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Promotion Policy and Firm Size

Ori Zax
Tel-Hai Academic College

Abstract

In contrast to the predictions of conventional economic theory, it is well documented that similar workers receive wages positively correlated with the size of the firm employing them. To explain these findings we augment the Waldman (1984) framework by adding a size variable and construct a dynamic model of promotion and obtain an equilibrium with a positive correlation between firm size and wages.

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Contact: Ori Zax - orizax@gmail.com.
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1 Introduction

A large body of empirical evidece shows that larger firms pay higher wages than smaller ones do (Abowd et al., (1999), Brown and Medoff, (1989), Bayard and Troske (1999)). At first glance, this contradicts conventional economic theory, which predicts the disappearance of such gaps after similar workers move from low-paying jobs to better-paying jobs.

Several theoretical justifications for the observed wage gaps have appeared in the economic literature. One class of explanations considers the moral-hazard problem and the related issue of supervision costs (Becker and Stigler (1974), Weiss (1990)). Search friction provides another explanation (Burdett and Mortensen (1998) and Postal-Vinay and Robin (2002)).

In this paper, we use Waldman (1984a) structure where he analyzes promotion decisions taking into account the information revealed by the hierarchy level of a worker. We examine the promotion process in an environment populated by firms of different sizes. We show that the wage gap is the result of differences in promotion policies among firms of different sizes. The main implication of the promotion policy is that managers in larger firms are better on average and, therefore, demand and receive higher wages than counterparts in smaller firms. Laborers, in turn, receive higher wages to compensate them for a lower promotion rate. Whereas most of the aforementioned models concentrate on "why larger firms overpay," our model focuses on "why workers at smaller firms are willing to be underpaid."

2 The model

We assume there is a fixed population (continuum) of firms of different size, i.e., employing different numbers of workers. We take the initial size distribution as given[1]. Each firm

[1] A large body of empirical evidence shows that the size distribution of firms is neither degenerate nor constant (Cabral and Mata (2003), Paolo and Generale (2008)).

Theoretical explanation for this phenomenon can be found at Lucas (1978), Waldman (1984b), Burdett and Mortensen (1998), Albuquerque and Hopenhayn (2004), Cooley and Quadrini (2001) and Cooley, Marimon and Quadrini (2004).

has two hierarchy levels (laborers and managers). Each worker (laborer or manager) lives for two periods. Workers' productivity depends on duration of their employment and position (hierarchy level). We assume the existence of a positive productivity premium for remaining with a firm for more than one period. Workers' ability is uniformly distributed on the interval $[0, 1]$. A_i denotes the ability of worker i. We assume that the worker's ability is unknown ex ante and is revealed to the host firm (the employer) at no cost at the end of the first period.thispagestyleempty

The production function is given by $F(z, m)$, where z represents the number of laborers (first-level workers), in efficiency units and m is the sum of the abilities of the managers (second-level workers). We assume that F is continuously differentiable and that $F_1, F_2 > 0, F_{12} > 0, F_{11}, F_{22} < 0$.

In the first period all workers are employed as laborers. In order to be a candidate for possible promotion it is necessary to have accumulated experience as a laborer. We assume that, at the end of the first period, the host firm observes the worker's ability level and decides who is to be promoted to the second level of the hierarchy, i.e., who will become a manager. In making this promotion decision, the firm also considers the wage that it will have to pay the promoted worker, a wage derived from the wage offers of competing firms. The promotion policy and the fact the worker has been promoted provide competing firms with additional information about the worker's abilities - information that will raise the offers that he may receive.

The firm's strategy for the second period consists of a threshold level for promotion, managerial wage level, laborer wage level, and wages offered to managers from other firms. In equilibrium, each firm's strategy maximizes its profits, taking into account the strategies that its competitors choose. Therefore, the strategy in the second period is a Nash equilibrium for the game that starts in the second period. The workers' strategy for the second period is to choose the firm that offers them the highest wage. We assume that in the case of a tie, workers will remain with their host firm. The workers' strategy for the first period is to choose the firm that offers him the highest lifetime wage. If more than one firm makes the highest offer, then the worker chooses one randomly.

As we show below, the profit generated by a promoted worker grows with his level of ability. This leads to an optimal promotion policy characterized by a threshold level, A^+, which is the ability level, from and above which a worker is promoted. Note that A^+, in view of the assumption of a uniform distribution on $[0,1]$ actually represents the proportion of non-promoted workers (laborers) and hence the probability of not being promoted.

The firm's laborers are divided into two groups. One group is the second period laborers consisting of laborers who are employed for a second period, of size A^+L. The other is the first period laborers, consisting of the "newcomers", of size L, who are employed by the firm for the first time. In terms of efficiency units, each member of the second period laborers equals 1, whereas those in the other group equal $\beta < 1$ efficiency units.

To get an expression of the sum of the abilities of the firm's internal managers (denoted by m), note that,

$$m = \alpha L \int_{A^+}^{1} A dA = \alpha L \frac{1 - A^{+2}}{2}$$

The sum of the managers' abilities is given by the product of the proportion of managers and their average ability. When the firm recruits external managers, we obtain

$$m = \alpha L \frac{1 - A^{+2}}{2} + \int_{\underline{L}}^{\bar{L}} o(L)\, dL$$

where $\bar{L}$ and $\underline{L}$ denote the the largest and smallest firm, respectively, $o(L)$ is the number of managers recruited from a firm of size L. The parameter $\alpha > 1$ reflects the relative advantage of an internal manager. We assume that α is large enough such that $o(L) = 0$ in equilibrium, for all L. Thus, if the firm employs both first and second period laborers, we obtain the following production function:

$$F\left(L\left(A^+ + \beta\right), \alpha L \frac{1 - A^{+2}}{2}\right) \qquad (1)$$

To find the promotion threshold level, we first calculate the manager's wage as a function of the threshold level, A^+. The manager's wage results from competition over him by his current employee and other potential employers.

We assume that one individual manager does not effect the sum of all abilities. We obtain that each firm's wage offer (which equals to the highest external wage the manager can achieve) is given by

$$\frac{1 + A^{+}(L)}{2} T \tag{2}$$

This wage is the product of two components: the expected ability that a manager in a firm of size L signals, and T - the alternative wage of ability unit which equals the highest marginal product obtainable in another firm, i.e., $T = F_2$, where F_2 equals the marginal product of ability unit in the largest firm[2]. Note that the wage of each manager equals the highest external wage offer he could get and hence all managers remain in their host firm (Note that each manager obtains an outside wage offer which is given by equation (2), and will quit the firm if she offers him a lower wage).

The wage of a second period laborer (a non-promoted worker), also given by his alternative wage, is constant across the firms and is denoted by k. The wage of a first period employee is discussed later. The analysis is made under the assumption that in equilibrium

$$F_1 - k > \beta F_1 - w_{1l} > 0 \text{ for all } L \tag{3}$$

This implies that the marginal profit from employing a second period laborer exceeds that of employing a first period laborer, hence the firm will employ both first and second period laborers.

To simplify matters, we assume that the alternative wage of a manager employed in the smallest firm is higher than k, the alternative wage of a non promoted worker, $\left(T\frac{1+A^{+}(\mathrm{L})}{2} > k\right)$. This is satisfied if differences among firms are not too profound. In the absence of this assumption, the alternative wage of managers employed in a small firm equals k, while the wage of a manager employed in a large firm is given by Equation (2).

We also assume that the number of workers employed by the smallest firm is large enough so that the distribution of workers' abilities throughout the population is a good approximation for the distribution of workers' abilities in each firm.

[2]Note that, the production function given by equation (1) yields a linear relationship between the expected ability and the expected marginal product of each manager.

In order to obtain an internal solution we assume that

$$F_1\left(L\beta, 0.5\alpha L\right) > F_2\left(\beta L, 0.5\alpha L\right)\ \ for\ all\ L \tag{4}$$

With this condition promoting all second period workers (choosing $A^+ = 0$) cannot be optimal since it would result in managers having lower marginal product than laborers. We also assume that $F_2\left(L\left(1+\beta\right), 0\right) = \infty$, hence in equilibrium $A^+ < 1$.

To find the optimal threshold level, A^+, we consider the profit of the firm as a function of A^+ (an optimum exists since profits are continuous in A^+, which is also bounded between zero and one). Hence, the firm maximizes

$$\pi = F\left(L\left(A^+ + \beta\right), \alpha L \frac{1 - A^{+2}}{2}\right) - A^+ L k - \frac{1 - A^{+2}}{2} T L - L w_{1l} \tag{5}$$

Differentiating with respect to A^+ yields the following first order condition:

$$F_1 - \alpha F_2 A^+ - k + T A^+ = 0$$

By performing implicit differentiation of the first-order condition, we obtain the following equation:

$$\frac{dA^+}{dL} = -\frac{F_{11}\left(A^+ + \beta\right) - \alpha^2 A^+ F_{22} \frac{1 - A^{+2}}{2} + F_{12}\alpha\left(\frac{1 - A^{+2}}{2} - A^+\left(A^+ + \beta\right)\right)}{F_{11}L - F_{12}\alpha L A^+ - \alpha F_2 - A^+ \alpha F_{21} L - A^{+2}\alpha^2 L F_{22} + T} \tag{6}$$

We denote the numerator by H.

If the alternative wage of a manager employed in the smallest firm is lower than k, the alternative wage of a non promoted worker, than Equation (6) becomes

$$\frac{dA^+}{dL} = -\frac{F_{11}\left(A^+ + \beta\right) - \alpha^2 A^+ F_{22} \frac{1 - A^{+2}}{2} + F_{12}\alpha\left(\frac{1 - A^{+2}}{2} - A^+\left(A^+ + \beta\right)\right)}{F_{11}L - F_{12}\alpha L A^+ - \alpha F_2 - A^+ \alpha F_{21} L - A^{+2}\alpha^2 L F_{22}}.$$

Using second order conditions, we obtain the following:

Theorem 1 *The proportion of managers decreases (increases) with firm size if the production technology satisfies $H > 0$ ($H < 0$).*

The above equation does not have a closed form solution for A^+. In appendix A we provide a numerical example using the standard Cobb-Douglas production function to generate the $H > 0$ scenario.

Intuitively, F_{11} appears in the first term and as it increases (in absolute value) the optimal proportion of managers increases. Thus, the faster the marginal productivity of labor decreases, the firm wishes to increase the proportion of managers. Similarly the rate at which the marginal productivity of labor changes (F_{22}) appears in the second term and as it increases, the optimal proportion of managers decreases. The cross-derivative of the production function with respect to laborers and managers (F_{12}) which appears in the third term will raise or lower the optimal proportion of managers the more sensitive is the marginal product of managers to the number of laborers dependent on the sign of $\left(\frac{1-A^{+2}}{2} - A^{+}\left(A^{+} + \beta\right)\right)$.

Building on Theorem 1 and assuming that $H > 0$, we obtain the following:

Corollary 1 *The smaller the firm, the less ability is attributed to the workers whom it promotes.*

Proof. According to Theorem 1, the proportion of workers promoted is larger in smaller firms. Thus, the threshold for promotion is lower in a smaller firm, meaning that the promoted workers in such a firm are less able (on average). ■

This lower level of ability translates into lower alternative wages, resulting in a positive relationship between firm size and wages paid to promoted workers in the second period. Note that since the percentage of those promoted is not constant across firms, the total wage cost in the second period does not necessarily rise.

Corollary 2 *When workers are risk-neutral and the promotion policy satisfies $A^{+}T > k$, larger firms pay higher wages in the first period (w_{1l}).*

Proof. We denote by $W(A^{+})$ the expected wage in the second period. If a worker is not promoted, the wage he receives is k, if he is promoted the wage he receives is $\frac{1+A^{+}}{2}T$, hence $W\left(A^{+}\right) = \left(1 - A^{+}\right)\frac{1+A^{+}}{2}T + A^{+}k = \frac{1-A^{+2}}{2}T + A^{+}k$.

Differentiating with respect to A^{+}, we obtain:

$\frac{dW}{dA^{+}} = -A^{+}T + k < 0$ (by assumption)

The risk neutrality of the workers implies that in equilibrium the total wage paid by the firm must equal the alternative that individuals could earn outside this industry, hence

$$W(A^{+}) + w_{1l} = Const$$

This means that an increase in the wage paid in the second period must be associated with a lower wage in the first period. Since A^+ is increasing in firm size and $W(A^+)$ is a decreasing function, w_{1l} is increasing in firm size as well. ■

Note that the restriction $A^+T > k$ is not overly severe, since $k < \alpha F_2 A^+$. (The wage of a worker is lower than the product of the marginal manager.) This phenomenon of higher wages paid by larger firms in the first period is even more pronounced when workers are risk-averse.

Having established the positive relationship between wages and firm size for managers and first-period workers, we now examine the wage of a second-period laborer. In the setup discussed thus far, it is indeed constant across firms. However, we can show that if firms are allowed to commit to pre-specified wage levels in the second period for laborers (instead of paying them their alternative wage) and workers are risk-averse and have no access to capital markets (i.e., cannot transfer income across periods), we again obtain a positive relationship between wage and firm size.

3 Conclusion

The positive correlation between firm size and wages is a well-documented empirical observation that is valid across countries and time.

In the present paper, we analyze the promotion decisions of different sized firms and obtain conditions under which larger firms employ a lower proportion of managers. This implies that a manager in a larger firm is better on average, hence will receive a higher wage. In order to maintain workers indifferent among the variety of firms, larger firms compensate their non-promoted employees.

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