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A note on bargaining over complementary pieces of information in networks

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Abstract

We consider two specific network structures, the star and the line, and study the set of bilateral alternating-offers bargaining processes for the pairs of linked agents. Agents have complementary information, bargain simultaneously over the price of their pieces of information, and benefit from their exchanges only after they finish all their negotiation processes. We propose meaningful distributions of the initial bargaining power of the agents according to the restrictions to negotiation imposed by the network, and study the resulting equilibrium prices and payoffs.

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1. Introduction

Consider a situation in which, for a given network, each pair of linked agents bargain over their endowments of information and suppose that each agent benefits from the exchange of information with her neighbors only when the bargaining with *all* of them finish. Situations of this sort arise naturally when agents face decision problems in which not knowing others' pieces of information decreases drastically the probability of picking the right action.[1] In order to obtain insights about how the network structure affects prices and final payoffs, one could propose more or less complex negotiation protocols.[2]

In this note, we propose a very simple protocol in which each pair of directly connected agents are engaged in an infinite horizon alternating-offers negotiation and all negotiation processes are simultaneous. Because each agent's negotiation with each neighbor is independent of her negotiation with any other neighbor, Rubinstein's (1982) logic can be directly applied to each given link in the network, taken as given the negotiation processes that the two agents have with other neighbors. As a consequence, for each given link, the agent who proposes first enjoys a first-mover advantage or initial bargaining power, exactly as in Rubinstein's (1982) seminal paper. Nevertheless, in our simple bargaining protocol, the fact that the agents are connected through a network entails important features that affect the determination of equilibrium prices and payoffs. On the one hand, the network imposes restrictions to who can negotiate with whom. On the other hand, the network allows also for a set of different possible configurations of the agents' initial bargaining power. In other words, each network describes a set of possible ways in which the first-mover advantage can be distributed among the agents. This note characterizes equilibrium prices and final payoffs for some meaningful distributions of the initial bargaining power for two specific network structures, the star and the line. Using the aforementioned protocol, we show that, although the relative negotiation power is constant within each given link, prices and payoffs are crucially affected by the network structure through the possible configurations of the first-mover advantage that it allows. The mechanism through which prices are affected in our model resembles that present in Polanski (2007). In that paper prices are influenced by the distribution of a local monopoly power, which depends on the network structure, while in our setup prices are influenced by the distribution of the first-mover advantage.

Unfortunately, analyzing a noncooperative game of bargaining for general network structures is usually intractable.[3] Given this modeling restriction, our focus on the star and the line is

[1] In practice, decision makers seeking advice in a certain field try often to consult all available experts in their neighborhood before taking their decisions. This is particularly the case if those experts hold pieces of information that are complementary among them. For example, Airbus reported that the main cause for incurring in 500,000 million losses in 2006 was that their French and German teams did not receive the complementary pieces of information that each had (regarding design software) before assembling parts of their A380 Jumbo.

[2] For example, Calvó-Armengol (1999) adapts Rubinstein's (1982) model of alternating offers to a three-agent network where a central agent negotiates for a fixed number of periods with a neighbor and then switches to the other neighbor for another fixed number of periods. One could also consider complex protocols in which agents indirectly connected in a network can be engaged in multilateral bargaining.

[3] In his book on social and economic networks, Jackson (2008) points out that the application of noncooperative game theory to study bargaining processes in networks "is generally intractable because of the large number of players and the rather open-ended bargaining protocol in many settings" (p. 412). A notable exception is the excellent analysis carried out recently by Manea (2011), which provides results for fairly general networks. Although this note does not consider more complex networks, these two simple structures are sufficient to provide

motivated as follows. First, both are connected networks with no cycles.[4] Second, the star is the canonical case where the population splits into (a) a set of agents with a single neighbor each, and (b) a single agent with everyone else as neighbor. On the other hand, the line is a typical case of connected network where all agents (except those two at the ends of the line) have the same number of neighbors. Also, indirect connections are present in the line in a very simple and neat way, and there is a single path connecting any pair of agents. Third, using *degree centrality* as centrality measure, the star provides a structure where one agent enjoys maximum centrality and bargains with a set of agents, each of whom has minimal centrality. On the other hand, in the line each agent (except those two at the ends) enjoys the same centrality. Thus, the star and the line provide a simple setting that serves as a benchmark to analyze how the network structure, together with the distributions of the first-mover advantage that it allows, influence prices and payoffs.

We use the star to study the influence only of direct connections and degree centrality on prices and final payoffs. Consequently, we consider two extreme distributions of the first-mover advantage, one in which the central agent has first-mover advantage with respect to each peripheral agent, and another in which each of the peripheral agents has first-mover advantage with respect to the central agent. By doing so, possible influences of indirect connections are ruled out. On the other hand, we use the line to analyze the influence of indirect connections. To avoid ex-ante asymmetries, we endow each agent with first-mover advantage with respect to her immediate successor.

Related literature on bargaining in networks considers three-agent networks with sequential bargaining rounds (Calvó-Armengol, 1999), networks of sellers and buyers (Corominas-Bosch, 2004), networks of information flows in cooperative settings (Polanski, 2007), and stationary networks in noncooperative settings (Abreu and Manea, 2012; Manea, 2011).

2. The Model

2.1. Network Notation

Consider a finite set of agents $N = \{1, \dots, n\}$, with $n \geq 3$. A *network* g on N is a collection of pairs from the set N and each pair $\{i, j\} \in g$ is a *link*. We will use the shorthand notation ij instead to denote a link. A network g restricts the agents' bargaining possibilities: two agents $i, j \in N$ can bargain with each other only if $ij \in g$. Let N_i^g denote the set of agent i's *neighbors* in network g. We will restrict our attention to two specific network structures, namely, the *star* and the *line*. Without loss of generality, we will specify the star network as $g_{\mathrm{S}} := \{12, 13, \dots, 1n\}$ and the line network as $g_{\mathrm{L}} := \{12, 23, \dots, (n-1)n\}$.

2.2. Bargaining in the Network

Each agent is initially endowed with one unit of information and obtains payoffs from her use of information. In addition, each agent receives also revenues from exchanging her endowment of information with her neighbors in the network. We assume that an agent's payoffs due to

clear insights into our research questions.

[4]While the existence of equilibrium in mixed strategies is guaranteed in our model, pure-strategy equilibrium may fail to exist in networks with cycles (under some distributions of the first-mover advantage). The interpretation of mixed-strategy equilibrium for our benchmark is unclear to us.

the use of information are strictly increasing in any other agent's piece of information. Thus, the pieces of information held by different agents are complementary.[5] Since information is a non-depletable good, it follows that each agent finally receives the unit of information initially owned by each of her neighbors, provided that an agreement is reached with each of them. Then, an agent's payoffs due to the use of information depends on her number of neighbors. Let v_i^g be the payoff that accrues to agent i from the use of information in network g. Also, let $V^g = \sum_{i=1}^n v_i^g$ be the total payoffs due to the use of information in the society when agents are connected through network g.

Each pair of linked agents in a network bargain over the relative price of their initial endowments of information following the *infinite horizon bargaining game of alternating offers* proposed by Rubinstein (1982). The time period for the bargaining process within any link is discrete and labelled by $t \in T$, where T is the set of positive integers. In each date $t \in T$, one of the agents proposes an agreement price and the other agent either accepts or rejects it.

To model the agents' interactions, we need to fix one of the agents in each given link ij as the first mover in that link. The first mover is the agent who starts proposing a price in the first period, so that she enjoys an initial favorable bargaining position, or *first-mover advantage*, with respect to the other agent in the link. Suppose that i is the first mover in link ij. Then, let $q_{ij} \in [0,1]$ be the relative price that summarizes the terms of transaction between agents i and j. Formally, we specify q_{ij} as the ratio between the price of agent i's information over the sum of prices of both agents' pieces of information. Therefore, $q_{ij} + q_{ji} = 1$ by construction. If the price offer is accepted, then the bargaining ends and the exchange takes place at the agreed price. If the price offer is rejected, then the play passes on to the next date, where the rejecting agent proposes in turn an agreement price. Bargaining continues in this way with no limits to the number of dates. Throughout the paper, we will follow the notational convention that if agent i is the first mover in link ij, then agents i and j negotiate over time by proposing each of them values for the price $q_{ij} \in [0,1]$.[6] Each agent is engaged from date $t = 1$ in a bilateral bargaining process as the one above described with each of her neighbors. The bargaining processes across different links are simultaneous and independent. Agents have perfect recall.

Given the bilateral bargaining processes between linked agents in the network, each agent receives the revenue from exchanging her initial endowment of information with her neighbors. Suppose that agent i is the first mover in the link ij. Then the payoff that accrues to agent i from the exchange of information with her neighbor j at price q_{ij} is the net revenue

$$r_i(q_{ij}) := q_{ij} \cdot 1 - (1 - q_{ij}) \cdot 1 = 2q_{ij} - 1, \tag{1}$$

and the payoff to agent j from trading with agent i is given by

$$r_j(q_{ij}) := (1 - q_{ij}) \cdot 1 - q_{ij} \cdot 1 = 1 - 2q_{ij}. \tag{2}$$

Note that $r_j(q_{ij}) = -r_i(q_{ij})$.

Agents are impatient and discount their future payoffs using a common discount factor $\delta \in (0,1)$. We assume that each agent receives the payoffs due both to the use and exchange

[5]Information structures where agents held complementary information about the state of the world and each agent values the pieces of information possessed by others are considered, among others, by Hagenbach and Koessler (2010), and Jiménez-Martínez (2006).

[6]Note that the bargaining in each link ij can be regarded as a "split-the-pie" game in which $(q_{ij}, 1 - q_{ij})$, with $q_{ji} = 1 - q_{ij}$, represents a possible division of the desirable pie.

of information at the date at which the bargaining processes with each of her neighbors ends. One way to interpret this assumption is by considering that each agent needs to aggregate all pieces of information gathered from her neighbors in order to be able to benefit from the use of information.

Assumption 1. For a given network g, each agent $i \in N$ can only benefit from the use and the exchange of information at the date in which the bargaining processes with all of her neighbors finish.

The above assumption implies that an agent's optimal bargaining decisions are related to her relative position in the network. The agent cares about the date of agreement for each of her neighbors and, therefore, her bargaining decisions for two different neighbors must be correlated. Such a correlation depends on the number of her neighbors, on the number of neighbors that each of her neighbors has, and so on.

Finally, note that in order to specify completely this game of pairwise negotiations for a given network, one needs to label a first mover for each link in the network. Specifically, given a network g, we partition each agent i's set of neighbors, N_i^g, into two sets, $\underline{N}_i^g$ and $\overline{N}_i^g$, where $\underline{N}_i^g$ denotes the set of agent i's neighbors who are second movers relative to agent i and $\overline{N}_i^g$ denotes the set of agent i's neighbors who are first movers with respect to agent i. Let $\underline{N}^g := \{\underline{N}_i^g\}_{i\in N}$ and $\overline{N}^g := \{\overline{N}_i^g\}_{i\in N}$ so that the pair $M(g) := (\underline{N}^g, \overline{N}^g)$ completely describes a distribution of the first-mover advantage associated with network g. We denote by $\Gamma_{M(g)}$ the game of pairwise negotiations that we have described for a given network g when $M(g)$ is the associated distribution of the first-mover advantage.

We introduce now formally the elements needed to specify final payoffs and to define equilibrium. Let A and R be two statements meaning, respectively, "Accept" and "Reject." Consider a network g and a given link $ij \in g$, where agent i is chosen (without loss of generality) as the first mover in the bargaining with agent j. A *strategy for agent i with respect to her neighbor j* is an infinite sequence with the form $s_{ij} = (q_{ij}^1, y^2, q_{ij}^3, y^4, \dots)$, where $q_{ij}^t \in [0,1]$ and $y^t \in \{A, R\}$ for each $t \in T$. In this case, a strategy for agent j respect to agent i is an infinite sequence with the form $s_{ji} = (y^1, q_{ij}^2, y^3, q_{ij}^4, \dots)$. Let $s_i = (s_{ij})_{j\in N_i^g}$ be a *strategy* for agent i and let $s = (s_i)_{i\in N}$ be a *strategy profile*.

Let $(s_{ij}, s_{ji})_\tau \in \mathbb{R}^2$ be the pair of coordinates in the τ-th position of the strategy pair (s_{ij}, s_{ji}). If $(s_{ij}, s_{ji})_\tau = (A, q_{ij}^\tau)$, then the price q_{ij}^τ is accepted by agent i at date τ. Analogously, if $(s_{ij}, s_{ji})_\tau = (q_{ij}^\tau, A)$, then the price q_{ij}^τ is accepted by agent j at date τ. Consider a strategy pair (s_{ij}, s_{ji}) and take a given date $t < \infty$. The acceptance date, starting from date t, in the bargaining process between agents i and j is given by

$$\tau_t^*(s_{ij}, s_{ji}) := \min_{\tau \geq t} \left\{ \tau \geq t : \text{either } (s_{ij}, s_{ji})_\tau = (A, q_{ij}^\tau) \text{ or } (s_{ij}, s_{ji})_\tau = (q_{ij}^\tau, A) \right\}.$$

Given Assumption 1, we are interested in the latest acceptance date for agent i across all her neighbors in the network. Starting from date t, this latest acceptance date is

$$\tau_t^*(i, s) := \max_{j \in N_i^g} \tau_t^*(s_{ij}, s_{ji}).$$

Note that the agreement dates $\tau_t^*(s_{ij}, s_{ji})$ specified above may not exist for each date t and each strategy profile s. This is the case when agents i and j do not reach an agreement starting from date t. If there is no agreement between agent i and one of her neighbors, the latest acceptance date $\tau_t^*(i,s)$ does not exist either. In this case, we write $\tau_t^*(i,s) = \infty$.

For a strategy profile s, let $u^g_{i,t}(s)$ be the value at time t of the discounted aggregate payoff to agent i due to her bargaining with her neighbors in network g. We assume that $u^g_{i,t}(s) = 0$ if $\tau_t^*(i,s) = \infty$. The interpretation is that agent i receives a zero payoff at a given date if she does not reach an agreement with any of her neighbors from that date onwards. If, instead, $\tau_t^*(i,s)$ is a finite integer, then agent i exchanges her endowment of information with each of her neighbors, and obtains her payoffs both from the use and the exchange of information. Thus,

$$u^g_{i,t}(s) := \begin{cases} 0 & \text{if } \tau_t^*(i,s) = \infty, \\ \delta^{\tau_t^*(i,s)-t}\left(v_i^g + \sum_{k \in \underline{N}_i^g} r_i(q_{ik}) + \sum_{l \in \overline{N}_i^g} r_i(q_{li})\right) & \text{if } t \leq \tau_t^*(i,s) < \infty. \end{cases} \tag{3}$$

Note that, for a strategy profile s, the final payoff to agent i in the game $\Gamma_{M(g)}$ is given by $u^g_{i,1}(s)$. Then, let us simply write $u^g_i(s) = u^g_{i,1}(s)$ to ease notation.

Definition 1. Given a network g and a distribution of the first-mover advantage $M(g)$ for that network, a *subgame perfect Nash equilibrium (SPE)* of the game $\Gamma_{M(g)}$ is a strategy profile s^* such that for each agent $i \in N$ and each date $t \in T$, we have $u^g_{i,t}(s^*) \geq u^g_{i,t}(s_i, s^*_{-i})$ for each s_i.

3. Main Results

As shown by Rubinstein (Conclusion 2, 1982), the bargaining game corresponding to each pair of linked agents has a unique subgame perfect equilibrium in which the reference agent proposes a certain price and her opponent accepts it in the first date. This price is characterized by a pair of equations which reflect intertemporal indifference requirements for each of the agents. Consider a network g and suppose that agent i is the first mover in the link $ij \in g$. Then, the indifference condition for agent i between exchanging her endowment at price q^*_{ij} in period $t = 2$ or at price $\tilde{q}_{ij}$ in period $t = 1$ is

$$\delta\left(v_i^g + r_i(q^*_{ij}) + \sum_{k \in \underline{N}_i^g \setminus \{j\}} r_i(q^*_{ik}) + \sum_{l \in \overline{N}_i^g} r_i(q^*_{li})\right) = v_i^g + r_i(\tilde{q}_{ij}) + \sum_{k \in \underline{N}_i^g \setminus \{j\}} r_i(q^*_{ik}) + \sum_{l \in \overline{N}_i^g} r_i(q^*_{li}),$$

and the indifference condition for agent j between exchanging her endowment at price $\tilde{q}_{ij}$ in period $t = 2$ or at price q^*_{ij} in period $t = 1$ is

$$\delta\left(v_j^g + r_j(\tilde{q}_{ij}) + \sum_{k \in \underline{N}_j^g} r_j(q^*_{jk}) + \sum_{l \in \overline{N}_j^g \setminus \{i\}} r_j(q^*_{lj})\right) = v_j^g + r_j(q^*_{ij}) + \sum_{k \in \underline{N}_j^g} r_j(q^*_{jk}) + \sum_{l \in \overline{N}_j^g \setminus \{i\}} r_j(q^*_{lj}).$$

Since an agreement is reached at $t = 1$ in equilibrium, we can use the two indifference requirements above, together with the implication $r_i(\tilde{q}_{ij}) = -r_j(\tilde{q}_{ij})$ and the definition of payoffs in (3), to obtain that, if s^* is a SPE of $\Gamma_{M(g)}$, then $u^g_i(s^*) = (1/\delta)u^g_j(s^*)$. Thus, independent of the network g and of the distribution of the first-mover advantage $M(g)$, in each given link, the first mover extracts a surplus of the second mover, exactly in the same way as in the Rubinstein's (1982) negotiation model with only two agents. This implication is intuitive since the two agents who negotiate in a given link take as independently given the rest of negotiation processes in which they are involved.

Nevertheless, given Assumption 1, an agent cares, in the overall game, about the negotiation processes with all her neighbors in the network. As a consequence, her optimal strategy depends on her position in the network. In short, equilibrium prices and payoffs are affected by the distribution of the first-mover advantage which, in turn, is conditioned by the network structure. Since bargaining processes are independent across pairs of linked agents, the SPE of our game $\Gamma_{M(g)}$ is characterized by a system of linear equations which is related both to the structure of the network g and to the distribution of the first-mover advantage $M(g)$. For a general network g, this system of equations can be expressed as $A \cdot q = b$, where q denotes a price vector, and A and b are, respectively, a matrix and a vector of constants which depend on g and on the chosen $M(g)$. The size of this system is given by the number of links in network g. If $A \cdot q = b$ has some solution q^*, then it corresponds to the price vector associated to a SPE in pure strategies of $\Gamma_{M(g)}$.[7] We show that the systems of equations associated to the star and the line (for some meaningful specifications of $M(g)$) have a unique solution, which implies uniqueness of the SPE for the corresponding games.

The following proposition characterizes equilibrium prices and payoffs for the star network for two extreme cases: (a) each of the peripheral agents is first mover in her link, and (b) the central agent is first mover with respect to each of the peripheral agents.

Proposition 1. *Consider the star network given by $g_S = \{12, 13, \ldots, 1n\}$. Suppose that either each of the peripheral agents is first mover in her link, i.e., $\underline{N}_j^{g_S} = \{1\}$ for each $j \in \{2, 3, \ldots, n\}$, or the central agent is first mover with respect to each of the peripheral agents, i.e., $\underline{N}_1^{g_S} = \{2, 3, \ldots, n\}$. Then, under Assumption 1, the game $\Gamma_{M(g_S)}$ has a unique SPE where the prices and the payoffs are given, respectively, by:*

(a) for $\underline{N}_j^{g_S} = \{1\}$ for each $j \in \{2, 3, \ldots, n\}$,

$$q_{j1}^* = \frac{1 - v_j^{g_S}}{2} + \frac{V^{g_S}}{2(n - 1 + \delta)},$$

$$u_1^{g_S}(s^*) = \left[\frac{\delta}{n - 1 + \delta}\right] V^{g_S}, \text{ and } u_j^{g_S}(s^*) = \left[\frac{1}{n - 1 + \delta}\right] V^{g_S} \quad \text{for each } j \in \{2, 3, \ldots, n\};$$

(b) for $\underline{N}_1^{g_S} = \{2, 3, \ldots, n\}$,

$$q_{1j}^{**} = 1 - \left(\frac{1 - v_j^{g_S}}{2} + \frac{\delta V^{g_S}}{2[1 + (n - 1)\delta]}\right),$$

$$u_1^{g_S}(s^{**}) = \left[\frac{1}{1 + (n - 1)\delta}\right] V^{g_S}, \text{ and } u_j^{g_S}(s^{**}) = \left[\frac{\delta}{1 + (n - 1)\delta}\right] V^{g_S} \text{ for each } j \in \{2, 3, \ldots, n\}.$$

Thus, a first proposer obtains a surplus from each of her neighbors. In particular, the payoff gain to any agent for each link in which she moves from proposing second to proposing first is $(1 - \delta)V^{g_S}/(1 + \delta)$. It follows that the agent with the highest centrality is able to extract

[7] Notice that the system $A \cdot q = b$ does not have a solution when the matrix A is singular. In this case, there is no SPE of $\Gamma_{M(g)}$ in pure strategies. For example, such a situation emerges when one considers a wheel network and a distribution of the first-mover advantage in which each agent is first mover relative to her immediate successor.

the highest surplus possible in this network when she enjoys the first-mover advantage with respect to each of the peripheral agents. The star network and the two distributions of the first-mover advantage studied in Proposition 1 do not allow for the advantage of a first mover to be transmitted across indirectly connected agents.

Can a network facilitate the propagation of the relative initial bargaining power of a first mover through indirect connections? The next proposition shows that the answer is affirmative and that this sort of propagation of the first-mover advantage affects equilibrium prices. Specifically, we consider that the agents are connected along a line in which they are ordered from left to right, and each agent is first mover relative to her immediate successor in the line according to this order.[8] Under this distribution of the first-mover advantage, all agents, except those at the two ends of the line, are symmetric with respect to their initial bargaining power (i.e., each of them is first mover relative to one neighbor and second mover relative to the other neighbor).

Proposition 2. *Consider the line network given by $g_{\mathrm{L}} = \{12, 23, \ldots, (n-1)n\}$. Suppose that, for each $i \in \{2, 3, \ldots, n-1\}$, we have $\underline{N}_i^{g_{\mathrm{L}}} = \{i-1\}$ and $\overline{N}_i^{g_{\mathrm{L}}} = \{i+1\}$. Then, under Assumption 1, the game $\Gamma_{M(g_{\mathrm{L}})}$ has a unique SPE such that each agent $i \in \{1, \ldots, n-1\}$ charges a price*

$$q^*_{i(i+1)} = \frac{1}{2} + \frac{\sum_{k=i+1}^{n} v_k^{g_{\mathrm{L}}} \sum_{j=0}^{i-1} \delta^j - \sum_{k=1}^{i} v_k^{g_{\mathrm{L}}} \sum_{j=i}^{n-1} \delta^j}{2 \sum_{j=0}^{n-1} \delta^j}$$

to each neighbor $i+1$ along the line. Moreover, in this SPE, each agent i's payoff is given by

$$u_i^{g_{\mathrm{L}}}(s^*) = \delta^{i-1} \left(\frac{1-\delta}{1-\delta^n} \right) V^{g_{\mathrm{L}}}.$$

Thus, each agent benefits not only from her position relative to her immediate successor but also from the position of the indirectly connected agents who are located at her right-hand side. Each agent extracts a surplus from the benefits that her neighbor enjoys from her own neighbor, and so on. Indirect connections play a key role in the equilibrium prices and payoffs.

4. Concluding Comments

While equilibrium shares depend crucially on the distribution of the first-mover advantage, the network describes the different ways in which the first-mover advantage can be distributed among the agents. In this sense, the network provides restrictions, as well as some degree of flexibility, over the bargaining processes that determine prices. For the star, we have intentionally distributed the first-mover advantage in a way such that indirect connections are irrelevant. Otherwise, it is easy to show that the gain from proposing first propagates through the indirectly linked agents along that path in the star, exactly as it does in the line. For the line, one could propose situations in which some agents in the line do not propose first in any of the links in which they are included. It would be interesting to analyze the equilibrium prices and payoffs in these cases.

Finally, starting with a model of bilateral negotiations between directly connected agents, one could possibly add a variety of more o less complex mechanisms through which the network

[8]Of course, the results in Proposition 2 continue to hold qualitatively if we reverse this order.

can influence equilibrium prices and payoffs. Perhaps, under some mechanisms, the initial advantage that a first mover has with respect to a neighbor could even be affected by the structure of the network. Of course, this would provide us with a model that relates the network structure to prices and payoffs. This note can be rather viewed as an attempt to use a very simple protocol (in which, for each given link, nothing goes beyond the insights provided by Rubinstein, 1982) to study carefully how the network structure can influence prices and payoffs, provided that the agents benefit from bargaining only when all their negotiation processes finish.

Appendix

Proof of Proposition 1. Consider the star network $g_S = \{12, 13, \ldots, 1n\}$ and fix a given link $1j \in g_S$.

(a) Suppose that the peripheral agent j is the first mover with respect to the central agent 1. Then, equation

$$\delta \left(v_j^{g_S} + r_j(q_{j1}^*) \right) = v_j^{g_S} + r_j(\tilde{q}_{j1}) \tag{4}$$

represents the indifference condition for agent j between trading her endowment of information with agent 1 at price q_{j1}^* in period $t = 2$ or at $\tilde{q}_{j1}$ price in period $t = 1$. On the other hand, equation

$$\delta \left(v_1^{g_S} + r_1(\tilde{q}_{j1}) + \sum_{k \neq 1, j} r_1(q_{k1}^*) \right) = v_1^{g_S} + r_1(q_{j1}^*) + \sum_{k \neq 1, j} r_1(q_{k1}^*) \tag{5}$$

represents the indifference condition for agent 1 between exchanging her endowment of information with agent j at price $\tilde{q}_{j1}$ in period $t = 2$ or at price q_{j1}^* in period $t = 1$. Consider the prices q_{k1}^*, for $k \neq 1, j$, as exogenously given for the moment. By applying the expressions for agents' revenue in (1) and (2) to agents 1 and j, and by substituting price $\tilde{q}_{j1}$ from equation (4) into equation (5), we obtain

$$(1+\delta) q_{j1}^* + \sum_{k \neq 1, j} q_{k1}^* = \frac{v_1^{g_S} - \delta v_j^{g_S} + n - 1 + \delta}{2}.$$

Now, consider simultaneously the bargaining processes for all links $1j$, $j \neq 1$. Then, we obtain $n-1$ equations as the one above, one equation for each $j \neq 1$. This gives us a linear system whose solutions are the prices q_{j1}^*, $j \neq 1$. Using matrix notation, we can be rewrite this system as $A \cdot q^* = b$, where

$$A = \begin{pmatrix} (1+\delta) & 1 & \ldots & 1 \\ 1 & (1+\delta) & \ldots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \ldots & (1+\delta) \end{pmatrix}, \quad q^* = \begin{pmatrix} q_{21}^* \\ q_{31}^* \\ \vdots \\ q_{n1}^* \end{pmatrix},$$

and

$$b = \frac{1}{2} \begin{pmatrix} v_1^{g_S} - \delta v_2^{g_S} + n - 1 + \delta \\ v_1^{g_S} - \delta v_3^{g_S} + n - 1 + \delta \\ \vdots \\ v_1^{g_S} - \delta v_n^{g_S} + n - 1 + \delta \end{pmatrix}.$$

Each bilateral bargaining process within the star network corresponds to an infinite-horizon alternating-offers process between two agents. Then, the existence of a unique solution to the system $A \cdot q^* = b$ above implies Rubinstein's (1982) conditions for the existence of a SPE for the collection of all bilateral alternating offers processes within the network. Application of Cramer's rule gives us

$$q_{j1}^* = \frac{1 - v_j^{g_S}}{2} + \frac{\sum_{k=1}^n v_k^{g_S}}{2(n-1+\delta)}.$$

By using the expression for payoffs in equation (3), we obtain that, in this SPE, the payoff to agent 1 is

$$u_1^{g_S}(s^*) = \left[\frac{\delta}{n-1+\delta}\right] \sum_{k=1}^n v_k^{g_S},$$

and the payoff to each peripheral agent $j \in \{2, 3, \dots, n\}$ is

$$u_j^{g_S}(s^*) = \left[\frac{1}{n-1+\delta}\right] \sum_{k=1}^n v_k^{g_S}.$$

(b) Suppose that the central agent 1 is first mover with respect to the peripheral agent j. Then, equation

$$v_1^{g_S} + r_1(\tilde{q}_{1j}) + \sum_{k \neq 1,j} r_1(q_{1k}^{**}) = \delta \left(v_1^{g_S} + r_1(q_{1j}^{**}) + \sum_{k \neq 1,j} r_1(q_{1k}^{**}) \right) \tag{6}$$

represents the indifference condition for agent 1 between exchanging her endowment of information with agent j at price $\tilde{q}_{1j}$ in period $t = 2$ or at price q_{1j}^{**} in period $t = 1$. Consider the prices q_{1k}^{**}, for $k \neq 1, j$, as exogenously given for the moment. Also, equation

$$v_j^{g_S} + r_j(q_{1j}^{**}) = \delta \left(v_j^{g_S} + r_j(\tilde{q}_{1j}) \right) \tag{7}$$

represents the indifference condition for agent j between trading her endowment of information with agent 1 at price q_{1j}^{**} in period $t = 2$ or at $\tilde{q}_{1j}$ price in period $t = 1$. By applying the expressions for agents' revenue in (1) and (2) to agents 1 and j, and by substituting price $\tilde{q}_{1j}$ from equation (7) into equation (6), we obtain

$$(1+\delta) q_{1j}^{**} + \delta \sum_{k \neq 1,j} q_{1k}^{**} = \frac{-\delta v_1^{g_S} + v_j^{g_S} + 1 + (n-1)\delta}{2}.$$

Now, consider simultaneously the bargaining processes for all links $1j$, $j \neq 1$. Then, we obtain $n-1$ equations as the one above, one equation for each $j \neq 1$. This gives us a linear system whose solutions are the prices q_{j1}^{**}, $j \neq 1$. Using matrix notation, we can rewrite this system as $A \cdot q^{**} = b$, where

$$A = \begin{pmatrix} (1+\delta) & \delta & \dots & \delta \\ \delta & (1+\delta) & \dots & \delta \\ \vdots & \vdots & \ddots & \vdots \\ \delta & \delta & \dots & (1+\delta) \end{pmatrix}, \quad q^{**} = \begin{pmatrix} q_{21}^{**} \\ q_{31}^{**} \\ \vdots \\ q_{n1}^{**} \end{pmatrix},$$

and

$$b = \frac{1}{2}\begin{pmatrix} -\delta v_1^{g_S} + v_2^{g_S} + 1 + (n-1)\delta \\ -\delta v_1^{g_S} + v_3^{g_S} + 1 + (n-1)\delta \\ \vdots \\ -\delta v_1^{g_S} + v_n^{g_S} + 1 + (n-1)\delta \end{pmatrix}.$$

Each bilateral bargaining process within the star network corresponds to an infinite-horizon alternating-offers process between two agents. Then, the existence of a unique solution to the system $A \cdot q^{**} = b$ above implies Rubinstein's (1982) conditions for the existence of a SPE for the collection of all bilateral alternating offers processes within the network. Application of Cramer's rule gives us

$$q_{1j}^{**} = \frac{1+v_j^{g_S}}{2} - \frac{\delta \sum_{k=1}^n v_k^{g_S}}{2[1+(n-1)\delta]} = 1 - \left(\frac{1-v_j^{g_S}}{2} + \frac{\delta \sum_{k=1}^n v_k^{g_S}}{2[1+(n-1)\delta]}\right).$$

By using the expression for payoffs in equation (3), we obtain that, in this SPE, the payoff to agent 1 is

$$u_1^{g_S}(s^{**}) = \left[\frac{1}{1+(n-1)\delta}\right] \sum_{k=1}^n v_k^{g_S},$$

and the payoff to each peripheral agent $j \in \{2,\ldots,n\}$ is

$$u_j^{g_S}(s^{**}) = \left[\frac{\delta}{1+(n-1)\delta}\right] \sum_{k=1}^n v_k^{g_S},$$

as stated. ∎

Proof of Proposition 2. Consider the line network $g_L = \{12, 23, \ldots, (n-1)n\}$. Fix link 12 and suppose that agent 1 is first mover with respect to agent 2. Then, equation

$$\delta\left(v_1^{g_L} + r_1(q_{12}^*)\right) = v_1^{g_L} + r_1(\tilde{q}_{12}) \tag{8}$$

gives us the indifference condition for agent 1 between exchanging her endowment of information with agent 1 at price q_{12}^* in period $t=2$ or at price $\tilde{q}_{12}$ in period $t=1$. Analogously, equation

$$\delta\left(v_2^{g_L} + r_2(\tilde{q}_{12}) + r_2(q_{23}^*)\right) = v_2^{g_L} + r_2(q_{12}^*) + r_2(q_{23}^*) \tag{9}$$

specifies the indifference condition for agent 2 between exchanging her endowment of information with agent 1 at price $\tilde{q}_{12}$ in period $t=2$ or at price q_{12}^* in period $t=1$. Take price q_{23}^* as exogenously given for the moment. By applying the expressions for agents' revenue in (1) and (2) to agents 1 and 2, and by substituting price $\tilde{q}_{12}$ from equation (8) into equation (9), we obtain

$$(1+\delta)q_{12}^* - q_{23}^* = \frac{v_2^{g_L} - \delta v_1^{g_L} + \delta}{2}.$$

Now, fix a link $i(i+1)$, connecting agents who are not at the ends of the line, i.e., $i \in \{2,\ldots,n-2\}$, and suppose that agent i is second mover with respect to agent $i-1$ and first mover with respect to agent $i+1$. Suppose for the moment that the bargaining prices corresponding to any other link are exogenously given. By proceeding analogously as done above for link 12, we obtain

$$-\delta q^*_{(i-1)i} + (1+\delta) q^*_{i(i+1)} - q^*_{(i+1)(i+2)} = \frac{v^{g_L}_{i+1} - \delta v^{g_L}_{i}}{2}.$$

Finally, by doing the analogous computations for link $(n-1)n$, we obtain

$$-\delta q^*_{(n-2)(n-1)} + (1+\delta) q^*_{(n-1)n} = \frac{v^{g_L}_{n} - \delta v^{g_L}_{n-1} + 1}{2}.$$

Now, consider simultaneously the bargaining processes across all links $i(i+1)$, $i \in \{1,\ldots,n-1\}$, in the network. Then, we obtain a linear system of $n-1$ equations with $n-1$ unknowns, $q^*_{i(i+1)}$, $i \in \{1,\ldots,n-1\}$. All prices are simultaneously obtained by solving this linear system. Using matrix notation, this system can be expressed as $A \cdot q^* = b$, where

$$A = \begin{pmatrix} (1+\delta) & -1 & 0 & \ldots & 0 & 0 \\ -\delta & (1+\delta) & -1 & \ldots & 0 & 0 \\ 0 & -\delta & (1+\delta) & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & (1+\delta) & -1 \\ 0 & 0 & 0 & \ldots & -\delta & (1+\delta) \end{pmatrix}, \quad q^* = \begin{pmatrix} q^*_{12} \\ q^*_{23} \\ q^*_{34} \\ \vdots \\ q^*_{(n-2)(n-1)} \\ q^*_{(n-1)n} \end{pmatrix},$$

and

$$b = \frac{1}{2} \begin{pmatrix} v^{g_L}_2 - \delta v^{g_L}_1 + \delta \\ v^{g_L}_3 - \delta v^{g_L}_2 \\ v^{g_L}_4 - \delta v^{g_L}_3 \\ \vdots \\ v^{g_L}_{n-1} - \delta v^{g_L}_{n-2} \\ v^{g_L}_n - \delta v^{g_L}_{n-1} + 1 \end{pmatrix}.$$

Each bilateral bargaining process within the line network corresponds to an infinite-horizon alternating-offers process between two agents. Then, the existence of a unique solution to the system $A \cdot q^* = b$ above implies Rubinstein's (1982) conditions for the existence of a PBE for the collection of all bilateral alternating offers processes within the network.

Application of Cramer's rule gives us

$$q^*_{i(i+1)} = \frac{1}{2} + \frac{\sum_{k=i+1}^{n} v^{g_L}_k \sum_{j=0}^{i-1} \delta^j - \sum_{k=1}^{i} v^{g_L}_k \sum_{j=i}^{n-1} \delta^j}{2 \sum_{j=0}^{n-1} \delta^j}.$$

By using the expression for payoffs in equation (3), we obtain that, in this PBE, the payoff to each agent $i \in 1,\ldots,n$ is given by

$$u^{g_L}_i(s^*) = \left[\frac{\delta^{i-1}}{\sum_{j=0}^{n-1} \delta^j} \right] \sum_{k=1}^{n} v^{g_L}_k = \delta^{i-1} \left(\frac{1-\delta}{1-\delta^n} \right) V^{g_L},$$

as stated. ∎

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