(A) and 5(B)) reveal the existing representational inequality in education among Indian social groups in rural and urban areas. These computed values further reveal that in urban areas the existing RI in education is more than in the rural areas. Therefore, in conclusion there exists inequality among the social groups in India in rural and urban areas in some dimensions of well-being. However, the disparities among the social groups are greater in urban areas than in rural areas.

Table – 3: Representational matrix of educational achievement

Educational categories	Rural (A)					Urban (B)				
	OC	OBC	SC	ST	C_i	OC	OBC	SC	ST	C_i
1. Illiterate and literate without formal schooling	0.07	0.126	0.067	0.055	0.318	0.069	0.088	0.042	0.016	0.215
2. Below primary and primary completed	0.083	0.122	0.058	0.056	0.319	0.099	0.105	0.044	0.023	0.271
3. Above primary and secondary completed	0.079	0.096	0.038	0.041	0.254	0.119	0.105	0.033	0.026	0.283
4. Above secondary and higher secondary completed	0.021	0.023	0.007	0.008	0.059	0.053	0.031	0.008	0.007	0.099
5. Above higher secondary	0.019	0.02	0.005	0.006	0.05	0.078	0.038	0.009	0.007	0.132
$R_j = \alpha_j$	0.272	0.387	0.175	0.166	1	0.418	0.367	0.136	0.079	1

Table – 4: Normative representational matrix of educational achievement

Educational Categories	Rural (A)				Urban (B)			
	OC	OBC	SC	ST	OC	OBC	SC	ST
1. Illiterate and literate without formal schooling	0.086	0.123	0.056	0.053	0.089	0.079	0.029	0.017
2. Below primary and primary completed	0.088	0.123	0.056	0.053	0.113	0.099	0.037	0.021
3. Above primary and secondary completed	0.069	0.098	0.044	0.042	0.116	0.104	0.039	0.022
4. Above secondary and higher secondary completed	0.016	0.023	0.01	0.009	0.039	0.035	0.013	0.007
5. Above higher secondary	0.014	0.019	0.008	0.008	0.054	0.049	0.018	0.011

Table – 5: Group-wise misrepresentation and society-wide measure of 'representational inequality' for categorical data

Educational categories	Rural (A)				Urban (B)			
	OC	OBC	SC	ST	OC	OBC	SC	ST
1. Illiterate and literate without formal schooling (η_{1j})	-0.186	0.024	0.196	0.037	-0.224	0.113	0.448	-0.058
2. Below primary and primary completed (η_{2j})	-0.056	-0.008	0.036	0.057	-0.123	0.061	0.189	0.095
3. Above primary and secondary completed (η_{3j})	0.145	-0.020	-0.136	-0.024	0.026	0.009	-0.154	0.181
4. Above secondary and higher secondary completed (η_{4j})	0.312	0	-0.33	-0.111	0.358	-0.114	-0.384	0
5. Above higher Secondary (η_{5j})	0.357	-0.052	-0.375	-0.25	0.024	-0.224	-0.5	-0.363
$\eta_j = (1/m)\sum_{i=1}^{m}\lvert\eta_{ij}\rvert$	0.211	0.026	0.214	0.096	0.151	0.104	0.335	0.14
Computed values of the Society-wide indices	$I_{RI}^{O1(R)}(\epsilon = 2) = 0.022$ $I_{RI}^{O1(R)}(\varepsilon = 1) = 0.121$ $I_{RI}^{O2(R)}(\epsilon = 2) = 0.025$ $I_{RI}^{O2(R)}(\epsilon = 1) = 0.109$				$I_{RI}^{O1(U)}(\epsilon = 2) = 0.034$ $I_{RI}^{O1(U)}(\varepsilon = 1) = 0.157$ $I_{RI}^{O2(U)}(\epsilon = 2) = 0.045$ $I_{RI}^{O2(U)}(\epsilon = 1) = 0.198$			

Appendix I (A):

Desirable axiomatic properties of RI measures in cardinal setting:

(i) Minimal and maximal inter-group inequality: Without any RI the index takes zero value and for maximum RI, the value of the index is one.

(ii) Anonymity: This principle comprises two sub-principles for RI measures, which are:

(a) Within-group anonymity principle: A measure of RI satisfies this property if its value is invariant with the permutations of the attribute assigned to different individuals within a group. If X_{ij} is the cardinal variable possessed by *i*th individual in jth group (where $i = 1, 2,.., s, s+1,.., n_j$ and $j = 1, 2,\ldots, K$), then this axiom states that interchanging the possession of X_{sj} and $X_{(s+1)j}$ by sth and (s+1)th individual of jth does not influence the value of the index.

(b) Group-identity-anonymity principle: A measure of RI satisfies this property if it is invariant after any permutation of the group identities which is associated with the individual attribute. In other words, this axiom states that the information relevant to assess RI is taken into account after observing the partition of the population into groups and the possession of the attributes by the members of these groups. If $\{X_{ij}\}$ is the set of the attribute possessed by the jth group (where $i = 1, 2,.., n_j$ and $j = 1, 2,\ldots, l, l+1,\ldots, K$), then any permutations of the attribute between *l*th and(*l*+1)th groups do not influence the value of the index.

(iii) Scale independent/invariance property (SI): This axiom is considered to check the consistency of the index to assess horizontal inequality in different settings. The index of RI satisfies scale independence property, if income or expenditure (the values of any cardinally measurable attribute) of all persons will rise with a given proportion λ, then the value of the index is unchanged.

(iv) Population replication invariance (PI): In case of between-group inequality measure the population replication invariance principle has three sub-principles, which are also relevant for RI measures:

(a) Population composition invariance principle (PCI): This property states that, with the replication of every individual in different groups by different rates $\lambda_1 \neq \lambda_2 \neq \ldots \neq \lambda_K$ (for K groups in the society), while the population compositions of the groups will be changed, then the value of the index is unchanged. If the functional form of an index is $I: \mathcal{D} \times \phi \rightarrow \mathcal{R}$ (where $\mathcal{D}$ and ϕ are the set of group means and population proportions of the groups), then the index does not satisfy this property.

(b) Total Population size invariance principle (TPI): If every individual in the population is replicated by a given proportion, then the degree of inequality should be unchanged. So, the individuals of different groups will be replicated by equal proportions $\lambda_1 = \lambda_2 = \cdots = \lambda_K = \lambda$.

(c) Group replication invariance principle (GRI): According to this property, if the groups will be replicated by a given proportion (obviously with same within group distribution of population), then there is no change in inequality value. If this property will not be fulfilled, then we could not compare the inequalities of different settings having different number of groups.

(v) Transfer principle: A measure of between group inequality satisfies this axiom in group inequality analysis, if its value is invariant with the transfer of income from a rich to a poor person within a group, however, its value declines through a transfer from a person of the rich group to another person of the poor group.

Appendix I (B):

Desirable axiomatic Properties of the measures of RI for categorical data:

The axiomatic properties (ii) and (iv) of RI measures in ordinal setting are identical with the cardinal data. The scale invariance axiom is not relevant for inequality measures for ordinal data. The property ‘minimal and maximal inequality’ should be changed here to some extent, in the context of categorical data.

(i) Minimal and maximal inter-group inequality: The value of the RI measure is equal to zero, when there is no between-group inequality, and it takes a positive value, when there is representational inequality.

For categorical data with non-permutable classes, the permutations of the classes influence the computed value of the measure of between-group inequality. The concept of RI fails to account the influence this permutation. So, an additional axiom should be incorporated for RI measure for categorical data to explain this value judgment.

(v) Symmetry in categories or classes of the well-being indicator (S): This property states that the value of the RI measure should be invariant with the exchange of places of some categories or classes of the well-being indicator, i.e., any permutations of the categories of the well-being indicator does not influence the value of the RI measure. In other words, the value of the RI measure does not depend on the labels of the categories of the well-being indicator. It only depends on the proportion of populations of the identity groups.

If there are m categories of the well-being indicator and K identity groups, then the following two representational matrices are derived for two cases of permutations of the categories of the well-being indicator.

$$[R] = \begin{matrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1K} & \boldsymbol{C_1} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2K} & \boldsymbol{C_2} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \gamma_{m1} & \gamma_{m2} & \cdots & \gamma_{mK} & \boldsymbol{C_m} \\ \boldsymbol{R_1} & \boldsymbol{R_2} & \cdots & \boldsymbol{R_K} & \end{matrix} \qquad [\bar{R}] = \begin{matrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1K} & \boldsymbol{C_1} \\ \gamma_{51} & \gamma_{52} & \cdots & \gamma_{5K} & \boldsymbol{C_5} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2K} & \boldsymbol{C_2} \\ \gamma_{m1} & \gamma_{m2} & \cdots & \gamma_{mK} & \boldsymbol{C_m} \\ \boldsymbol{R_1} & \boldsymbol{R_2} & \cdots & \boldsymbol{R_K} & \end{matrix}$$

A ‘representational inequality’ measure for categorical data satisfies this property, if its value is invariant after transforming R into $\bar{R}$.

The RI measures are independent of the labels of the classes of the categorical variable, *i.e.*, these measures do not responsive to the exchange of places of the classes of the categorical variable. Hence, any overrepresentation of a group in one (top) category does not compensate the underrepresentation in another (bottom) category in the context of RI setting.

The concept of RI actually assesses horizontal inequality according to the deviations of the actual representations of the groups from their normative representations at different levels of the attribute. Any misrepresentation, *i.e.*, over or underrepresentation is equally objectionable irrespective the categories of the ordinal attribute (bottom or top). So, instead of the exchange principle, which is a variant of the transfer principle for ordinal data, we could consider the ‘balanced bilateral transfer principle’ (BBTP) for characterizing the RI measures for categorical data (according to Reddy and Jayadev, 2011b).

(vi) Balanced bilateral transfer principle (BBTP): Considering two groups *k* and *s*, if group *k* is over represented at level ‘*a*’ and under represented at level ‘*b*’ of the attribute, when the group s is under represented at ‘*a*’ and over represented at ‘b’ levels, then a transfer of population mass π (some number of persons) of the *k*th group from ‘*a*’ to ‘*b*’ and transfer of equal population mass π of *s*th group from ‘*b*’ to ‘*a*’, will reduce misrepresentations of *k*th and *s*th groups at ‘*a*’ and ‘*b*’ resulting the reduction of representational inequality. This transfer is known as progressive balanced bilateral transfer.

Appendix II:

Complete segregation and clustering, and maximum possible value of RI measure:

According to Reddy and Jayadev (2011a), complete segregation and non-overlapping (or clustering) are two distinct notions. Former is associated with 'representational inequality' and the latter is associated with the 'sequence inequality'. If there is complete segregation of the groups, then at each income level there is only one group. In case of complete separation or clustering the groups are concentrated in different parts of the distribution of the attribute. Therefore, clustering is possible only when groups are completely segregated.

The computed value of the RI measure 'D' will be changed by reassigning the individual observations of the original distribution to the groups, keeping the relative sizes and number of the groups unchanged. The reason behind this change in the value of 'D' is the change in the dispersions of group means around the population mean. Through repeated reassignment of the observations to the groups, it is found that the dispersions of the group means become maximum and the computed value of the index 'D' is maximum, when the groups are non-overlapping or concentrated in different parts of the distribution of the attribute, since the differences between the population shares and proportional possession of the attribute by the distinct groups are maximum under this circumstance. Hence, we use the non-overlapping distributions across groups as the reference for maximal RI computed by 'D'.

Method to compute the maximum possible between-group inequality:

If there are K number of identity groups in any society, then to compute the maximum possible between group inequality we arrange the groups in an ascending order according to the computed values of the group mean of the attribute. Keeping this order of the identity groups (let the order of the groups be: l, 3,…, 4, K,…,1 ,2, for the group means $\mu_l < \mu_3 < \cdots < \mu_4 < \mu_K < \cdots < \mu_2$) and the relative sizes of the groups unchanged, we assign the lowest n_l observations to the lth group, then we assign the second lowest n_3 observations to the third group and in such a way we assign all observations of the attribute to the groups. Assigning the observations in this manner the groups become non-overlapping, we have to compute the group means of the attribute and between-group inequality again. The computed value of between-group inequality latter on such arrangement of the observations is greater than other computed values of between-group inequality for different cases of permutations of the observations with the original distribution of the attribute.

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