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Developing a two way error component estimation model with disturbances following a special autoregressive (4) for quarterly data

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Abstract

This paper provides an estimation method for a two way error component regression model where the time-varying disturbances are serially correlated, following a special AR (4) process for quarterly data. The variance-covariance matrix of the compound error terms and its spectral decomposition are also derived, allowing the computation of the Generalized Least Square (GLS) estimates and residuals. The Best Quadratic Unbiased (BQU) Estimates of the variance components are proposed, as well as estimates of all parameters involved in the resulting feasible GLS method.

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1. Introduction

According to Balestra and Nerlove (1966), most panel data studies use a random error component specification to model the disturbances found in regression equations. Efforts to account for individual and time-specific effects from a random errors perspective led authors like Wallace and Hussain (1969), Nerlove (1971) and Amemiya (1971) to develop the two-way errors component model.

The classical error component models, either the one way or two way models, assume equi-correlation among the compound errors (see Baltagi, 2005). It is evident that, most economic relationships, especially behavioral functions like investment or consumption, cannot accommodate such a restrictive assumption, knowing that an unobserved shock this period can affect the investor's or consumer's decisions for at least the next few periods. Therefore time-varying disturbances may contain their own correlation patterns, spreading shocks in the overall regression equation. As a result, the equi-correlation assumption is no longer justified. Ignoring such serial correlation when it is present, results in consistent but inefficient estimates of the regression coefficients and biased standard errors (see Greene, 2003).

MacDonald and MacKinnon (1985) argued that the autoregressive (AR(p)) specification is more popular than the MA(q) specification in empirical applications, not because it is more plausible, but rather because it is easier to compute (see Baltagi and Li, 1994). In this perspective, Baltagi and Li (1991, 1992)[1] obtained a simple transformation that changes the autoregressive error component disturbances into spherical disturbances. They show how this transformation can be applied when there is some remaining disturbances in the one way model, follow an AR(1), AR(2), or a special AR(4) process for quarterly data. The last one is the subject of our interest. The two-way error component model with serially correlated error terms is considered by Revankar (1979) and Karlsson and Skoglund (2004). However, in their model, only the time-varying disturbance is assumed to be serially correlated, following an AR(1) process.

Brou et al. (2011) also consider a similar model, with an autocorrelation (AR(1)) in the time specific effect and in the remainder error term. Therefore, the contribution of this paper is to handle the double autocorrelation for the special AR(4) that allows both time-varying disturbances of the compound error to be serially correlated, but each independently following a particular AR(4) process. This type of autoregressive process is chosen because of its relevance for many quarterly data encountered in finance and business cycles analysis.

The remainder of the paper proceeds as follows. In Section 2, we consider simple structures of the disturbances following a particular AR (4) process for quarterly data. In section 3, we explain the structure of the variance-covariance matrix of the transformed error terms and its spectral decomposition. Section 4 presents the GLS transformations of the original data aimed at correcting for the serial correlation in the particular context of the two-way structure. Section 5 derives estimations of the correlation parameter ρ and the BQU estimates of the variances. Some final remarks are given in section 6.

[1] Baltagi and Li (1991, 1992) treat the one-way error component model by considering the presence of serial autocorrelation in the genuine error term.

2. Specification and Assumptions of the Model

First, we present the formulas for a particular AR(4) process for quarterly data[2]. Second, we assume that both the remaining disturbances v_{it} and time effect λ_t follow this particular process.

2.1. The AR (4) process for quarterly data

Consider the following two way error component model (see Baltagi, 2005),

$$y_{it} = \alpha + X_{it}^{'}\beta + u_{it}, \quad i = 1,\cdots,N; t = 1,\cdots,T \tag{1}$$

where α is the intercept and β is a k x 1 vector of slope coefficients, *i*, denote individuals and *t*, time periods.

$$u_{it} = \mu_i + \lambda_t + v_{it}, \quad i = 1,\cdots,N; \; t = 1,\cdots,T \tag{2}$$

Quarterly data process for double autocorrelation with time-varying disturbances is expressed as:

$$\begin{aligned} &\lambda_t = \rho_\lambda \lambda_{t-4} + \varepsilon_t, \; |\rho_\lambda| < 1, \; \varepsilon_t \sim IIN\left(0, \sigma_\varepsilon^2\right) \\ &v_{it} = \rho_v v_{i,t-4} + e_{it}, \; |\rho_v| < 1, \; e_{it} \sim IIN\left(0, \sigma_e^2\right) \end{aligned} \tag{3}$$

The individual-specific effect is assumed to be spherical, that is, $\mu_i \sim IIN\left(0, \sigma_\mu^2\right)$, and ρs are hypothesized as being different for each time-varying disturbances as in Brou et al. (2011). However, in this study, the ρs can be assumed to be the same.

2.2. Assumptions of the Model of our interest

Unlike other studies, we assume that both time-varying disturbances exhibit the same autocorrelation scheme, i.e. the same ρ measures the serial correlation ($\rho = \rho_v = \rho_\lambda$). In this case the time varying errors become: $v_{it} = \rho v_{i,t-4} + e_{it}, |\rho| < 1, \quad e_{it} \sim IIN\left(0, \sigma_e^2\right)$ for the remaining error term and $\lambda_t = \rho\lambda_{t-4} + \varepsilon_t, \; \varepsilon_t \sim IIN\left(0, \sigma_\varepsilon^2\right)$ for the time specific effect.

The restriction on ρ is due to the difficulty to transform the quarterly data into Moving Average (MA) process. This transformation allows the use of Pesaran (1973) orthogonal matrix and easies the handling of the double autocorrelation (see Brou et al., 2011).

We also assume for the double quarterly autocorrelation model that $\mu_i s$, $v s$ and λs are pairwise independent. Likewise, εs and es are also independent. Moreover, we assume that $Cov(v_{it}, v_{jt}) = 0, \forall t, \; \forall i \neq j$, stating that the correlation in the remaining disturbance is not spread among contemporaneous individuals. For convergence purpose and under the

[2] Thomas and Wallis (1971) are among the first authors who suggested a particular AR (4) process. They argue that when quarterly data are used, a fourth-order process may be appropriate. However, instead of a general fourth-order process, they suggest that only the disturbances in corresponding quarters of each year should be correlated.

stationarity assumption, the initial values are defined as $\nu_{i0} \sim N\left(0, \frac{\sigma_e^2}{1-\rho^2} = \sigma_\nu^2\right)$ *and* $\lambda_0 \sim N\left(0, \frac{\sigma_\varepsilon^2}{1-\rho^2} = \sigma_\lambda^2\right)$.

In vector form, $u = \left(\mathbf{I_N} \otimes i_T\right)\mu + \left(i_N \otimes \mathbf{I_T}\right)\lambda + \nu$ (4)

where i_T and i_N are vectors of one's of dimension T and N respectively, I_N and I_T are identity matrices of dimension N and T respectively. $\mu = \left(\mu_1, \cdots, \mu_N\right)'$, $\lambda = \left(\lambda_1, \cdots, \lambda_T\right)'$ and $\nu = \left(\nu_{11}, \cdots, \nu_{1T}, \cdots, \nu_{N1}, \cdots, \nu_{NT}\right)'$, respectively. Thus, u has mean zero and variance-covariance matrix

$$\mathbf{\Sigma} = E\left(uu'\right) = \mathbf{\Sigma_\nu} + \sigma_\mu^2\left(\mathbf{I_N} \otimes \left(i_T i_T'\right)\right) + \left(i_N i_N'\right) \otimes \mathbf{\Sigma_\lambda}, \tag{5}$$

where,

$$\mathbf{\Sigma_\lambda} = E\left(\lambda\lambda'\right) = \sigma_\lambda^2 \begin{pmatrix} 1 & \rho_\lambda(1) & \cdots & \rho_\lambda(T-1) \\ \rho_\lambda(1) & 1 & & \vdots \\ \vdots & & \ddots & \rho_\lambda(1) \\ \rho_\lambda(T-1) & \cdots & \rho_\lambda(1) & 1 \end{pmatrix} = \sigma_\lambda^2 \mathbf{\Gamma_\lambda} \tag{6a}$$

and $\mathbf{\Sigma_\nu}$ is defined as follows

$$\mathbf{\Sigma_\nu} = E\left(\nu\nu'\right) = E\left[\begin{pmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_N \end{pmatrix}\begin{pmatrix} \nu_1' & \nu_2' & \cdots & \nu_N' \end{pmatrix}\right] = \begin{bmatrix} E\left(\nu_1\nu_1'\right) & E\left(\nu_1\nu_2'\right) & \cdots & E\left(\nu_1\nu_N'\right) \\ E\left(\nu_2\nu_1'\right) & E\left(\nu_2\nu_2'\right) & \cdots & E\left(\nu_2\nu_N'\right) \\ \vdots & \vdots & \ddots & \vdots \\ E\left(\nu_N\nu_1'\right) & E\left(\nu_N\nu_2'\right) & \cdots & E\left(\nu_N\nu_N'\right) \end{bmatrix} \quad \text{with}$$

$$E\left(\nu_i\nu_i'\right) = E\left[\begin{pmatrix} \nu_{i1} \\ \nu_{i2} \\ \vdots \\ \nu_{iT} \end{pmatrix}\begin{pmatrix} \nu_{i1} & \nu_{i2} & \cdots & \nu_{iT} \end{pmatrix}\right] = \begin{bmatrix} E\left(\nu_{i1}^2\right) & E\left(\nu_{i1}\nu_{i2}\right) & \cdots & E\left(\nu_{i1}\nu_{iT}\right) \\ E\left(\nu_{i2}\nu_{i1}\right) & E\left(\nu_{i2}^2\right) & \cdots & E\left(\nu_{i2}\nu_{iT}\right) \\ \vdots & \vdots & \ddots & \vdots \\ E\left(\nu_{iT}\nu_{i1}\right) & E\left(\nu_{iT}\nu_{i2}\right) & \cdots & E\left(\nu_{iT}^2\right) \end{bmatrix}$$

that is, $$E\left(\nu_i\nu_i'\right) = \sigma_\nu^2 \begin{pmatrix} 1 & \rho_\nu(1) & \cdots & \rho_\nu(T-1) \\ \rho_\nu(1) & 1 & & \vdots \\ \vdots & & \ddots & \rho_\nu(1) \\ \rho_\nu(T-1) & \cdots & \rho_\nu(1) & 1 \end{pmatrix} = \sigma_\nu^2 \mathbf{\Gamma_\nu} \qquad \forall i = 1, \ldots, N.$$

and $E\left(\nu_i\nu_j'\right) = \mathbf{0} \quad \forall i \neq j$ since $E\left(\nu_{it}\nu_{js}\right) = 0 \quad \text{for} \;\; i \neq j, \;\; \forall t, s.$

Consequently, $$\mathbf{\Sigma_\nu} = \sigma_\nu^2 \begin{bmatrix} \mathbf{\Gamma_\nu} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma_\nu} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Gamma_\nu} \end{bmatrix} = \sigma_\nu^2\left(\mathbf{I_N} \otimes \mathbf{\Gamma_\nu}\right). \tag{6b}$$

In this paper, we assume that λ_t and ν_{it} exhibit the same autocorrelation scheme, as that of a special AR (4) process for quarterly data, $\rho_\lambda(h) = \rho_\nu(h) \;\forall\; h = 1, \ldots, T-1$. Therefore, $\mathbf{\Gamma_\lambda} = \mathbf{\Gamma_\nu} = \mathbf{\Gamma}$.

From $\lambda_t = \rho\lambda_{t-4} + \varepsilon_t$ and $v_{it} = \rho v_{i,t-4} + e_{it}$, we obtain $\rho_\lambda(h) = \rho_v(h) = \begin{cases} \rho^{|h|/4} & \text{if } \dfrac{h}{4} \text{ is an integer} \\ 0 & \text{otherwise.} \end{cases}$

Hence,

$$\mathbf{\Gamma} = Toeplitz\left(1,0,0,0,\rho,0,0,0,\rho^2,0,\ldots,\frac{\gamma_{T-1}}{\gamma_0}\right) \text{ where } \frac{\gamma_{T-1}}{\gamma_0} = \begin{cases} \rho^{(T-1)/4} & \text{if } \dfrac{T-1}{4} \text{ is an integer} \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

The correlation coefficient is now:

$$Correl(u_{it}, u_{js}) = \begin{cases} \left[\sigma_\mu^2 + \rho^{|t-s|/4}\left(\sigma_\lambda^2 + \sigma_v^2\right)\right] / \left(\sigma_\mu^2 + \sigma_\lambda^2 + \sigma_v^2\right) & \text{for } i = j,\ t \neq s \text{ and } \exists\, k \in \mathbb{Z} : t = 4k + s \\ \sigma_\mu^2 / \left(\sigma_\mu^2 + \sigma_\lambda^2 + \sigma_v^2\right) & \text{for } i = j,\ t \neq s \text{ and } t \neq 4k + s \quad \forall k \in \mathbb{Z} \\ \sigma_\lambda^2 / \left(\sigma_\mu^2 + \sigma_\lambda^2 + \sigma_v^2\right) & \text{for } i \neq j \text{ and } t = s \\ \rho^{|t-s|/4}\sigma_\lambda^2 / \left(\sigma_\mu^2 + \sigma_\lambda^2 + \sigma_v^2\right) & \text{for } i \neq j,\ t \neq s \text{ and } \exists\, k \in \mathbb{Z} : t = 4k + s \\ 0 & \text{for } i \neq j,\ t \neq s \text{ and } t \neq 4k + s \quad \forall k \in \mathbb{Z} \\ 1 & \text{for } i = j,\ t = s. \end{cases}$$

Unlike the classical assumption, we may note that, the correlation coefficient actually depends on the time length $\left|\dfrac{t-s}{4}\right|$. This confirms the assumption that, omitted or unobserved variables can affect dependent variables at least within the next few periods.

3. Variance-Covariance of the Transformed Errors and Its Spectral Decomposition

When the time-varying disturbances terms follow a special AR(4) process for quarterly data, the transformation used to correct for the autocorrelation can be defined, as in Baltagi (2005), by:

$$\begin{aligned} u_{it}^* &= \sqrt{1-\rho^2}\, u_{it} && \text{for } t = 1,2,3,4 \\ u_{it}^* &= u_{it} - \rho u_{i,t-4} && \textit{for } t = 5,6,\ldots,T. \end{aligned}$$

The corresponding Prais-Winsten transformation matrix is therefore

$$\mathbf{C} = \begin{pmatrix} \sqrt{1-\rho^2} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{1-\rho^2} & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \sqrt{1-\rho^2} & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \sqrt{1-\rho^2} & 0 & 0 & \cdots & 0 \\ -\rho & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & -\rho & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & -\rho & 0 & 0 & & \cdots & 1 \end{pmatrix} \quad (8)$$

The variance-covariance matrix of the transformed errors is,

$$\begin{aligned} \mathbf{\Sigma}^* &= \left(\mathbf{I_N} \otimes \mathbf{C}\right)\Sigma\left(\mathbf{I_N} \otimes \mathbf{C}'\right) \\ &= \left(\mathbf{I_N} \otimes \mathbf{C\Sigma_v C}'\right) + \sigma_\mu^2\left(\mathbf{I_N} \otimes \mathbf{C}\left(i_T i_T'\right)\mathbf{C}'\right) + \left(\left(i_N i_N'\right) \otimes \mathbf{C\Sigma_\lambda C}'\right). \end{aligned} \quad (9)$$

Following Baltagi and Li (1991), we set $i_T^{\alpha}=\left(\alpha,\alpha,\alpha,\alpha,i_{T-4}^{'}\right)$ where $\alpha=\sqrt{(1+\rho)/(1-\rho)}$, and $\mathbf{J}_{\mathbf{T}}^{\boldsymbol{\alpha}}=\left(i_T^{\alpha}\right)\left(i_T^{\alpha}\right)^{'}$, $\mathbf{J}_{\mathbf{N}}=i_N i_N^{'}$. We then obtain,

$$\begin{cases}\mathbf{C}\boldsymbol{\Sigma}_{\mathbf{v}}\mathbf{C}^{'}=\sigma_e^2\mathbf{I}_{\mathbf{T}}\\ \mathbf{C}\boldsymbol{\Sigma}_{\boldsymbol{\lambda}}\mathbf{C}^{'}=\sigma_\varepsilon^2\mathbf{I}_{\mathbf{T}}\\ \mathbf{C}i_T=(1-\rho)i_T^{\alpha}.\end{cases} \tag{10}$$

Therefore, $$\boldsymbol{\Sigma}^{*}=\sigma_e^2\left(\mathbf{I}_{\mathbf{N}}\otimes\mathbf{I}_{\mathbf{T}}\right)+\sigma_\mu^2\left(1-\rho\right)^2\left(\mathbf{I}_{\mathbf{N}}\otimes\mathbf{J}_{\mathbf{T}}^{\alpha}\right)+\sigma_\varepsilon^2\left(\mathbf{J}_{\mathbf{N}}\otimes\mathbf{I}_{\mathbf{T}}\right) \tag{11}$$

In order to get idempotent matrices, we make the following transformation: $\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}=\dfrac{i_T^{\alpha}i_T^{\alpha'}}{d^2}$, $\overline{\mathbf{J}}_{\mathbf{N}}=\dfrac{i_N i_N^{'}}{N}$ where $d^2=i_T^{\alpha'}i_T^{\alpha}=4\alpha^2+T-4$. We then use Wansbeek and Kapteyn (1982, 1983) approach, in the expression of $\boldsymbol{\Sigma}^{*}$, that is $\mathbf{I}_{\mathbf{N}}$ is replaced by $\mathbf{E}_{\mathbf{N}}+\overline{\mathbf{J}}_{\mathbf{N}}$ where $\mathbf{E}_{\mathbf{N}}=\mathbf{I}_{\mathbf{N}}-\overline{\mathbf{J}}_{\mathbf{N}}$; $\mathbf{J}_{\mathbf{N}}$ is replaced by $\mathbf{N}\overline{\mathbf{J}}_{\mathbf{N}}$; $\mathbf{J}_{\mathbf{T}}^{\boldsymbol{\alpha}}$ is replaced by $d^2\overline{\mathbf{J}}_{\mathbf{t}}^{\boldsymbol{\alpha}}$; and $\mathbf{I}_{\mathbf{T}}$ is replaced by $\mathbf{E}_{\mathbf{T}}^{\boldsymbol{\alpha}}+\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}$ where $\mathbf{E}_{\mathbf{T}}^{\boldsymbol{\alpha}}=\mathbf{I}_{\mathbf{T}}-\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}$ to obtain;

$$\begin{aligned}\boldsymbol{\Sigma}^{*}&=\sigma_e^2\left[\left(\mathbf{E}_{\mathbf{N}}+\overline{\mathbf{J}}_{\mathbf{N}}\right)\otimes\left(\mathbf{E}_{\mathbf{T}}^{\boldsymbol{\alpha}}+\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}\right)\right]+d^2(1-\rho)^2\sigma_\mu^2\left[\left(\mathbf{E}_{\mathbf{N}}+\overline{\mathbf{J}}_{\mathbf{N}}\right)\otimes\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}\right]+N\sigma_\varepsilon^2\left[\overline{\mathbf{J}}_{\mathbf{N}}\otimes\left(\mathbf{E}_{\mathbf{T}}^{\boldsymbol{\alpha}}+\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}\right)\right]\\ &=\sigma_e^2\left(\mathbf{E}_{\mathbf{N}}\otimes\mathbf{E}_{\mathbf{T}}^{\boldsymbol{\alpha}}\right)+\left(\sigma_e^2+d^2(1-\rho)^2\sigma_\mu^2\right)\left(\mathbf{E}_{\mathbf{N}}\otimes\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}\right)+\left(\sigma_e^2+N\sigma_\varepsilon^2\right)\left(\overline{\mathbf{J}}_{\mathbf{N}}\otimes\mathbf{E}_{\mathbf{T}}^{\boldsymbol{\alpha}}\right)+\\ &\quad+\left(\sigma_e^2+d^2(1-\rho)^2\sigma_\mu^2+N\sigma_\varepsilon^2\right)\left(\overline{\mathbf{J}}_{\mathbf{N}}\otimes\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}\right).\end{aligned}$$

The spectral decomposition of $\boldsymbol{\Sigma}^{*}$ is $$\boldsymbol{\Sigma}^{*}=\sum_{i=1}^{4}\gamma_i\mathbf{Q}_{\mathbf{i}} \tag{12}$$

with, $\gamma_1=\sigma_e^2$, $\gamma_2=\sigma_e^2+d^2(1-\rho)^2\sigma_\mu^2$, $\gamma_3=\sigma_e^2+N\sigma_\varepsilon^2$, $\gamma_4=\sigma_e^2+d^2(1-\rho)^2\sigma_\mu^2+N\sigma_\varepsilon^2$ and $\mathbf{Q}_{\mathbf{1}}=\mathbf{E}_{\mathbf{N}}\otimes\mathbf{E}_{\mathbf{T}}^{\boldsymbol{\alpha}}$, $\mathbf{Q}_{\mathbf{2}}=\mathbf{E}_{\mathbf{N}}\otimes\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}$, $\mathbf{Q}_{\mathbf{3}}=\overline{\mathbf{J}}_{\mathbf{N}}\otimes\mathbf{E}_{\mathbf{T}}^{\boldsymbol{\alpha}}$, $\mathbf{Q}_{\mathbf{4}}=\overline{\mathbf{J}}_{\mathbf{N}}\otimes\overline{\mathbf{J}}_{\mathbf{T}}^{\boldsymbol{\alpha}}$.

4. GLS Transformation

However, at this step, the overall disturbances $u*$ is still not spherical. This issue can be overcome by a GLS approach. Following Fuller and Battese (1974), where the new transformation matrix could be $\sigma_e\boldsymbol{\Sigma}^{*-1/2}$. From the spectral decomposition of $\boldsymbol{\Sigma}^{*}$, it follows that: $$\sigma_e\boldsymbol{\Sigma}^{*-1/2}=\sum_{i=1}^{4}\frac{\sigma_e}{\gamma_i}\mathbf{Q}_{\mathbf{i}}=\mathbf{Q}_1+\sum_{i=2}^{4}\frac{\sigma_e}{\gamma_i}\mathbf{Q}_{\mathbf{i}} \tag{13}$$

Premultiplying the Prais-Winsten transformed observations $y^{*}=\left(\mathbf{I}_{\mathbf{N}}\otimes\mathbf{C}\right)y$ by $\sigma_e\boldsymbol{\Sigma}^{*-1/2}$, one gets $y^{**}=\sigma_e\boldsymbol{\Sigma}^{*-1/2}y^{*}$. Its typical elements are given by:

$$y_{it}^{**}=\begin{cases}y_{it}^{*}-\theta_1\alpha b_i-\theta_2\bar{y}_{\bullet t}^{*}+\theta_3\alpha b & if\quad t=1,2,3,4\\ y_{it}^{*}-\theta_1 b_i-\theta_2\bar{y}_{\bullet t}^{*}+\theta_3 b & if\quad t=5,....,T\end{cases} \tag{14}$$

where $$\theta_1=1-\frac{\sigma_e}{\gamma_2^{\frac{1}{2}}},\quad \theta_2=1-\frac{\sigma_e}{\gamma_3^{\frac{1}{2}}},\quad \theta_3=\theta_1+\theta_2+\frac{\sigma_e}{\gamma_4^{\frac{1}{2}}}-1,\ b=\sum_{i=1}^{N}b_i \tag{15}$$

and $$b_i=\frac{1}{d^2}\left(\alpha\sum_{t=1}^{4}y_{it}^{*}+\sum_{t=5}^{T}y_{it}^{*}\right), \tag{16}$$

as defined by Baltagi (2005). The b_i s are weighted averages of Prais-Winsten transformed observations with a special weight $\frac{\alpha}{d^2}$ to the first four observations.

Likewise the one-way AR(1), AR(2) and AR(4) serially correlated error component model (see Baltagi, 2005), the estimation of the two-way model can be done through two steps: (i) firstly, by applying the Prais-Winsten transformation as it is usually done in the time-series literature, and (ii) lastly by subtracting a pseudo-average from these transformed data.

This procedure can be reduced to a one-step one, since y^{**} can be directly expressed in terms of y as $y^{**} = \sigma_e \Sigma^{*\frac{-1}{2}} (\mathbf{I_N} \otimes \mathbf{C}) y$ with typical elements given as

$$y_{it}^{**} = \begin{cases} \sqrt{1-\rho^2}\left(y_{it} - \theta_2 \bar{y}_{\bullet t}\right) - \theta_1 \alpha b_i + \theta_3 \alpha b & if \quad t = 1,2,3,4 \\ \left(y_{it} - \theta_2 \bar{y}_{\bullet t}\right) - \rho\left(y_{i,t-4} - \theta_2 \bar{y}_{\bullet, t-4}\right) - \theta_1 b_i + \theta_3 b & if \quad t = 5,....,T \end{cases} \tag{17}$$

where the b_i s are now seen as weighted average of the original observations:

$$b_i = \frac{1}{d^2}\left[(1+\rho)\sum_{t=1}^{4} y_{it} + \sum_{t=5}^{T}\left(y_{it} - \rho y_{it-4}\right)\right] \tag{18}$$

After the transformation of the model, one can seek Best Quadratic Unbiased Estimates (BQUE) and parameters estimates.

5. BQU Estimates and Parameters Estimation

To get the BQU estimates of the variance components; Σ^* being the variance-covariance matrix of the model, by applying Balestra (1973) results and from the spectral decomposition, we obtain,

$$\mathbf{Q_i} u^* \sim N(0, \gamma_i \mathbf{Q_i}), \text{ for } i = 1, \cdots, 4. \tag{19}$$

The best quadratic unbiased estimator of γ_i is equal to $\hat{\gamma}_i = \frac{u^{*'} \mathbf{Q_i} u^*}{trace(\mathbf{Q_i})}$, for all *i*.

Thus,
$$\begin{cases} \hat{\sigma}_e^2 = \dfrac{u^{*'}\left(\mathbf{E_N} \otimes \mathbf{E_T^\alpha}\right)u^*}{(N-1)(T-1)} \\[2ex] \hat{\sigma}_e^2 + \hat{d}^2(1-\hat{\rho})^2 \hat{\sigma}_\mu^2 = \dfrac{u^{*'}\left(\mathbf{E_N} \otimes \mathbf{\bar{J}_T^\alpha}\right)u^*}{(N-1)} \\[2ex] \hat{\sigma}_e^2 + N\hat{\sigma}_\varepsilon^2 = \dfrac{u^{*'}\left(\mathbf{\bar{J}_N} \otimes \mathbf{E_T^\alpha}\right)u^*}{(T-1)} \\[2ex] \hat{\sigma}_e^2 + \hat{d}^2(1-\hat{\rho})^2 \hat{\sigma}_\mu^2 + N\hat{\sigma}_\varepsilon^2 = \dfrac{u^{*'}\left(\mathbf{\bar{J}_N} \otimes \mathbf{\bar{J}_T^\alpha}\right)u^*}{1^2}. \end{cases} \tag{20}$$

From a practical point of view, we face several unknown parameters: $\rho, \sigma_e, \sigma_\varepsilon, \sigma_\mu, \sigma_e, \gamma_2, \gamma_3$ and γ_4. We first need an estimate of the AR (4) parameter ρ. Following Baltagi and Li (1997) recommendation in the one-way serially correlated model, an estimator of ρ based on the autocovariance function $q_s = E\left(u_{it} u_{i,t-s}\right)$ will be derived.

From $u_{it} = \mu_i + \lambda_t + v_{it}$, $v_{it} = \rho v_{i,t-4} + e_{it}$ and $\lambda_t = \rho\lambda_{t-4} + \varepsilon_t$; we obtain:

$$q_s = E\left(u_{it}u_{i,t-s}\right) = \begin{cases} \sigma_\mu^2 + \rho^{|t-s|/4}\left(\sigma_\lambda^2 + \sigma_\nu^2\right) & \text{if } \exists\, k \in \mathbb{Z} : t = 4k + s \\ \sigma_\mu^2 & \text{otherwise} \end{cases}$$

One deduces:

$$q_0 = E\left(u_{it}^2\right) = \sigma_\mu^2 + \sigma_\lambda^2 + \sigma_\nu^2 ; q_1 = E\left(u_{it}u_{i,t-1}\right) = \sigma_\mu^2 \; and \; q_4 = E\left(u_{it}u_{i,t-1}\right) = \sigma_\mu^2 + \rho\left(\sigma_\lambda^2 + \sigma_\nu^2\right).$$

Hence, $\dfrac{q_4 - q_1}{q_0 - q_1} = \rho$. We get an estimator $\hat{\rho}$ as $\hat{\rho} = \dfrac{\tilde{q}_4 - \tilde{q}_1}{\tilde{q}_0 - \tilde{q}_1}$, where $\tilde{q}_s = \dfrac{1}{N(T-s)} \sum_{i=1}^{N} \sum_{t=s+1}^{T} \hat{u}_{it}\hat{u}_{it-s}$ and $\hat{u}_{it}$ denote the OLS residuals of model (4).

One can obtain feasible estimates of the variance components by replacing, in (20), the true disturbances u^* by OLS residuals, as suggested by Wallace and Hussain (1969), or by the within-residuals (Amemiya 1971). Although OLS estimates are unbiased, they are asymptotically inefficient with biased standard errors (Amemiya, 1971). In contrast, the within-estimators are unbiased and asymptotically efficient, as any feasible GLS estimators (see Baltagi, 2005 and Prucha, 1984). Since the Within-regression uses only part of the available data, Swamy and Arora (1972) suggest a feasible GLS estimator in three steps.

Following Swamy and Arora (1972), we suggest running three least squares regressions by transforming the data by some Q_i's matrices. The first one consists in transforming the Prais-Winsten data by $\mathbf{Q_1} = \mathbf{E_N} \otimes \mathbf{E_T^{\alpha}}$. It yields an estimate of σ_e^2:

$$\hat{\hat{\gamma}}_1 = \hat{\hat{\sigma}}_e^2 = \left[y^{*'}Q_1y^* - y^{*'}Q_1X^*\left(X^{*'}Q_1X^*\right)^{-1}X^{*'}Q_1y^* \right] \Big/ \left[(N-1)(T-1) - K \right] \tag{21}$$

The second regression transforms the Prais-Winsten data by $\mathbf{Q_2} = \mathbf{E_N} \otimes \mathbf{\bar{J}_T^{\alpha}}$ and estimate γ_2, as:

$$\hat{\hat{\gamma}}_2 = \left[y^{*'}Q_2y^* - y^{*'}Q_2X^*\left(X^{*'}Q_2X^*\right)^{-1}X^{*'}Q_2y^* \right] \Big/ \left[(N-1) - K \right] \tag{22}$$

With $\hat{\hat{\sigma}}_\mu^2 = \left(\hat{\hat{\gamma}}_2 - \hat{\hat{\sigma}}_e^2\right) \Big/ \left[\hat{d}^2(1-\hat{\rho})^2 \right] = \left(\hat{\hat{\gamma}}_2 - \hat{\hat{\sigma}}_e^2\right) \Big/ f\left(\hat{\rho}\right)$ and $f\left(\hat{\rho}\right) = \left(\dfrac{4(1+\hat{\rho})}{1-\hat{\rho}} + T - 4 \right)(1-\hat{\rho})^2$.

Hence, $f(\hat{\rho}) = (T-8)\hat{\rho}^2 - 2(T-4)\hat{\rho} + T$.

The third regression uses $\mathbf{Q_3} = \mathbf{\bar{J}_N} \otimes \mathbf{E_T^{\alpha}}$ to obtain an estimate of $\gamma_3 = \sigma_e^2 + N\sigma_\varepsilon^2$, expressed as

$$\hat{\hat{\gamma}}_3 = \left[y^{*'}Q_3y^* - y^{*'}Q_3X^*\left(X^{*'}Q_3X^*\right)^{-1}X^{*'}Q_3y^* \right] \Big/ \left[(T-1) - K \right] \tag{23}$$

and $\hat{\hat{\sigma}}_\varepsilon^2 = \left(\hat{\hat{\gamma}}_3 - \hat{\hat{\sigma}}_e^2\right) \Big/ N$ (24)

The parameter $\hat{\hat{\gamma}}_4$ is then deduced as $\hat{\hat{\gamma}}_4 = \hat{\hat{\sigma}}_e^2 + f\left(\hat{\rho}\right)\hat{\hat{\sigma}}_\mu^2 + N\hat{\hat{\sigma}}_\varepsilon^2$ (25)

We are now able to deduce estimates of the $\theta_i s$ as they appear in equations (15). We then implement the GLS procedure to get the estimated coefficients and complete the estimation of our two-way serially correlated AR (4) error component model for quarterly data.

6. Final Remarks

This paper investigated the issue of serial correlation in a two-way random effect model, when all time-varying components of the error term exhibit the same serial correlation of a special AR(4) type. Through the spectral decomposition of the variance-covariance matrix of the transformed errors, a feasible GLS estimation procedure was suggested, with clear estimates of all parameters involved in its implementation.

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