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# Preemption and rent equalization in the adoption of new technology: comment

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## Abstract

In this comment, we show that the existence of the preemption equilibrium in Fudenberg and Tirole (Review of Economics Studies, vol. 52, PP. 383-401, 1985)'s continuous-time games of timing is not guaranteed under their assumptions.

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# 1. Comment

We direct this comment to the paper by Fudenberg and Tirole (1985). The authors develop a new framework for modeling continuous-time games of timing, and show that in a duopoly, the threat of preemption in the adoption of new technology will equalize two firms' rents. The results of preemption and rent equalization have been adopted by a number of papers: Fudenberg and Tirole (1987), Tirole (1988), Choi (1996), Weeds (2002), Adner and Zemsky (2005), Ostrovsky and Schwarz (2005), Honoré and Paula (2010), and Shen and Villas-Boas (2010). However, by constructing a counterexample, we show that the existence of the *preemption equilibrium* is not guaranteed under Assumptions 1 and 2 in their paper.

We adopt Fudenberg and Tirole (1985)'s notation throughout our comments, and consider the following duopoly case of the model. Two identical firms, denoted by firm 1 and firm 2, exist in the industry. At time 0, a cost reducing innovation is announced. For $i \in \{1, 2\}$, let $\pi_0(m)$ be the net cash flow of firm $i$ when $m$ firm(s) have adopted the innovation, but firm $i$ has not. Let $\pi_1(m)$ be firm $i$'s net cash flow when $m$ firm(s) including $i$ have adopted. $T_i$ denotes firm $i$'s adoption date; and $c(t)$ is the present value of the cost of implementing the innovation on line by time $t$. Without loss of generality, suppose that firm $i$ is the $i$-th to adopt, then we can represent firm $i$'s payoff, $V^i(T_i, T_j)$, as follows.

$$V^i(T_i, T_j) = \begin{cases} \int_0^{T_i} \pi_0(0)e^{-rt}dt + \int_{T_i}^{T_j} \pi_1(1)e^{-rt}dt + \int_{T_j}^{\infty} \pi_1(2)e^{-rt}dt - c(T_i) & \text{if } T_i \leq T_j; \\ \int_0^{T_j} \pi_0(0)e^{-rt}dt + \int_{T_j}^{T_i} \pi_0(1)e^{-rt}dt + \int_{T_i}^{\infty} \pi_1(2)e^{-rt}dt - c(T_i) & \text{if } T_i > T_j, \end{cases} \tag{1}$$

where $j \in \{1, 2\}$ and $j \neq i$, and $r$ is the constant common interest rate.

We next introduce the duopoly version of the two assumptions they impose on the firm's net cash flow and adoption cost of the innovation.

**Assumption 1.**

(i) $\pi_0(0) \geq \pi_0(1) > 0$ and $\pi_1(1) \geq \pi_1(2) > 0$, and

(ii) $\pi_1(1) - \pi_0(0) > \pi_1(2) - \pi_0(1)$.

**Assumption 2.**

(i) $\pi_1(1) - \pi_0(1) \leq -c'(0)$.

(ii) $\inf_{t \geq 0}\{c(t)e^{rt}\} < [\pi_1(2) - \pi_0(1)]/r$.

(iii) For all $t \geq 0$, $(c(t)e^{rt})' < 0$ and $(c(t)e^{rt})'' > 0$.

Based on these two assumptions, we define the adoption dates $T_1^* < T_2^*$ to be the solutions of the following first-order conditions: for $m = 1, 2$,

$$[\pi_1(m) - \pi_0(m-1)]\, e^{-rT_m^*} + c'(T_m^*) = 0.$$

Moreover, we define[1]

$$L(t) = \begin{cases} V(t, T_2^*) & \text{if } t < T_2^*; \\ V(t,t) & \text{if } t \geq T_2^*, \end{cases}$$

and

$$F(t) = \begin{cases} V(T_2^*, t) & \text{if } t < T_2^*; \\ V(t,t) & \text{if } t \geq T_2^*, \end{cases}$$

to be the leader's and the follower's payoffs, respectively, when the former preempts the latter at time $t$, and let $M(t) = V(t,t)$ be the payoff of both firms when they adopt together at time $t$. Finally, let $\hat{T}_2 = \text{argmax}_{t \in \mathbb{R}_+} M(t)$.

Under Assumptions 1 and 2, the authors offer a necessary condition for the existence of the $(T_1, T_2^*)$-*diffusion equilibrium*[2] (i.e., there exists a unique preemption time $T_1$ in $(0, T_1^*)$ such that $L(T_1) = F(T_1)$)[3], and then show in their Proposition 2 that the $(T_1, T_2^*)$-*diffusion equilibrium* always exists.[4] We show that under Assumptions 1 and 2, the existence of $T_1$ is not guaranteed by constructing a numerical example.

**The counterexample.** Let $\theta, r, k \in \mathbb{R}_{++}$ be such that $\theta + r - k > 0$. Let the related parameters be defined as follows so that they satisfy Assumptions 1 and 2: $c(t) = e^{-(\theta+r)t}$, $\pi_0(0) = 4k/3$, $\pi_0(1) = k$, $\pi_1(1) = \theta + r + k$, and

[1]The function $V(t, t')$ is defined as the righ-hand side of (1) with replacing $T_i$ and $T_j$ by $t$ and $t'$, respectively.

[2]The $(T_1, T_2^*)$-diffusion equilibrium exhibits that one of the two firms adopts at $T_1$ and the other firm adopts at $T_2^*$ with probability one.

[3]To show this, they first prove that $L(t) - F(t)$ is strictly quasi-concave, and then they improperly use Assumption 2(i) to claim that $L(0) < F(0)$. On the other hand, they also show that $L(T_2^*) = F(T_2^*)$ (from the definitions of $L(t)$ and $F(t)$), and $L(T_1^*) > F(T_1^*)$ (from their proposition 1). Therefore, These imply that there must exist a unique $T_1 \in (0, T_1^*)$ such that $L(T_1) = F(T_1)$ since $L(t)$ and $F(t)$ are continuous functions of $t$.

[4]Two cases are analyzed in their Proposition 2. If $L(T_1^*) > M(\hat{T}_2)$, the authors show that the $(T_1, T_2^*)$-*diffusion equilibrium* is the unique equilibrium of the timing game; otherwise, *diffusion equilibrium* and *joint-adoption equilibrium* coexist.

$\pi_1(2) = \theta + r + k/3$. It is easy to see that Assumptions 1 and 2 are satisfied under this setting.

We now show that $L(0) > F(0)$. Recall that $\forall t \in [0, T_2^*]$,

$$L(t) \equiv \int_0^t \pi_0(0)e^{-rs}ds + \int_t^{T_2^*} \pi_1(1)e^{-rs}ds + \int_{T_2^*}^{\infty} \pi_1(2)e^{-rs}ds - c(t); \qquad (2)$$

and

$$F(t) \equiv \int_0^t \pi_0(0)e^{-rs}ds + \int_t^{T_2^*} \pi_0(1)e^{-rs}ds + \int_{T_2^*}^{\infty} \pi_1(2)e^{-rs}ds - c(T_2^*). \qquad (3)$$

Thus,

$$\begin{aligned} L(0) - F(0) &= \int_0^{T_2^*} [\pi_1(1) - \pi_0(1)]e^{-rs}ds - c(0) + c(T_2^*) \\ &= \theta e^{-rT_2^*} \int_0^{T_2^*} [e^{rt} - e^{-\theta t}]dt \\ &> 0, \end{aligned}$$

where the inequality follows from $T_2^* > 0$.[5] Finally, since $L(0) > F(0)$ and the strict quasi-concavity of $L(t) - F(t)$, $L(T_1^*) > F(T_1^*)$ together with $L(T_2^*) = F(T_2^*)$ implies the non-existence of $T_1$.[6]

We now use two numerical results to show that the counterexample includes two cases in Proposition 2 of Fudenberg and Tirole (1985). Let $(\theta, r, k) =$

[5] To see that $T_2^* > 0$ in this example, since $T_2^*$ is defined to be the solution of the following equation:

$$[\pi_1(2) - \pi_0(1)]e^{-rt} + c'(t) = 0,$$

(i.e., $(\theta + r + 2k/3)e^{-rt} - (\theta + r)e^{-(\theta+r)t} = 0$ in this example) by solving this equation, we obtain that

$$T_2^* = -\frac{1}{\theta} \ln \left(1 - \frac{2k}{3(\theta + r)}\right) > 0.$$

Similarly,

$$\begin{aligned} T_1^* &= -\frac{1}{\theta} \ln \left(1 - \frac{k}{3(\theta + r)}\right); \text{ and} \\ \hat{T}_2 &= -\frac{1}{\theta} \ln \left(1 - \frac{k}{(\theta + r)}\right). \end{aligned}$$

Hence, we conclude that $\hat{T}_2 > T_2^* > T_1^* > 0$ in this example.

[6] To see this, suppose, by contradiction, that there exists $T_1 \in (0, T_1^*)$ such that $L(T_1) = F(T_1)$. Since $L(T_1^*) > F(T_1^*)$, there must exist two disjoint intervals in $[0, T_1^*]$ such that the function $L(t) - F(t)$ is decreasing w.r.t. $t$ in the first one, and increasing w.r.t. in the other. Note that $L(T_2^*) = F(T_2^*)$ implies that $L(t) - F(t)$ must decrease in a subinterval of $(T_1^*, T_2^*]$, which is a contradiction of the strictly quasi-concavity of $L(t) - F(t)$.

$(2, 0.05, 1.5)$. It can be show that $L(T_1^*) \leq M(\hat{T}_2)$ (i.e. Case B of Proposition 2 in Fudenberg and Tirole (1985).) Let $(\theta, r, k) = (1, 2.1, 2)$. It can be shown that $L(T_1^*) > M(\hat{T}_2)$ (i.e. Case A of Proposition 2 in Fudenberg and Tirole (1985).)[7] Figures 1 and 2 depict the dynamics of $L(t)$, $F(t)$ and $M(t)$ in these two case, respectively. We can see, from the two figures, that $L(0) > F(0)$. Thus the $(T_1, T_2^*)$-*diffusion equilibrium* does not exist.

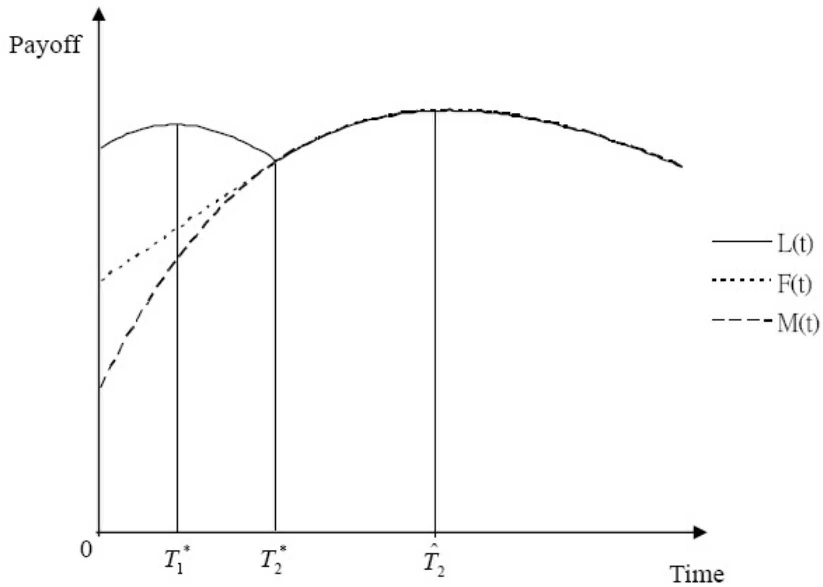


Figure 1: The Case of $L(T_1^*) \leq M(\hat{T}_2)$

[7]In fact, $L(T_1^*) = 50.36 < 50.38 = M(\hat{T}_2)$ and $L(T_1^*) = 1.3 > 1.28 = M(\hat{T}_2)$ in the first and second cases, respectively.

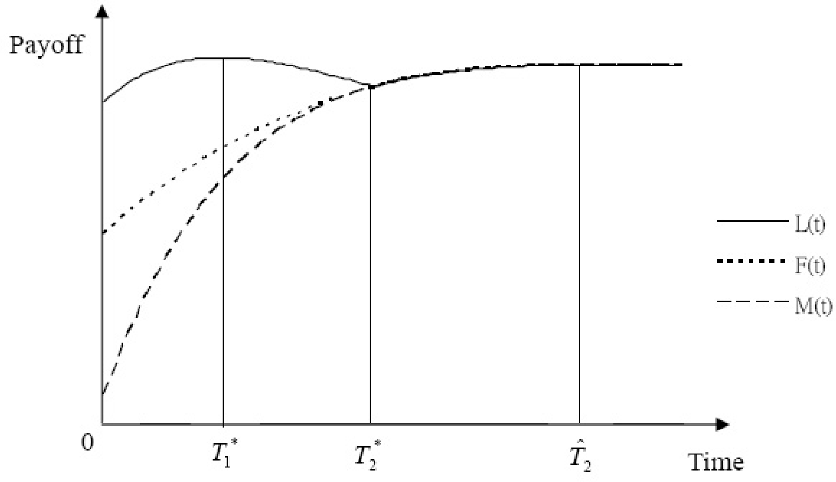


Figure 2: The Case of $L(T_1^*) > M(\hat{T}_2)$

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