Economics Bulletin

Volume 33, Issue 2

A new strategy to estimate time-to-build completion rates

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Abstract

Time-to-build completion rates can be estimated individually, under the restrictions that they are positive values summing up to one. As an alternative strategy, we may use the beta distribution as a restriction and calculate the corresponding completion rates collectively. The estimation results of a time-to-build model indicate that imposing the beta distribution may successfully preclude unrealistic completion pattern and improve fit of the model to data.

I am grateful to anonymous referees and various seminar participants for comments and useful suggestions.
Citation: Yong-gook Jung, (2013) "A new strategy to estimate time-to-build completion rates", *Economics Bulletin*, Vol. 33 No. 2 pp. 1073-1081.
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Submitted: December 03, 2012. **Published:** April 22, 2013.

Economics Bulletin, 2013, Vol. 33 No. 2 pp. 1073-1081

1. Introduction

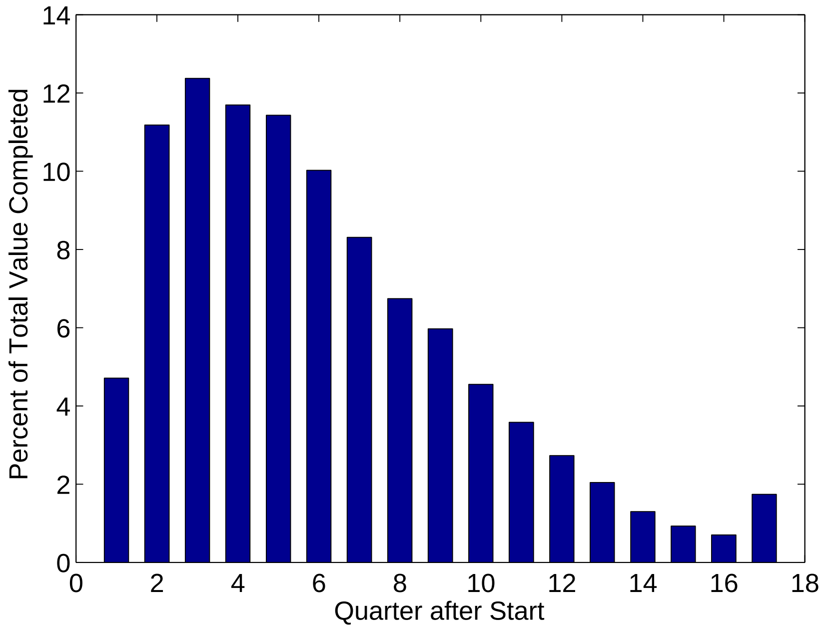
More than half of non-residential fixed investment is the expenditure for building structure, which typically takes several quarters of time.[1] Therefore, many macroeconomic studies explicitly consider time-to-build, since Kydland and Prescott (1982) started to specify investment lag in real business cycle model.[2]

The proportion of total costs in a particular period of a multi-period investment project could be regarded as "completion rate." One of the most common assumptions about completion rates is that equal amount of costs is paid over time-to-build periods. This assumption is used by Kydland and Prescott (1982), Gomme *et al.* (2001), Casares (2006), and Edge (2007). However, empirical evidence suggests that aggregate completion rates have an asymmetric distribution. Montgomery (1995) calculates the completion rates for U.S. private nonresidential structures using survey data from the U.S. Department of Commerce. He finds that more resources are required in the earlier stages of an aggregate investment project, as in Figure 1. His study implies that a uniform completion pattern may not be appropriate to describe the fluctuations in aggregate investment.

This paper proposes the use of the beta distribution in the estimation of time-to-build completion rates. Section 2 describes a simple time-to-build model to be estimated. Section

[1] Mayer (1960) claims that 15 months are required on average to complete a typical construction project. Almon (1968) estimates that there is a seven-quarter lag from the time of appropriation to investment expenditure. According to Montgomery (1995), the value-weighted construction periods for nonresidential structures are between five and six quarters.

[2] For example, Wen (1998), Gomme *et al.* (2001), Christiano and Todd (2002), and Christiano and Vigfusson (2003), Casares (2006), and Edge (2007) consider investment lags explicitly.



* Source: Montgomery (1995)

Figure 1: Value-Weighted Average Completion Pattern, 1961-1991

3 introduces two alternative restrictions on the completion rates. The model from section 2 is estimated using different restrictions as discussed in section 3. Section 4 reports those estimation results. Section 5 concludes.

2. A Time-to-Build Model

A standard real business cycle model is used as a building block. The features of the model are as follows: First, the time-to-build specification proposed by Kydland and Prescott (1982) is employed. Second, variable capital utilization is allowed, so that the representative household can deal with the demand for capital service without changing the level of capital stock. This assumption is necessary for time-to-build models since new capital stock would not be ready shortly after a shock.

The representative household owns capital stock and provides its service to the representative firm. A unit of investment in period t yields productive capital with the lag of J periods. The scale of investment project j period away from completion in period t is represented by $S_{j,t}$ for $j = 1, 2, \cdots, J$. The law of motion that describes the evolution of the incomplete investment projects is given by

$$S_{j-1,t+1} = S_{j,t}, \tag{1}$$

for $j = 2, 3, \cdots, J$. Capital stock evolves as follows:

$$K_{t+1} = (1 - \delta)K_t + S_{1,t}. \tag{2}$$

It is assumed that a constant fraction ω_j of resources is expended on $S_{j,t}$, for $j = 1, 2, \cdots, J$. We regard the parameter ω_j as the rate of completion in the $(J - j + 1)th$ stage of an investment project.[3] The total investment outlays in period t are represented by

$$V_t I_t = \sum_{j=1}^{J} \omega_j S_{j,t}, \tag{3}$$

where I_t and V_t are investment spending and the level of the investment-specific productivity shock, respectively. The household can save investment spending as V_t becomes greater. The investment-specific productivity shock is assumed to follow an autoregressive process:

$$\ln V_{t+1} = \rho_v \ln V_t + \varepsilon_{v,t+1}, \qquad 0 < \rho_v < 1, \tag{4}$$

where the zero-mean, serially-uncorrelated innovation ε_v is assumed to be normally distributed with the standard deviation of σ_v.

As assumed by Christiano *et al.* (2005), the service of capital (Q) depends on the unit of capital (K) and the utilization rate of it (u):

$$Q_t = u_t K_t. \tag{5}$$

[3] The interpretation of ω_j is based on the following observation: Expenditure on nonresidential structures in the NIPA (National Income and Product Account) is measured by the increased value of the structures during the survey period. Therefore, we interpret a fraction of resources expended on a stage of an investment project as completion rate.

The household has to pay for the cost of adjusting the utilization rate, $\gamma(u_t) K_t$, which is an increasing, convex function of u_t. We assume that $\gamma(1) = 0$ and $\gamma''(1)/\gamma'(1) = \sigma_\gamma$.

The source of household income is the sum of the net rental income $([R_t u_t - \gamma(u_t)]K_t)$ and the labor income $(W_t H_t)$, where R and W denote rental rate and wage rate. Therefore, the household's budget constraint is

$$C_t + I_t \leq [R_t u_t - \gamma(u_t)] K_t + W_t H_t, \tag{6}$$

where C_t and H_t represent consumption and labor supply, respectively. Given the budget constraint, the objective of the household is to maximize the following function:

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \ln(C_\tau) + \eta \ln(1 - H_\tau) \Big\}. \tag{7}$$

The representative firm produces the final good using Cobb-Douglas technology:

$$Q_t^\alpha (Z_t H_t)^{1-\alpha} \geq Y_t, \quad 0 < \alpha < 1. \tag{8}$$

The neutral productivity shock (Z) follows an autoregressive process:

$$\ln Z_{t+1} = \rho_z \ln Z_t + \varepsilon_{z,t+1}, \quad 0 < \rho_z < 1, \tag{9}$$

where ε_z is assumed to be normally distributed with a zero mean and the standard deviation of σ_z.

3. Restrictions for Completion Rates

If we want to estimate the completion rate ω_j's individually, then the following restrictions are necessary:

$$\tilde{\omega}_j \geq 0, \tag{10}$$

$$\omega_j = \frac{\tilde{\omega}_j}{\sum_{j=1}^{J} \tilde{\omega}_j}, \tag{11}$$

for $j = 1, 2, \cdots, J$. These restrictions imply that $\tilde{\omega}_{j-1}$ and $\tilde{\omega}_{j+1}$ can take on any positive values without regard to the value of $\tilde{\omega}_j$. Unrealistic distributions of completion rates could emerge under these restrictions. They can be very different from a uniform distribution or the distribution suggested by Montgomery (1995).[4]

As an alternative, we can estimate the completion rates collectively. The necessary conditions for relevant ω_j's, $0 \leq \omega_j \leq 1$ and $\sum_{j=1}^{J} \omega_j = 1$, indicate that completion pattern has the property of a probability distribution. Therefore, we can restrict completion rates to take on the shape of a specific probability distribution. Then, we can estimate the parameters of the distribution to calculate completion pattern.

The probability distribution should have the following properties: (1) It is a continuous function to preclude the possibility of unrealistic completion patterns. (2) It has a finite

[4] For instance, a saw-tooth shape.

support. (3) And it is flexible enough to take on symmetric, asymmetric, and uniform shapes. As Casella and Berger (2002) notes, the beta distribution is the only "named" probability density function which meets these requirements. If we estimate the parameters of the beta distribution, then we can calculate the completion rates collectively. Given the estimated parameters ttb_1 and ttb_2, the completion rate ω_j is computed as:

$$\omega_j = \int_{(j-1)/J}^{j/J} \frac{1}{B(ttb_1, ttb_2)} x^{ttb_1 - 1}(1-x)^{ttb_2 - 1} dx, \tag{12}$$

for $j = 1, \ldots, J$, where $B(ttb_1, ttb_2)$ denotes the beta function

$$B(ttb_1, ttb_2) = \int_0^1 x^{ttb_1 - 1}(1-x)^{ttb_2 - 1} dx. \tag{13}$$

Intuitively, if we cut the probability density function of the beta distribution into J pieces of the same width, then ω_j is the area of the jth piece. Since ω_j represents the completion rate for the $(J+1-j)th$ stage of an investment project, the completion pattern is the mirror image of the corresponding shape of the beta distribution. As shown in Figure 2, we can generate any plausible time-to-build completion patterns by varying the combination of ttb_1 and ttb_2. For instance, when $ttb_1 = ttb_2 = 1$, completion rates are uniformly distributed. On the other hand, when $ttb_1 = 1.5$ and $ttb_2 = 0.7$, the completion pattern takes on an asymmetric shape.

4. Estimation Results

To evaluate the performances of the two restrictions, the model described in section 2 is estimated using two sets of the United State's time series data: Output and investment, which correspond to the variables Y_t and I_t. The source of the data is the NIPA (National Income and Product Accounts). Output is calculated by the sum of domestic consumption and investment. Consumption and investment are measured by the consumption of nondurable goods and services and by the private nonresidential fixed investment, respectively. The sample ranges from 1960:Q1 to 2007:Q4. Nominal output and investment values are divided by the civilian noninstitutional population of 16 years or older and the implicit consumption deflator.[5] The data are detrended by using the Hodrick-Prescott filter.

Five parameters are fixed in the estimation procedure. Time-to-build period (J) is fixed at six following Montgomery (1995). The discount factor β is set at 0.99. The household's weight on leisure is set at $\eta = 1.5$ in order to make the steady state value of labor supply (H) around 0.3. The depreciation rate (δ) is set at 0.025 arriving at 10 percent of the annual rate. Capital's share in production (α) is set at 0.33.

The other parameters are estimated by maximum likelihood method and the results are summarized in Table 1. First column shows the result when $\tilde{\omega}_j$'s are estimated individually. Estimates of $\tilde{\omega}_1$, $\tilde{\omega}_3$, and $\tilde{\omega}_5$ are relatively inaccurate considering the standard errors. The second column summarizes the estimates of the parameters when the beta distribution is imposed. The estimates of common parameters $\{\sigma_\gamma, \rho_v, \rho_z, 100\sigma_v, 100\sigma_z\}$ are not much dif-

[5] Since output and investment are measured by consumption good in the model, it is more consistent to use the consumption deflator than the GDP deflator as a price index.

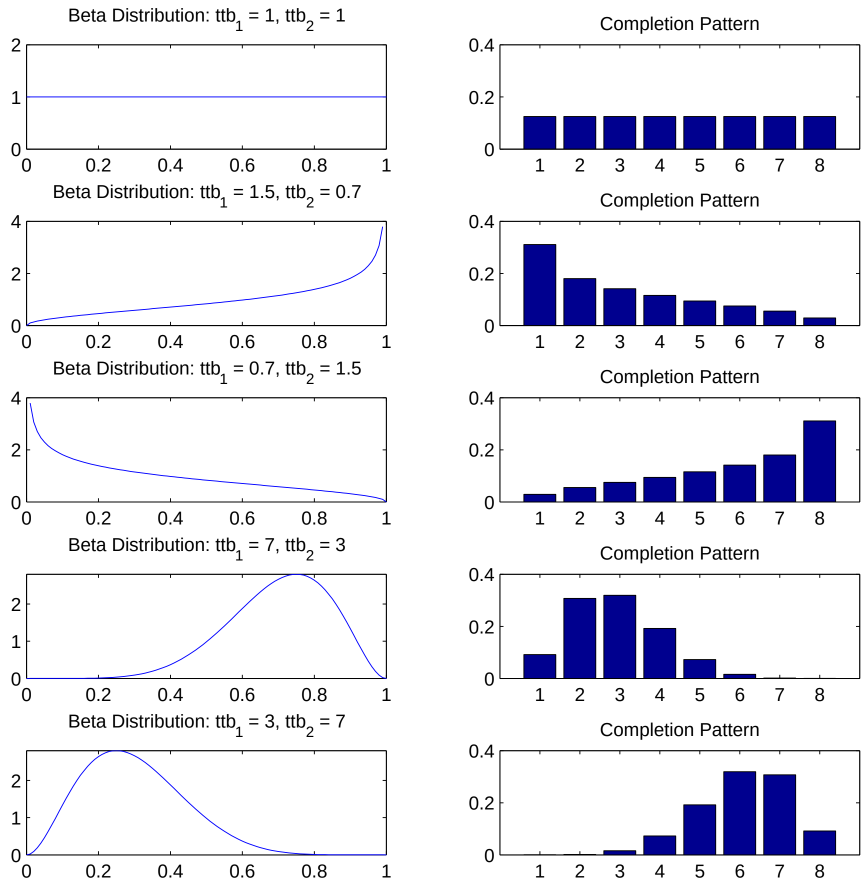


Figure 2: Beta distribution and Completion Pattern

Table 1: Estimates of Parameters*

Parameter	Minimal Restriction	Beta Distribution
σ_γ	0.4363 (0.5685)	0.4384 (0.3518)
ρ_v	0.8727 (0.1206)	0.8299 (0.1242)
ρ_z	0.9773 (0.0343)	0.9839 (0.0285)
$100\sigma_v$	0.3592 (0.0838)	0.4131 (0.0933)
$100\sigma_z$	0.3908 (0.1139)	0.3922 (0.0849)
$\tilde{\omega}_1$	0.4974 (0.4047)	–
$\tilde{\omega}_2$	0.5606 (0.2833)	–
$\tilde{\omega}_3$	0.6002 (0.4880)	–
$\tilde{\omega}_4$	0.8147 (0.3219)	–
$\tilde{\omega}_5$	0.2029 (0.3934)	–
$\tilde{\omega}_6$	0.4217 (0.2538)	–
ttb_1	–	1.5609 (0.1526)
ttb_2	–	1.6132 (0.1365)
Log Likelihood	-444.47	-436.64

* Standard errors in parentheses.

ferent over the two restrictions. However, the parameters of the beta distribution $\{ttb_1, ttb_2\}$ are very significant considering the standard errors.

Figure 3 displays the implied completion patterns from the estimation. Upper panel is the case that the minimal restrictions are imposed. The completion rates of first, second, and third quarter are 13.6, 6.6, and 26.3 percent, respectively. As a result, the completion pattern takes on a saw-tooth shape, which can hardly be justified. On the other hand, as shown in the lower panel, the completion pattern takes on a bell shape when the beta distribution is imposed. The alternative restriction scheme successfully precludes unrealistic completion patterns.

To compare the fits of the model with different restrictions, AIC (Akaike information criterion) and BIC (Bayesian information criterion) values are calculated.[6] When minimal

[6]The information criteria are calculated as follows: $AIC = -2\log L + 2K$ and $BIC = -2\log L + K(\log N)$,

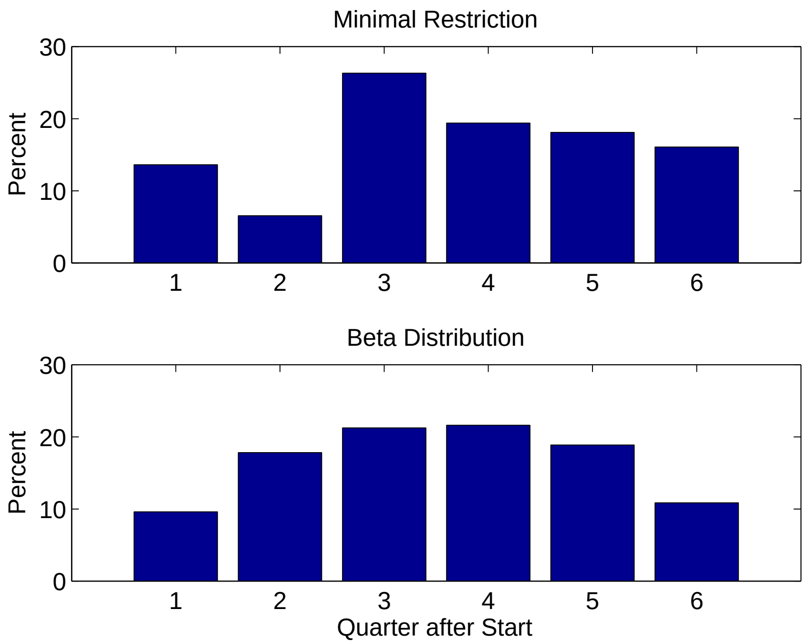


Figure 3: Implied Completion Patterns

restrictions are imposed, AIC and BIC values are 910.94 and 946.77. On the other hand, AIC and BIC values are 887.27 and 910.08 when the beta distribution is used as a restriction. The model selection criteria indicate that imposing the beta distribution is the better strategy to improve the fit of time-to-build model to data. The worse fit of the minimal restriction case could be ascribed to the insignificant estimates of the completion rates. If we restrict the model with the beta distribution, then we may reduce the uncertainty associated with the completion rates.

5. Conclusion

It is difficult to infer aggregate time-to-build completion pattern because of the heterogeneity of capital stock. Therefore, imposing a proper restriction may be helpful to get a tight estimate of the completion pattern. The completion rates of an investment project have the property of a probability distribution. Yet the minimal set of restrictions imposed on the rates is not strong enough to preclude unrealistic estimates. This paper proposes the use of the beta distribution as an alternative restriction. If the parameters of the beta distribution are estimated, then completion rates can be calculated collectively. The estimation results of a time-to-build model indicate that imposing the beta distribution may successfully preclude unrealistic completion pattern and improve fit of the model to data.

where $\log L$ is log likelihood, K is the number of estimated parameters, and N is the number of sample size. The preferred model is the one with the minimum AIC and BIC values.

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