Taylor rules and the effects of debt-financed fiscal policy in a monetary growth model

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Abstract

We explore the long-run implications of adopting a Taylor-type interest-rate rule in a simple monetary growth model in which budget deficits are financed partly by unbacked government debt. Because monetary policy is accommodative only when it is passive, the Taylor principle, which requires monetary policy to be active, itself generates a negative relationship between output and inflation. As a result, a permanent increase in government consumption becomes contractionary. Thus, policy makers face a choice between implementing an activist fiscal policy and following the Taylor principle.
1. Introduction

Since the publication of Taylor (1993), many researchers have integrated a Taylor-type interest-rate rule into a variety of dynamic general equilibrium monetary models to verify or challenge the so-called Taylor principle, according to which, for stability, the central bank must be “active”, which means that it raises (cuts) the nominal interest rate by more than one percent if the inflation rate increases (decreases) by one percent.

To date, one of the major criteria for evaluating a policy rule is whether the policy guarantees uniqueness of rational expectations equilibrium or introduces self-fulfilling sunspot fluctuations. Thus, the Taylor principle is often tested on the ground of uniqueness of steady-state equilibrium or determinacy of the steady-state equilibrium (Leeper, 1991; Benhabib et al., 2001a, 2001b; Carlstrom and Fuerst, 2001).

Since the global financial crisis of 2007 and the recession that followed, there has been a resurgence of interest, among economists and policy-makers, in the effects of debt-financed government spending. Motivated by this observation, we study whether the Taylor principle must be satisfied in the times of fiscal expansion. In particular, we study the effects of debt-financed fiscal policy when the central bank follows a Taylor-type interest-rate rule.

To do so, it is important to start with a model in which public debt is not neutral. Schabert (2004) studied a New Keynesian model in which public debt provides transaction services, and found that monetary policy should not be too aggressive, or the effect of fiscal spending will be reduced. Ascari and Rankin (2010) studied a New Keynesian model with finitely-lived agents that is similar to Leith and Wren-Lewis (2000), and argued that a permanent increase in public debt decreases steady-state output. Using the basic New Keynesian model, Woodford (2011) summarized how the government spending multiplier depends on the monetary policy response.

In this paper, we build and study a flexible-price overlapping generations model with money, public debt, and capital accumulation that is similar to Schreft and Smith (1997, 1998) and Bhattacharya et al. (1997). Our study is closely related to Schabert (2004), Ascari and Rankin (2010), and Woodford (2011), but we obtain some new results because in our neoclassical growth model, monetary policy has real effects through investment. The key feature of our model is that, in any steady-state equilibrium, inflation promotes capital accumulation if and only if monetary policy is passive. In other words, the Taylor principle itself generates a negative relationship between output and inflation. As a result, under an active monetary policy, an increase in government spending translates into a higher nominal interest rate and lower capital and output.

The main results are as follows. First, for uniqueness of a steady-state equilibrium, monetary policy cannot be either too active or too passive, because multiple steady-state equilibria can arise in either case. In other words, the elasticity of the nominal interest rate with respect to a change in inflation must be close to one to guarantee uniqueness. Second, although fiscal policy increases output under a passive monetary policy, the output effect becomes negligible as the elasticity gets closer to one (to ensure uniqueness of steady state).
2. The Model

Consider an economy consisting of an infinite sequence of two-period-lived overlapping generations, an initial old generation, and an infinitely-lived government.\(^1\) Let \(t = 1, 2, \ldots\) index time. At each date \(t\), a new generation of a unit measure is born. Each agent is endowed with one unit of labor when young and is retired when old. In addition, the initial old agents are endowed with \(K_1 > 0\) units of capital and \(M_0 > 0\) units of fiat money.

There is a single final good produced using the Cobb-Douglas production function \(Y_t = AK_t^\alpha N_t^{1-\alpha}\) with \(A \geq 1\) and \(0 < \alpha < 1/2\), where \(K_t\) denotes the capital input and \(N_t\) denotes the labor input. Let \(k_t \equiv K_t/N_t\) denote the capital-labor ratio. Then, the intensive production function is \(f(k_t) = Ak_t^\alpha\). It is easy to see that \(f(0) = 0, f' > 0 > f''\), and the Inada conditions hold. The final good can either be consumed in the period it is produced, or stored to yield capital in the next period. For expository reasons, capital is assumed to depreciate 100% between periods.

Factor markets are perfectly competitive. Thus, factors of production receive their marginal product. Let \(r_t\) and \(w_t\) denote the rental rate of capital and the real wage rate. Each young agent supplies his or her labor endowment inelastically in the labor market. Then, profit maximization requires \(r_t = f'(k_t)\) and \(w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t)\). For the Cobb-Douglas specification, \(r_t = \alpha Ak_t^{\alpha-1}\) and \(w(k_t) = (1 - \alpha)Ak_t^\alpha\).

In order to focus on agents’ portfolio choice, we follow Schreft and Smith (1997, 1998) and Bhattacharya et al. (1997) to assume that all individuals save all their income. As a means of saving, agents may hold money and non-monetary assets. In order to motivate the demand for money as a liquid asset, we divide each period into two subperiods. The non-monetary assets, denoted by \(Z_t\), are assumed to yield a gross nominal return of \(I_{t+1} \geq 1\) in the next period. However, the non-monetary assets cannot be liquidated until the second subperiod. Money, whose nominal interest rate is zero, is assumed to be the only liquid asset in this economy. Thus, the only distinction between money and non-monetary assets is that non-monetary assets must be held a little longer (Kudoh, 2007).

We assume that each individual wishes to consume in both subperiods. Let \(c_{1t}\) and \(c_{2t}\) denote the consumption of the final good in the first and second subperiods, respectively, by an old agent born in period \(t\). The individual’s objective function is \(\phi u(c_{1t}) + (1 - \phi) u(c_{2t})\), where \(\phi\) captures the relative weight of utility between the two subperiods. Throughout, we use the following specification: \(u(c) = [1 - \rho]^{-1}c^{1-\rho}\) with \(\rho \neq 1\) and \(\rho > 0\). Because the individual cannot liquidate non-monetary assets in the first subperiod, the agent faces a cash-in-advance constraint: \(p_{t+1}c_{1t} \leq M_t\). The individual’s budget constraint when young is \(M_t + Z_t = p_tw_t - T_t\), where the consumer takes \(T_t\) as given. Similarly, the budget constraint when old is \(p_{t+1}c_{1t} + p_{t+1}c_{2t} = M_t + I_{t+1}Z_t\). The cash-in-advance constraint binds as long as the (net) nominal interest rate is positive (i.e., \(I_t > 1\)). Under the binding cash-in-advance constraint, we obtain \(p_{t+1}c_{2t} = I_{t+1}Z_t = I_{t+1}[p_tw_t - T_t - M_t]\). Thus, a young individual’s

\(^1\)The detail of the model is presented in the working paper version of the paper (Kudoh and Nguyen, 2010). See also Schreft and Smith (1997, 1998) and Bhattacharya et al. (1997).
maximization problem is given by
\[
\max_{M_t} \left\{ \phi \frac{[M_t/p_{t+1}]^{1-\rho}}{1-\rho} + (1-\phi) \frac{[(p_t w_t - T_t - M_t) I_{t+1}/p_{t+1}]^{1-\rho}}{1-\rho} \right\}. \tag{1}
\]

The first-order condition for this problem yields the following money demand function:
\[
M_t = \gamma (I_{t+1}) (p_t w_t - T_t), \tag{2}
\]
\[
\gamma (I_{t+1}) \equiv \left[ 1 + \left( \frac{1-\phi}{\phi} \right)^{1/\rho} \right] I_{t+1}^{1/\rho-1}. \tag{3}
\]

It is easy to establish that \(\gamma'(I) < 0\) holds for \(\rho \in (0, 1)\), \(\lim_{I \to \infty} \gamma(I) = 0\) and \(\lim_{I \to 0} \gamma(I) = 1\) hold for \(\rho \in (0, 1)\), and \(\gamma(1) = [1 + ((1-\phi)/\phi)^{1/\rho}]^{-1}\). Throughout, we focus on the case in which \(\rho \in (0, 1)\) so that the money demand function possesses the standard property that \(\gamma'(I) < 0\).

We let \(G_t\) denote the government spending, \(T_t\) denote the amount of tax revenue, \(I_t \geq 1\) denote the gross nominal interest rate, and \(B_t^g\) denote the amount of government bonds issued in period \(t\). The fiscal authority’s budget constraint is \(G_t + I_t B_{t-1}^g = T_t + B_t^g\) for \(t \geq 2\) and \(G_1 = T_1 + B_1^g\) for \(t = 1\). We assume that the government simply consumes \(G_t\) and that \(G_t\) does not affect the utility of any generation or the production process at any date. It follows that
\[
g_t = \tau_t + b_t^g - R_{t+1} b_{t-1}^g, \tag{4}\]
where \(g_t = G_t/p_t\), \(\tau_t = T_t/p_t\), \(b_t^g = B_t^g/p_t\), and \(R_{t+1} \equiv I_{t+1} p_t/p_{t+1}\). Because bonds and capital are competing financial assets in this economy, the non-arbitrage condition requires the rates of return on these assets to be the same in equilibrium. Thus, \(R_t = f'(k_t)\).

If \(B_t^m\) denotes the monetary authority’s demand for government bonds, then the budget constraint for the central bank is \(B_t^m = I_t B_{t-1}^m + M_t - M_{t-1}\) for \(t \geq 1\), where \(B_t^m\) is the amount of government bonds purchased by the central bank through open market operations. It follows that
\[
b_t^m = I_t \frac{p_{t-1}}{p_t} b_{t-1}^m + m_t - \frac{p_{t-1}}{p_t} m_{t-1}, \tag{5}\]
where \(b_t^m = B_t^m/p_t\) and \(m_t = M_t/p_t\). In what follows, we let \(\Pi_t \equiv p_t p_{t-1}\).

In this paper, we consider the following policy rules. The fiscal authority chooses the entire path for the real government spending. To simplify the analysis, we assume \(g_t = g\) for all \(t\). We assume that the tax is set to be proportional to the real wage rate: \(\tau_t = \theta w_t\), where \(0 \leq \theta < 1\) is an exogenous tax rate.

Following Leeper (1991) and Carlstrom and Fuerst (2001), we assume that the central bank follows a Taylor-type (1993) feedback rule:
\[
I_t = I^* (\frac{\Pi_t}{\Pi^*})^\beta. \tag{6}\]
for $\beta > 0$, and $I^*$ for $\beta = 0$, where $I^*$ and $\Pi^*$ are the implicit targets for $I_t$ and $\Pi_t$.\footnote{The expression (6) does not have a term that relates the output gap to the nominal interest rate. It is important to note that in our flexible-price economy, the output gap is, by construction, zero (see, e.g., Woodford, 2003). Further, according to the estimates of Clarida, Gali, and Gertler (1998), the coefficient on the output gap is quite small for many central banks.} We choose these targets to be consistent with the natural real interest rate, which is the growth rate of the economy. Thus, $I^*/\Pi^* \equiv R^* = 1$ in this economy. The level of $\beta$ is of paramount importance in the analysis. Linearizing (6) yields $(\Pi/I)dI/d\Pi = \beta$. Thus, $\beta$ is the elasticity that captures the degree of aggressiveness of monetary policy.

3. Equilibrium

The equilibrium conditions for the asset markets are as follows. First, dividing (2) by $p_t$ yields

$$m_t = (1 - \theta) \gamma(I_{t+1})w(k_t),$$

(7)

which turns out to be the market clearing condition for money. The capital market equilibrium requires $Z_t = B_t + p_tK_{t+1}$. Dividing it by $p_t$ yields

$$b_t + k_{t+1} = (1 - \theta) [1 - \gamma(I_{t+1})]w(k_t).$$

(8)

The bond market equilibrium requires $B_t + B_t^m = B_t^q$, where $B_t$ is the bond holdings by the household. In real terms, we have

$$b_t + b_t^m = b_t^q.$$  

(9)

We substitute the government budget constraint (4) and the central bank’s budget constraint (5) into (9) to obtain the consolidated government budget constraint:

$$g - \theta w(k_t) = b_t - R_t b_{t-1} + m_t - \frac{p_{t-1}}{p_t} m_{t-1}.$$  

(10)

Throughout the paper, we focus on the steady-state equilibria, in which all real variables are invariant over time. It is easy to show that the monetary policy rule (6), the Fisher equation, and the arbitrage condition between bonds and capital ($R_t = f(k_t)$) reduce to

$$I = \left( \frac{I^{*1/\beta}}{\alpha \Pi^*} \right) \frac{\beta}{1 - \beta} k^{\beta(1 - \alpha)} \equiv i(k),$$

(11)

which summarizes the equilibrium relationship between the nominal interest rate and capital under various degrees of monetary policy activeness.

Similarly, we substitute the market clearing conditions for money (7) and capital (8) into the consolidated government budget constraint (10) to obtain

$$\mu(I) = \frac{Ak^\alpha - g - k}{\alpha(1 - \alpha)(1 - \theta)A^2} \equiv \eta(k),$$

(12)
where \( \mu(I) \equiv 1 - \gamma(I) + \gamma(I)/I \). Thus, a steady state equilibrium is determined by a solution to the system of equations (11) and (12). It follows from the properties of the function \( \gamma(I) \) that \( \mu(1) = 1 \), and for \( \rho \in (0,1) \), \( \lim_{I \to 0} \mu(I) = \infty \) and \( \lim_{I \to \infty} \mu(I) = 1 \). This suggests that the function \( \mu(\cdot) \) is generally U-shaped. However, because \( I \) is the (gross) nominal interest rate, the value of \( I \) we study can be limited to a range that is close to one. In addition, we exclude the scenario of a negative nominal interest rate \( (I < 1) \) from our analysis. Finally, it is easy to establish that \( \mu'(I) < 0 \) holds for \( I \in [1,1/(1-\rho)] \), which identifies the region of \( I \) in which the function \( \mu(\cdot) \) is monotonic and therefore invertible. Throughout this paper, we limit our attention to the region \( I \in [1,1/(1-\rho)] \). It follows from the expression (12) that

\[
I = \mu^{-1}(\eta(k)) \equiv \Omega(k).
\]

It is now evident that the steady-state equilibria are completely characterized diagrammatically by the intersections of the two loci defined by (11) and (13). To proceed, we need to study the shapes of the two loci.

We start with the first locus, (11). From (11), it is easy to obtain

\[
i'(k) = \left( \frac{I^{1/\beta}}{\alpha \Pi^*} \right)^{1-\beta} \beta(1 - \alpha)k^{\beta(1-\alpha)} -1,
\]

from which it is easy to establish that the \( i(k) \)-locus is downward sloping under an active monetary policy \( (\beta > 1) \) and is upward sloping under a passive monetary policy \( (\beta < 1) \). To summarize, the nominal interest rate and output are negatively related if and only if monetary policy is active. The intuition is as follows. Under an active monetary policy, the central bank reacts strongly to a change in the inflation rate, implying that the nominal interest rate changes more than the inflation rate. Thus, the Fisher equation \( (R = I/\Pi) \) implies that the nominal interest rate and the real interest rate are positively related. Therefore, the nominal interest rate and the stock of capital are negatively related along the \( i(k) \)-locus under an active monetary policy. Because the nominal interest rate changes less than the inflation rate under a passive monetary policy, the nominal interest rate and the stock of capital are positively related along the \( i(k) \)-locus.

We now proceed to studying the configuration of the \( \Omega(k) \)-locus. To do so we first need to study the properties of the function \( \eta(k) \). It is easy to verify that, because \( \alpha < 1/2 \), the equation \( \eta(k) = 0 \) has three roots: \( k = 0 \) and the roots of \( Ak^\alpha - k - g = 0 \). In addition, we note that \( \eta(0) = 0 \) and \( \lim_{k \to 0} \eta(k) = -\infty \). From the definition of \( \eta(k) \), it is easy to obtain

\[
\eta'(k) = \frac{Ak^\alpha - 2k - \frac{1-2\alpha}{1-\alpha}g}{\alpha(1-\theta)A^2k^{2\alpha}}.
\]

It is easy to verify that \( \lim_{k \to 0} \eta'(k) = -\infty \) and \( \lim_{k \to \infty} \eta'(k) = -\infty \). Let \( k' \) and \( \bar{k}' \) \( (k' < \bar{k}') \) denote the two distinct solutions to the numerator of (14). Then, \( \eta'(k) > 0 \) holds for \( k \in (k', \bar{k}') \). Since we limit our attention to \( I \in [1,1/(1-\rho)] \), this will limit the region of \( k \) as well. \( I \in [1,1/(1-\rho)] \) implies that \( \mu(I) \in [\mu(1/(1-\rho)), \mu(1)] \). Let \( \bar{k} \) and \( \bar{k} \) denote the two distinct solutions to \( \mu(1/(1-\rho)) = \eta(k) \). It is then easy to verify that \( \eta(k) > 0 \) for any \( k \in [\bar{k}, \bar{k}] \). Further, it is evident that an increase in \( g \) shifts the \( \eta(k) \)-locus downward.
We now study the $\Omega(k)$-locus. It is easy to verify that $\Omega_0(k) = \eta_0(k)/\mu_0(I)$ and $\Omega_00(k) = \eta_{00}(k)/\mu_0(I)$. Thus, the configuration of $\Omega(k)$ can be deduced from that of $\eta(k)$. Noticing $\mu_0(I) < 0$ for $I \in [1, 1/(1 - \rho)]$, it is straightforward to obtain the configuration of $\Omega(k)$,
which is depicted in Figure 2. It is easy to verify that an increase in \( g \) shifts the locus upward. It is important to note that part of the \( \Omega(k) \)-locus can be below \( I = 1 \). However, we can safely exclude such a scenario by considering the case where \( g \) is sufficiently high.

Having established the configurations of the two loci, (11) and (13), we are now in the position to find the steady-state equilibria.

**Proposition 1** There is a unique steady state equilibrium if \( k^* \) satisfies \( k < k^* < \bar{k} \), where

\[
 k^* \equiv \left( \frac{1}{1 - \rho} \right)^{\frac{1 - \beta}{\beta(1 - \alpha)}} \left( \frac{\alpha A \Pi^*}{\bar{I}^{1/\beta}} \right)^{\frac{1}{1 - \alpha}}.
\]

If \( k^* \) satisfies \( k < k^* < \bar{k} \), then there are at most two steady state equilibria.

We construct a proof of proposition 1 in what follows using diagrams. Since we limit our analysis to \( I \in [1, 1/(1 - \rho)] \), we define \( \bar{k} \) and \( k^* \) to be the solutions to \( i(k) = 1 \) and \( i(k) = 1/(1 - \rho) \), respectively. In particular, \( \bar{k} \equiv (\alpha A \Pi^*/\bar{I}^{1/\beta})^{1/(1 - \alpha)} \). Figure 2 depicts a case in which \( k^* \) satisfies \( k < k^* < \bar{k} \). Since the function \( i(k) \) is monotonic for any \( \beta \geq 0 \), it is evident from the figure that the steady state is uniquely determined.

To present sharp results, in what follows we preclude the polar scenarios in which monetary policy is too active and too passive, to focus on the case of a unique steady state equilibrium. In this case, \(|i'(k)| > |\Omega'(k)|\) holds at the steady state (Figure 3). From (11) and (13), we obtain the following:

\[
 \frac{dk}{dg} = \frac{\partial \eta(k)/\partial g}{\mu'(I) \{i'(k) - \Omega'(k)\}},
\]

where \( \partial \eta(k)/\partial g < 0 \) and \( \mu'(I) < 0 \). It follows that \( dk/dg > 0 \) if and only if \( i'(k) > \Omega'(k) \).

**Proposition 2** Suppose \( k^* \) satisfies \( k < k^* < \bar{k} \). If monetary policy is active, then an increase in \( g \) reduces \( k \) and increases \( I \) and \( \Pi \). If monetary policy is passive, then it increases \( k \), \( I \), and \( \Pi \).

These results are illustrated in Figure 3. An upward shift in the \( \Omega(k) \)-locus causes the economy to move along the \( i(k) \)-locus. Since the locus is upward sloping under an active monetary policy, the economy moves from point A to point B. More formally, (16) implies that \( dk/dg > 0 \) if and only if \( i'(k) > \Omega'(k) \). Under an active monetary policy, \( i'(k) < 0 \). Because we focus on the unique equilibrium, we have \(|i'(k)| > |\Omega'(k)|\). It follows that \( i'(k) < \Omega'(k) \). Therefore, we obtain \( dk/dg < 0 \) under an active monetary policy.

The intuition is as follows. An increase in government spending requires an increase in either the direct tax revenue, the seigniorage, or the revenue from bonds. The monetary policy rule (6) and the Fisher equation imply \( R = I^*(\Pi^*)^{-\beta} \Pi^{\beta-1} \), so an increase in inflation reduces capital accumulation if and only if monetary policy is active. Thus, under an active monetary policy, an increase in the government’s need for revenue increases the inflation rate, which increases both the nominal and the real interest rates. In other words, when the
central bank is a tough inflation fighter, an increase in government spending will result in higher nominal and real interest rates, reducing capital and output.

When monetary policy is passive, higher inflation reduces the real interest rate and increases capital and output. In this case, an increase in government spending increases both the inflation rate and the nominal interest rate. The overall effect on the real interest rate is negative, so the stock of capital and output increase. In Figure 3, the economy moves from point A' to point B'. More formally, (16) implies that \( \frac{dk}{dg} > 0 \) if and only if \( i'(k) > \Omega'(k) \). Under a passive monetary policy, \( i'(k) > 0 \). Because we focus on the unique equilibrium, we have \( |i'(k)| > |\Omega'(k)| \). It follows that \( i'(k) > \Omega'(k) \). Therefore, we obtain \( \frac{dk}{dg} > 0 \) under a passive monetary policy.

**Corollary 3** Under a passive monetary policy, the output effect of fiscal policy is positive, but it becomes negligible as \( \beta \) approaches unity.

To see this, consider (11), from which we can show that as \( \beta \) goes to unity, \( k \) approaches \( (\alpha A)^{1/(1-\alpha)} \). In other words, the \( i(k) \)-locus become a vertical line. Because the level of capital is determined without any reference to \( g \), there is no output effect of fiscal policy. Thus, an increase in \( g \) increases the nominal interest rate without any effect on output. The implication is important. For a positive output effect, the Taylor principle must be violated. However, to prevent multiple equilibria, \( \beta \) must be close to unity, in which case the output effect become negligible.
4. Conclusion

Because monetary policy is accommodative if and only if it is passive, a permanent increase in debt-financed government spending under an active monetary policy is contractionary. Thus, policy makers face a choice between implementing an activist fiscal policy and following the Taylor principle. In addition, even under an accommodative monetary policy rule, there is a trade-off between uniqueness of steady-state equilibrium and the strength of the output effect of fiscal spending.

References


