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On the Implications of the Markowitz Model of Utility embodying Gain Seeking Preferences for Odds on Betting and Bookmakers choice of Spread or Odds Betting

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Abstract

We demonstrate in a parametric formulation of the Markowitz model of utility that unless agents are initially gain seeking they will not bet on heavily odds on favorites for a given negative expected rate of return. The model supports Sauer's (1998) observation that it may not be profitable to make a market in contests involving heavy odds on favorites with implications for bookmakers choice of spread or odds markets in sports betting.

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1.Introduction

Sauer (1998) noted that in lopsided horse races, so that the favorite would be heavily odds on, bookmakers in the U.K. often do not offer odds against the favorite but make a market in the long shots. This important observation suggests that it may not be profitable to make a market in contests involving heavy odds on favorites. It follows that if a high proportion of lopsided matches can be anticipated in some sports Sauer has provided a potential rationale as to why bookmakers might prefer spreads in some sports betting markets, such as the NFL (US football) and the NBA (basketball), as opposed to odds betting in the NHL (Hockey) and the MLB (baseball)[1].

Given the popularity of sports betting[2] or casino betting, where betting at odds on or even odds is typical, it is perhaps surprising that little attention has been given to formal modeling of the risk preferences required of agents to explain betting on odds favorites. In the non-expected models of utility of Markowitz (1952) or Cumulative Prospect Theory (CPT) of Tversky and Kahneman (TK) (1992) the representative agent does not bet on odds on favorites and there is no discussion of odds on betting.

In this note we analyze the issues employing a parametric formulation of the Markowitz (1952) model of utility. We chose this model since agents are not required to exhibit probability distortion in order to gamble at actuarially unfair odds as is the case with other non-expected utility models such as CPT or rank-dependent utility of Quiggin (1982).[3] Both Markowitz and TK assumed that their representative agent was loss averse. Whilst experimental evidence supports this assumption for the majority of agents it also reveals that around 12-25% of agent's initially exhibit gain seeking behavior, (see for example Abdellaoui et al. (2007(a)), Abdellaoui et al. (2013) and Harinck et al. (2007)).

Whilst gain seeking behavior may be an attribute of preferences, per-se, it is also a parsimonious method of modeling any utility of gambling. Both survey and empirical evidence reveal that a significant proportion of respondents who gamble mention the pleasure of participation in gambling and that the volume of bets increases when a match is televised, (see for example The Wager (2000) and Paul and Weinbach (2010)). It appears plausible to assume that the fun of participation in gambling implies that initially any monetary gains obtained are more pleasurable than the same amount of monetary losses. We show that agents who initially exhibit gain seeking preferences in a parametric

[1] Woodland and Woodland (1991) purport to show that the optimal wager sizes of their representative agents are greater in spread than odds markets. However Woodland and Woodland assume in their model solutions that the subjective expected return for the total bet size is equal for betting on either the long-shot or favorite, not expected return per unit staked. If their assumption is replaced with that of equal subjective returns per unit staked the implications of their analysis are reversed.

[2] Estimates of wagering on sporting events in the US go as high as $380 billion annually (National Gambling Impact Study Commission, 1999). The American Gaming Association reports that in 2010, $2.76 billion was legally wagered in Nevada's sports books; and the National Gambling Impact Study Commission (NGISC) estimated that illegal wagers are as much as $380 billion annually. http://www.americangaming.org/industry-resources/research/fact-sheets/sports-wagering

[3] Sauer (1998) raised the issue of why individuals would persist in gambling, when positive subjective expected rates of return are the rationale, given on average they lose money. It is not generally appreciated how high these subjective rates of return can be. For instance employing the parametric formulation of Tversky and Kahneman (1992) of CPT an agent betting on a number at U.S. roulette has a positive subjective return of 239.3% as opposed to an objective return of -5.3%.

specification of the Markowitz model may prefer to bet on odds on chances at actuarially unfair odds when stake size is determined optimally and expected return is the same at all odds. This implication is not surprising. However the analysis also reveals that, for a bet utility will exhibit a maximum with respect to odds and this maximum can occur in the odds-on domain[4]. Further for sufficiently odds on chances the agent will not bet providing theoretical support for Sauer's observation.
The rest of the note is structured as follows. In section 2 we set out a parametric form of the Markowitz model embodying gain seeking preferences. In section 3 we set out some implications of the model. In section 4 we evaluate employing historical data the proportion of matches in four major U.S. sports betting markets that could be anticipated to be lopsided. The last section is a brief conclusion.

Section 2 A Parametric Model of The Markowitz Model of Utility and the Optimal Stake

We employ a parametric version of the Markowitz model based on the expo-power utility function of Saha (1993) that was first suggested by Abdellaoui et al. (2007(b)) and employed by Peel and Law (2009) in a different context. Employing the expo-power function expected utility, Eu, for a one prize gamble is given by.

$$Eu = p(1-e^{-r\rho s^n O^n}) - (1-p)k(1-e^{-\rho s^n}) \qquad (1)$$

Where p is the objective probability, O is odds, s is stake size, r, ρ, k and n are positive constants. In (1) when $n > 1$ we obtain the form of value function hypothesized by Markowitz. The agent is initially risk-loving then risk-averse over gains, as $\frac{(n-1)}{rn}$ is greater or less than $\rho(sO)^n$; and initially risk-averse then risk-loving over losses, as $\frac{(n-1)}{n}$. is greater or less than ρs^n. The degree of gain seeking or loss aversion, defined as the ratio of the utility of gains and losses of the same amount,[5] lies between $\frac{r}{k}$, as gains and losses tend to zero and $\frac{1}{k}$ when gains and losses are infinitely large.
The Markowitz specification of the value functions has considerable empirical support, (see for example Levy and Levy (2002)).

The optimal stake is obtained from (1) as

[4] This important property can also be derived in CPT or RDU models but the derivation is more complicated
[5] The definition of loss aversion employed by both Markowitz (1952) and Kahneman and Tversky (1979).

$$s = \left(\frac{\log[\frac{rpO^n}{k(1-p)}]}{\rho(rO^n - 1)} \right)^{\frac{1}{n}} \qquad (2)$$

or

$$s = \left(\frac{\log[\frac{r(1+\mu)O^n}{k(O-\mu)}]}{\rho(rO^n - 1)} \right)^{\frac{1}{n}} \qquad (3)$$

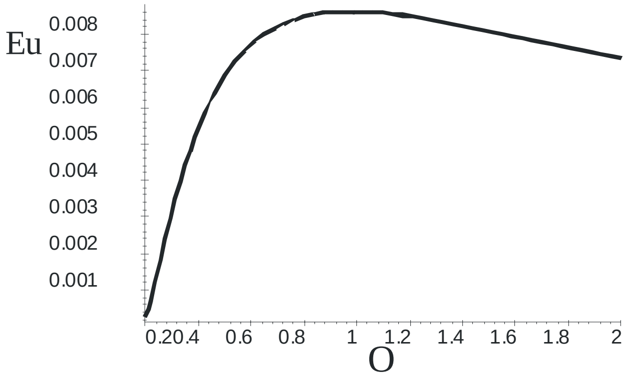
where $\mu = pO - (1-p)$ is the expected return for a one unit stake. The second order condition for stake size to maximize utility is given by $rO^n - 1 > 0$.

Equation (3) has a number of implications. First, gain seeking preferences at the origin, $\frac{r}{k} > 1$, are a necessary condition for an agent to bet at even or odds on at actuarially unfair odds, $\mu < 0$, since otherwise the numerator in (3) is negative. Second, the numerator of (3) is zero for sufficiently odds on favorites, so agents will not bet on them[i]. Third, larger values of n, which increase the risk seeking segment of the value function over gains, reduce the attractiveness of betting on odds on chances, ceteris paribus. Fourth, the parameter ρ is a scaling factor and determines how quickly the upper and lower bonds of utility are reached. Stake size increases, as ρ decreases, ceteris paribus.

Section 3 Some Implications of the Model

To illustrate the implications of the model we plot both expected utility and the optimal stake as odds vary for a given expected rate of return. First we consider an agent with parameters r=67, k=50, n=1.01 and $\rho = 0.000003$. In Figure (1) we observe that this agent obtains maximum utility betting at odds of O=10/11 when expected return per unit is $\mu = -0.045454$ which is the same as the expected return as spread betting in the NFL.

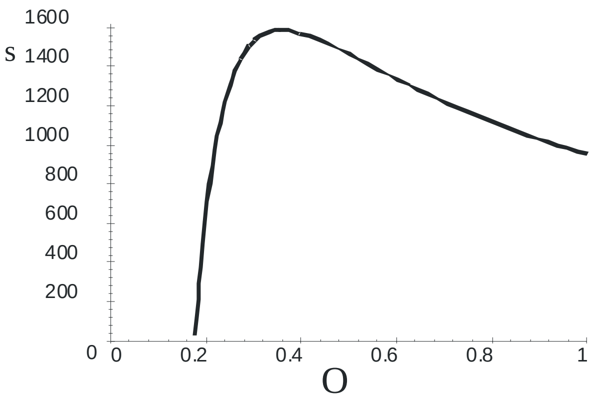
Figure (1) **An agent who obtains maximum expected utility betting at odds of 10/11**



Parameters r=67, k=50, n=1.01, $\rho = 0.000003$, $\mu = -0.045454$

In Figure (2) we plot the optimal stake for different odds and observe that the agent stakes $1007 at O=10/11 but would stake most in an odds market at odds of approximately O.4. In addition this agent will not stake at odds less than approximately O=0.2.

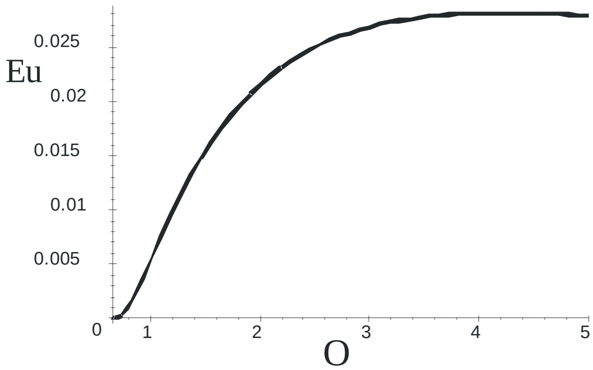
Figure (2) **The optimal stake of the agent in Figure (1) at different odds and the same expected rate of return**



Parameters: 0=10/11 s=1007, 0=0.4 s=1464, $\mu = -0.045454$

In Figure (3) we plot expected utility for the parameters r=64, k=50, n=1.35 and ρ=0.000083 with expected return $\mu = -0.045454$. We observe that this agent obtains maximum utility at odds of 4.3 and will not bet at odds of less than O=0.65.

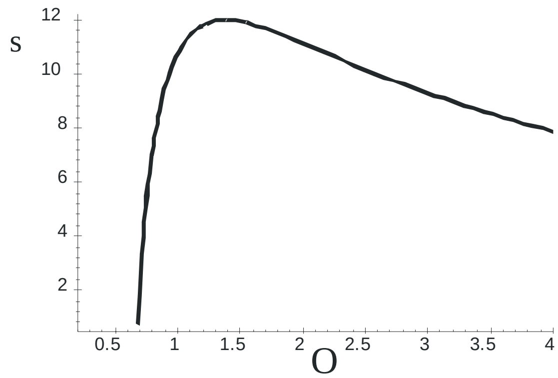
Figure (4) The expected utility of an agent who would prefer a longer odds gamble but will bet at odds on



Parameters: r=64, k=50, n=1.35, $\rho = 0.000083$, $\mu = -0.045454$.

Figure (4) reveals that the agent with parameters as in Figure 3 will stake more on the spread bet, ($11), than they would stake in an odds market at odds of 4.3/1, ($8.5).

Figure (4) The optimal stake of the agent in Figure (3) at different odds and the same expected rate of return



Values: O=10/11 s=$11, O=4.3 s=$8.5, $\mu = -0.045454$.

The analysis above implies that a bookmakers preference for a spread or odds market has to balance (a) the revenues gained by been able to make a market in spreads in lopsided matches (b) the revenues gained (lost) from punters who would stake more (less) in spread market than an odds market, as illustrated in Figures (2) and (4).

Section 4 Some Analysis of Competitive Balance and Odds in Four Major Sports in the US

Spread betting appears to be the predominant choice of betting market in US sports when a significant proportion of lopsided matches can be anticipated. In the NFL between 1985 and 2009 there were 5966 games of which 10.34% had a spread greater than ten (maximum 24). Whilst predominantly a spread market a few bookmakers[6] have recently quoted odds for NFL matches. The odds corresponding to a spread of ten are -$500 $400, which means that you stake $500 to win $100 or stake $100 to win $450. Consequently 10.34% of games would have exhibited extreme favorites with odds less than O=0.2.

In the NBA (basket ball) between 1990 and 2009 there were 21566 games of which 15.2% had a spread greater than ten and 5% a spread greater than thirteen (maximum 24). A spread of ten equates to odds of around -$450 $325 implying that the percentage of games that could be expected to be lopsided, with odds less than O=0.2, was in excess of ten percent.

In the NHL (hockey) odds market between 1996 and 2007 there were 11296 games with the heaviest favorite odds in the 20 cent line -$320 $260 or O=0.3125. 4.52% of games had the favorite odds less than -$245 $205 or O<0.408 (See Woodland and Woodland (2011)).

In the MLB (baseball)[7] between 1979 and 1989 there were 24603 games in the dime line. The heaviest favorite was -$300 $260 or O=0.333. In 973 matches (3.95%) the favorite odds were lie between 0.333 and 0.5 (See Woodford and Woodford (1994). The analysis of Paul et al (2009) suggests that similar percentages occurred in the period 1990-2006. Between 2009-2011 there were 7246 matches the heaviest favorite was -$385 $355 or O=0.259. In 1.794% of these matches the favorite was more odds on than O=0.4[8].

Section 5 Conclusion

We demonstrated in a parametric version of the Markowitz model of utility that agents exhibiting gain seeking preferences will bet at a given negative expected rate of return per unit stake, on odds on favorites but not on heavily odds on favorites. We reported empirical evidence which suggests that bookmakers prefer spreads to odds markets where a significant number of matches can be expect to be lopsided. The model and the empirical evidence provides support for Sauer's conjecture.

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[6] http://www.predictem.com/nba/moneyline.php

[7] Data for NFL, NBA and MLB was obtained through the generosity of Paul Weinbach and Alan Bowman.

[8] A referee notes that baseball and hockey have a tighter range in the margin of victory (compared to NFL and NBA) which may also be relevant as a contributory factor to the thrust of the analysis.

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