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Capital Accumulation and Social Welfare in Fiscal Federalism and the Unitary System

Wempi Saputra
Graduate School of Economics, Nagoya University

Abstract

This paper presents an analysis on capital accumulation and social welfare in fiscal federalism and the unitary system by using an overlapping generations model. We introduce three possible cases pertaining to how government tax policy towards individuals could be formulated: the government imposes tax on young and old generations under fiscal federalism (case A); the government imposes tax only on young generation under fiscal federalism (case B) as well as under the unitary system (case C). We show that, the level of steady-state capital accumulation as well as social welfare in case A is greater than those in cases B and C if certain conditions are satisfied. Our results suggest that, fiscal federalism is more preferable than the unitary system.

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1. Introduction

This paper presents an analysis on capital accumulation and social welfare under fiscal federalism and the unitary system by using an overlapping generations model. Specifically, we demonstrate a simple model to clarify the levels of steady-state capital accumulation and social welfare under the two systems by introducing three possible cases pertaining to how government's tax policy towards individuals could be formulated:[1)]

a. Case A: the government imposes a head tax on both young and old generations under fiscal federalism;
b. Case B: the government imposes a head tax only on young generation under fiscal federalism;
c. Case C: the government imposes a head tax only on young generation under the unitary system.

The basic theoretical principle of fiscal federalism is perhaps due to Tiebout (1956) who hypothesized that competition among communities might result in an efficiency level of the public good provision at the local level, if fully mobile households could choose a jurisdiction or a locality that provides the best fiscal packages, which met their preferences. This conjecture is further being elaborated by Oates (1972, 1993, 1999), and being supported by, among others, Bird (1993), Gramlich (1993), Brueckner (1999, 2006) and Saputra (2010). However, in the opposing view, some researchers argued that, for instance, due to the existence of market failures and the redistribution income problems (Bewley, 1981) and potential distortion of the local taxation, (Gordon, 1983), the Tiebout and Oates conjecture in favor of fiscal federalism might no longer hold.

Despite few attempts at theoretical analyses, there has been substantial research in the empirical arena with similarly divergent views. Akai and Sakata (2002), Thiesen (2003), Stansel (2005), Iimi (2005), Weingast (1995), Lin and Liu (2000), and Jin, Qian, and Weingast (2005) basically found the positive relationship between decentralization and economic growth and further argued that fiscal federalism strengthen the economic growth and local fiscal incentives do matters in supporting local market development. On the other hand, Zhang and Zou (1998), Davoodi and Zou (1998), Xie, Zou and Davoodi (1999) argued that fiscal decentralization of government spending is associated with the lower economic growth or even harmful for growth. Woller and Phillips (1998) reported there is no significant relationship between the ratios of sub-national revenue and expenditure to total revenue and expenditure while Thornton (2007) concluded that the impact of fiscal decentralization on economic growth is not statistically significant.

The objective of this paper is to fill the gap in the ongoing theoretical literatures of fiscal federalism that focuses on the dynamic aspects of the steady-state levels of capital accumulation and social welfare. This analysis, to our knowledge, is not well established in academic literatures. Our basic model relies on the work of Brueckner (1999) and exhibits a similar pattern to that of Brueckner (2006). We differ from the former study in two respects. First, we formulate the behavior of the government under the two systems in maximizing social welfare by introducing three possible cases of the government's taxing policy toward individuals (in order to finance the public goods provision), where previous models described both individuals and government in a simultaneous-move Nash game. In addition, in the social welfare comparisons between the two systems, we intend to not only establish which is superior between fiscal federalism and the unitary system, but we also try to investigate the most preferable system; namely, which yields the highest capital stock accumulation and social welfare levels. We depart from the latter study by abstracting our analysis from human capital and economic growth.

We report two main findings: first, the level of steady-state capital accumulation in case A is greater than that under cases B and C, while the level of steady-state capital accumulation in case B is equivalent to that in case C. Our first finding provides another interpretation on the understanding of steady-state level of capital accumulation in both systems, as previously argued by Brueckner (1999). In fact, he claimed that the steady-state level of capital accumulation in fiscal federalism is higher (lower) than that of under the unitary system if the young generation has a lower (higher) demand for public goods (which will influence the savings level). In this formulation, our finding suggests that, in the two cases under fiscal federalism (case A and case B), the steady-state level of capital accumulation is greater than, or at least equivalent to that under the unitary system (case C). Second, the social welfare level in case A is greater than that in case B if the level of interest rate in case A, r_A, is greater than or equal to the population growth rate, n. In our formulation, this golden rule welfare condition—the condition in which the marginal product of capital in the steady state is defined to be equal to the population growth rate—is a sufficient condition which makes the level of social welfare in the case A greater than that of case B, which contradicts Brueckner's (1999). In addition, the social welfare level in case A is greater than that in case C if the level of interest rate in case A is greater than or equal to the rate of time preference, ρ, while this rate of time preference must be greater than or equal to the population growth rate. In the comparison between case A and C, the condition of $r_A = \rho$ means that individuals' rate of time preference just equal to the interest rate at which they choose their level of consumption stream, as in the spirit of Olson and Bailey (1981). In this case, there is a stable level of consumption. Although the condition of $r_A > \rho$ is more consistent to the common condition in the real world since in almost cases, capital has a positive net marginal product, the condition that individuals choose a level of consumption stream if the interest rate just equal to the rate of time preference clearly holds for multi-period as well as two-period cases (Samuelson, 1937). Finally, by following the condition of $r_A = \rho$, $\rho = n$ implies that $r_A = n$. If this condition is satisfied, then we might conclude that the social welfare level under fiscal federalism is greater than that of under the unitary system. Please notice that the aforementioned two comparisons (between case A and B, and between case A and C) are the results of distinctive formulation in terms of head tax burden between both generations. In the comparison between case B and C in which a head tax is imposed on the same generation, we might interpret that the golden rule welfare condition as being derived from $r_A = \rho$, which implies $\rho = n$, is a sufficient condition to have the social welfare level under fiscal federalism being equal to that of under the unitary system. Thus, the social welfare level in case B is lower than or equal to that in case C if and only if $\rho \geq n$.[1] These results suggest that, in terms of capital accumulation and social welfare, case A is preferable among the three cases.

The rest of the paper is organized as follows. Section 2 presents the model, section 3 provides equilibrium characteristics and the solutions of the model, section 4 presents the comparison between the two systems and section 5 concludes the paper.

2.The Model

Our basic framework relies on the model of Brueckner (1999), modified to include the three possible cases on how government's taxing policy could be formulated. In our model, each region is populated by two generations, the young and the old, who are assumed to live for two periods. When young, individual works and divides the resulting labor income between consumption, saving and a head tax payment. Then, during the old period, the individual consumes the savings and any interest she earns, pays a head tax and dies. In all three cases, we assume that the population grows at a constant rate n, where $n > 0$. In this case, we assume that the population of the young generation is as large as $(1+n)$ of the old population. The consumption of both generations is divided into consumption of private numeraire goods, x_i, and of public goods, g_i. Hereafter, the subscripts $i = A, B, C$ denote cases A, B, and C, respectively. Following Brueckner (1999), public goods are provided by the government and could be consumed by both generations. In this case, the difference between fiscal federalism and the unitary system is that, under fiscal federalism, each generation is living in a segregated homogenous community; while in the unitary system, both generations are living together in the same community. In this federalist system, the public goods

[1] We thank the referee for his/her valuable improvement suggestion to show that the condition of $V_C > V_B$ holds when $\rho > n$, as an additional result of our analysis.

like police protection and recreational activities for instance, could then be provided specifically following a specific demand from young and old generations.

The assumption of a segregate community under the federalist system can be criticized due to its lack of realism, deriving from the facts that most communities are usually inhabited by both young and old generations. Nevertheless, we hope that, this formulation might have some practical relevance and might present a benchmark for any related policies on this ground (Brueckner,1999). For all public goods provisions, we abstract from the constraint of capacity and congestion.

The public goods are produced by using certain units of private goods, and are financed by a head tax, τ_i, imposed on both young and old generations. In addition to the subscript i mentioned earlier, we also use the time subscripts index t and $t+1$, throughout this paper, which denote the periods, and the superscripts y and o, which denote the young and old, respectively. The per-capita consumption of private goods are $x_{i,t}^{y}$ (for young individuals born at t in case i) and $x_{i,t+1}^{o}$ (for old individuals born at t in case i), while analogous definitions also apply to the head taxes, $\tau_{i,t}^{y}$ and $\tau_{i,t+1}^{o}$, for both young and old respectively:

2.1 Individual behavior

2.1.1. Case A

Under the federalist system, the local government can differentiate the level of public goods' demand between the two generations. The respective budget constraints for the young and old are

$$w_{A,t} - \tau_{A,t}^{y} = x_{A,t}^{y} + s_{A,t}, \qquad (1) \qquad (1 + r_{A,t+1})s_{A,t} - \tau_{A,t+1}^{o} = x_{A,t+1}^{o}, \qquad (2)$$

where $s_{A,t}$ and $w_{A,t}$ are, respectively, the level of savings and wages of the young individual at t, while $r_{A,t+1}$ is the level of interest rate.

From (1) and (2), we can obtain the lifetime budget constraint for individuals as

$$w_{A,t} - \tau_{A,t}^{y} - \frac{\tau_{A,t+1}^{o}}{1 + r_{A,t+1}} = x_{A,t}^{y} + \frac{x_{A,t+1}^{o}}{1 + r_{A,t+1}}. \qquad (3)$$

The formulation of lifetime utility function follows the Brueckner (2006) type. The utility function is separable for both generations, in which, utility of the old is discounted by a rate of time preference, ρ. For simplicity, we define: $\rho_A = \rho_B = \rho_C = \rho; \alpha_A = \alpha_B = \alpha_C = \alpha$. Thus, the utility function of generation t individual is assumed to be a log-linear utility function and can be given as

$$U = \alpha \log x_{A,t}^{y} + (1-\alpha)\log g_{A,t}^{y} + \tfrac{1}{1+\rho}[\alpha \log x_{A,t+1}^{o} + (1-\alpha)\log g_{A,t+1}^{o}], \qquad (4)$$

in which $0 < \alpha < 1$. In this function, $g_{A,t}^{y}$ and $g_{A,t+1}^{o}$ denote, respectively, the consumption of public goods by the young and old born at t. Under this function, individuals maximize their utility subject to budget constraint as described in equation (3). By defining λ as a Lagrange-multiplier and performing an optimization procedure, we can obtain

$$\frac{\alpha}{x_{A,t}^{y}} = \lambda, \qquad (5) \qquad \frac{1}{1+\rho}\frac{\alpha}{x_{A,t+1}^{o}} = \frac{\lambda}{1 + r_{A,t+1}}. \qquad (6)$$

By rearranging equations (5) and (6), we get

$$x_{A,t}^{y} = \frac{1+\rho}{1 + r_{A,t+1}} x_{A,t+1}^{o}, \qquad (7)$$

in which, by using (7) and (3), we can derive the $x_{A,t}^{y}$ and $x_{A,t+1}^{o}$, respectively as

$$x_{A,t}^{y} = \frac{1+\rho}{2+\rho}\left(w_{A,t} - \tau_{A,t}^{y} - \frac{\tau_{A,t+1}^{o}}{1 + r_{A,t+1}}\right), \qquad (8) \qquad x_{A,t+1}^{o} = \frac{1}{2+\rho}\left((1 + r_{A,t+1})(w_{A,t} - \tau_{A,t}^{y}) - \tau_{A,t+1}^{o}\right). \qquad (9)$$

The equations (8) and (9) describes the individuals' behavior, in which the level of consumption for young and old depends on the level of discount rate, the rate of time preference, wage rate and head taxes, and the interest rate. Accordingly, an individual's saving function can be obtained by using (8) or (9) and (3), which can be stated as

$$s_{A,t} = \frac{1}{2+\rho}\left(w_{A,t} - \tau^{y}_{A,t} + \frac{1+\rho}{1+r_{A,t+1}}\tau^{o}_{A,t+1} \right). \tag{10}$$

2.1.2. Case B

In this case, the budget constraint for the young generation remains the same as that of equation (1), adjusted to have a 'B' subscript, while the budget constraint for the old can be rearranged as

$$(1+r_{B,t+1})s_{B,t} = x^{o}_{B,t+1}. \tag{11}$$

By following optimization procedures similar to those explained above, we can derive $x^{y}_{B,t}, x^{o}_{B,t+1}$ and $s_{B,t}$ as follows:

$$x^{y}_{B,t} = \frac{1+\rho}{2+\rho}(w_{B,t} - \tau^{y}_{B,t}), \quad (12) \qquad x^{o}_{B,t+1} = \frac{1+r_{B,t+1}}{2+\rho}\left(w_{B,t} - \tau^{y}_{B,t}\right), \quad (13) \qquad s_{B,t} = \frac{1}{2+\rho}\left(w_{B,t} - \tau^{y}_{B,t}\right). \quad (14)$$

2.1.3. Case C

Since the formulation of case C is similar to that of case B, we can get:

$$x^{y}_{C,t} = \frac{1+\rho}{2+\rho}(w_{C,t} - \tau^{y}_{C,t}), \quad (15) \qquad x^{o}_{C,t+1} = \frac{1+r_{C,t+1}}{2+\rho}\left(w_{C,t} - \tau^{y}_{C,t}\right), \quad (16) \qquad s_{C,t} = \frac{1}{2+\rho}\left(w_{C,t} - \tau^{y}_{C,t}\right). \quad (17)$$

2.2. Firm's production function

In each system, firm produces goods, pays wages for the labor input, $L_{i,t}$, and makes rental payments for the capital input, $K_{i,t}$. Technology is represented by a production function: $Y_{i,t} = K_{i,t}{}^{\beta} L_{i,t}{}^{1-\beta}$, which exhibits constant returns to scale ($0 < \beta < 1$). The per-capita term of the production function is

$$y_{i,t} = k_{i,t}{}^{\beta}, \tag{18}$$

where the output-labor ratio and capital-labor ratio, respectively, are: $y_{i,t} \equiv Y_{i,t} / L_{i,t}; k_{i,t} \equiv K_{i,t} / L_{i,t}$.

Then, the profit maximizing condition of a representative firm yields:

$$r_{i,t} = \beta k_{i,t}{}^{\beta-1}, \quad (19) \qquad w_{i,t} = (1-\beta)k_{i,t}{}^{\beta}, \quad (20)$$

where $r_{i,t}$ and $w_{i,t}$ both describe the factor prices of production inputs.

2.3. Government's behavior

2.3.1. Case A

In case A, regional government imposes head taxes for both generations. The regional government chooses the optimal values of $g^{y}_{A,t}$ and $g^{o}_{A,t+1}$ by considering the behavior of individuals' born at t. Public goods provision are financed by head taxes imposed on both young and old generations. Thus, the budget constraint of government will be:

$$ag^{y}_{A,t} = \tau^{y}_{A,t}, \quad (21) \qquad ag^{o}_{A,t+1} = \tau^{o}_{A,t+1}, \quad (22)$$

where a is a linear technology parameters in the production of public goods and assumed to be equivalent in all three cases.

2.3.2. Case B

The related budget constraint of government is

$$a\left(g^{y}_{B,t} + \frac{g^{o}_{B,t+1}}{1+n} \right) = \tau^{y}_{B,t}, \tag{23}$$

where, in this case, the head tax is only imposed on the young, while public goods are provided for both generations.

2.3.3. Case C

The related budget constraint of government is

$$ag_{C,t}\left(\frac{2+n}{1+n}\right)=\tau^{y}_{C,t}. \tag{24}$$

where, in this case, a head tax is only imposed on the young, while the common level of public goods, g, is provided for both generations. From this equation, the value of $(2+n)/(1+n)$ describes a population share of the young. Since we assume that the young population is $1+n$ as large as the old population, then, holding the old population at a given date at N, the population of the young is $(1+n)N$. The total population is $(2+n)N$.

3.Equilibrium characteristics

3.1. Capital accumulation

Capital market clearing condition is defined such that a total saving of the young generation is equal to a capital stock in the next period. This condition could be stated as

$$s_{i,t}=(1+n)k_{i,t+1}. \tag{25}$$

3.1.1. Case A

Substituting (10) into (25), and by using (19) and (20), we can derive the dynamic behavior of capital stock as follows:

$$k_{A,t+1}=\frac{1}{(1+n)(2+\rho)}\left((1-\beta)k_{A,t}{}^{\beta}-\tau^{y}_{A,t}+\frac{1+\rho}{1+\beta k_{A,t+1}{}^{\beta-1}}\tau^{o}_{A,t+1}\right). \tag{26}$$

3.1.2. Case B

In this case, equation (25) could be rearranged by incorporating (14) to obtain

$$k_{B,t+1}=\frac{1}{(1+n)(2+\rho)}\left((1-\beta)k_{B,t}{}^{\beta}-\tau^{y}_{B,t}\right). \tag{27}$$

3.1.3. Case C

By following a similar pattern of case B above and using equation (17), we can obtain

$$k_{C,t+1}=\frac{1}{(1+n)(2+\rho)}\left((1-\beta)k_{C,t}{}^{\beta}-\tau^{y}_{C,t}\right). \tag{28}$$

To further analyze the capital stock accumulation in a steady-state, let $k_{i,t+1}=k_{i,t}=k^{*}_{i};\tau^{y}_{i,t}=\tau^{y}_{i};\tau^{o}_{i,t+1}=\tau^{o}_{i,t}=\tau^{o}_{i}$, where $k^{*}_{i},\tau^{y}_{i},\tau^{o}_{i}$ represent the steady-state values of, respectively, the capital stock and the head tax for the young and old in case i. We can then reformulate the equations (26), (27) and (28) to get the steady-state levels of capital stock under cases A, B and C, respectively:

$$k^{*}_{A}=\frac{1}{(1+n)(2+\rho)}\left((1-\beta)k^{*\beta}_{A}-\tau^{y}_{A}+\frac{1+\rho}{1+\beta k^{*\,\beta-1}_{A}}\tau^{o}_{A}\right), \tag{29}$$

$$k^{*}_{B}=\frac{1}{(1+n)(2+\rho)}\left((1-\beta)k^{*\,\beta}_{B}-\tau^{y}_{B}\right), \tag{30}$$ $$k^{*}_{C}=\frac{1}{(1+n)(2+\rho)}\left((1-\beta)k^{*\,\beta}_{C}-\tau^{y}_{C}\right), \tag{31}$$

3.2. The levels of public goods and head taxes

3.2.1. Case A

Since the regional government's decision must also be arranged in the steady-state, we first recall the individuals' behavior as stated in the equations (8) and (9) and rearrange them by replacing the level of wage rate and interest rate in those equations with (19) and (20) to obtain:

$$x_A^y = \frac{1+\rho}{2+\rho}((1-\beta)k_A^{*\beta} - \tau_A^y - \frac{\tau_A^o}{1+\beta k_A^{*\beta-1}}), \tag{8'}$$

$$x_A^o = \frac{1}{2+\rho}\left((1+\beta k_A^{*\beta-1})((1-\beta)k_A^{*\beta} - \tau_A^y) - \tau_A^o\right). \tag{9'}$$

The budget constraints of government must also be rearranged to its steady-state value as follows:

$$a g_A^y = \tau_A^y, \tag{21'}$$ $$a g_A^o = \tau_A^o. \tag{22'}$$

The regional government's objective function is to maximize welfare of individuals by choosing the appropriate levels of public goods. This function could be formulated as

$$V_A = \alpha \log x_A^y + (1-\alpha)\log g_A^y + \frac{1}{1+\rho}\left(\alpha \log x_A^o + (1-\alpha)\log g_A^o\right). \tag{32}$$

Inserting (8)', (9)', (21)' and (22)'into (32), we obtain:

$$\begin{aligned} V_A = {} & \alpha \log\left(\frac{1+\rho}{2+\rho}\left((1-\beta)k_A^{*\beta} - \tau_A^y - \frac{\tau_A^o}{(1+\beta k_A^{*\beta-1})}\right)\right) + (1-\alpha)\log\frac{\tau_A^y}{a} \\ & + \frac{1}{1+\rho}\left(\alpha \log\left(\frac{1}{2+\rho}\left((1+\beta k^{*\beta-1}_A)((1-\beta)k_A^{*\beta} - \tau_A^y) - \tau_A^o\right)\right) + (1-\alpha)\log\frac{\tau_A^o}{a}\right). \end{aligned} \tag{33}$$

By performing an optimization problem in respect to τ_A^y and τ_A^o, we can get

$$\frac{2+\rho}{1+\rho}\frac{\alpha(1+\beta k_A^{*\beta-1})}{\left((1+\beta k_A^{*\beta-1})((1-\beta)k_A^{*\beta} - \tau_A^y)\right) - \tau_A^o} = \frac{(1-\alpha)}{\tau_A^y}, \tag{34}$$

$$\frac{\alpha(2+\rho)}{\left((1+\beta k_A^{*\beta-1})((1-\beta)k_A^{*\beta} - \tau_A^y)\right) - \tau_A^o} = \frac{(1-\alpha)}{\tau_A^o}, \tag{35}$$

in which, after following some algebraic processes, we obtain

$$\tau_A^y = \frac{(1-\alpha)(1+\rho)}{(2+\rho)}(1-\beta)k_A^{*\beta}, \tag{36}$$ $$\tau_A^o = \frac{(1-\alpha)(1+\beta k_A^{*\beta-1})}{(2+\rho)}(1-\beta)k_A^{*\beta}. \tag{37}$$

Thus, the levels of public good for young and old, respectively, are as follows:

$$g_A^y = \frac{(1-\alpha)(1+\rho)}{a(2+\rho)}(1-\beta)k_A^{*\beta}, \tag{38}$$ $$g_A^o = \frac{(1-\alpha)(1+\beta k_A^{*\beta-1})}{a(2+\rho)}(1-\beta)k_A^{*\beta}. \tag{39}$$

3.2.2. Case B

In this case, we first reformulate equations (12) and (13) by following similar procedures to that of case A above. This formulation yields

$$x_B^y = \frac{1+\rho}{2+\rho}\left((1-\beta)k_B^{*\beta} - \tau_B^y\right), \tag{12'}$$ $$x_B^o = \frac{1+\beta k_B^{*\beta-1}}{2+\rho}\left((1-\beta)k_B^{*\beta} - \tau_B^y\right). \tag{13'}$$

In order to solve the regional government's decision, we follow equation (33) by considering the budget constraint as stated in (23). This formulation becomes

$$\begin{aligned} V_B = {} & \alpha \log\frac{1+\rho}{2+\rho}\left((1-\beta)k_B^{*\beta} - a\left(g_B^y + \frac{g_B^o}{1+n}\right)\right) + (1-\alpha)\log g_B^y \\ & + \frac{1}{1+\rho}\left(\alpha \log\frac{1+\beta k^{*\beta-1}_B}{2+\rho}\left((1-\beta)k_B^{*\beta} - a\left(g_B^y + \frac{g_B^o}{1+n}\right)\right) + (1-\alpha)\log g_B^o\right). \end{aligned} \tag{40}$$

Note that, in this case, we reformulate the regional government to choose the level of public goods and, by using these values, we can determine the level of a head tax for the young. Performing standard optimization procedures with respect to g_B^y and g_B^o, we can obtain

$$g_B^y = \frac{(1-\alpha)(1+\rho)}{a(2+\rho)}(1-\beta)k_B^{*\beta}, \quad (41) \qquad g_B^o = \frac{(1-\alpha)(1+n)}{a(2+\rho)}(1-\beta)k_B^{*\beta}, \quad (42)$$

and, accordingly, the level of τ_B^y is

$$\tau_B^y = (1-\alpha)(1-\beta)k_B^{*\beta}. \quad (43)$$

3.2.3. Case C

We follow the same pattern in case B above by replacing the related value of a head tax as stated in equation (24). In this case, we might only work on g_C and τ_C. This formulation might be stated as

$$\begin{aligned} V_C &= \alpha\log\frac{1+\rho}{2+\rho}\left((1-\beta)k_C^{*\beta} - ag_C\left(\frac{2+n}{1+n}\right)\right) + (1-\alpha)\log g_C \\ &+ \frac{1}{1+\rho}\left(\alpha\log\frac{1+\beta k_C^{*\beta-1}}{2+\rho}\left((1-\beta)k_C^{*\beta} - ag_C\left(\frac{2+n}{1+n}\right)\right) + (1-\alpha)\log g_C\right). \end{aligned} \quad (44)$$

Solving this problem, we can get the values of g_C and τ_C as follows:

$$g_C = \frac{(1-\alpha)(1+n)}{a(2+n)}(1-\beta)k_C^{*\beta}, \quad (45) \qquad \tau_C = (1-\alpha)(1-\beta)k_C^{*\beta}. \quad (46)$$

3.3. The Steady State of Capital Accumulation

We can rearrange the steady-state capital accumulation stated in equation (29) by using (36) and (37) to get k_A^* as follows

$$k_A^* = \left(\frac{(1-\beta)}{(1+n)(2+\rho)}\right)^{\frac{1}{1-\beta}}. \quad (47)$$

For the case B, by inserting the corresponding head tax value in (43) into (30), we can obtain k_B^* which is equal to

$$k_B^* = \left(\frac{\alpha(1-\beta)}{(1+n)(2+\rho)}\right)^{\frac{1}{1-\beta}}. \quad (48)$$

Finally, by following the similar pattern of case B, and by using equations (31) and (46), we obtain

$$k_C^* = \left(\frac{\alpha(1-\beta)}{(1+n)(2+\rho)}\right)^{\frac{1}{1-\beta}}. \quad (49)$$

4.The comparisons between the two systems

4.1.The comparison of steady-state capital accumulation

We make three comparisons, which are: between k_A^* and k_B^*, k_A^* and k_C^*, and k_B^* and k_C^*.

4.1.1. Between k_A^* and k_B^*

By subtracting (47) by (48), we can get

$$k_A^* - k_B^* = \left(\frac{(1-\beta)}{(1+n)(2+\rho)}\right)^{\frac{1}{1-\beta}} - \left(\frac{\alpha(1-\beta)}{(1+n)(2+\rho)}\right)^{\frac{1}{1-\beta}}. \quad (50)$$

It is easy to see that since $1 > \alpha$, $k_A^* > k_B^*$. This means that the level of steady-state capital stock accumulation in case A is greater than that of case B.

4.1.2. Between k_A^* and k_C^*

Considering the result stated in (50), since the value of $k_B^* = k_C^*$, it follows that $k_A^* > k_C^*$.

4.1.3. Between k_B^* and k_C^*

It is clear that the level of $k_B^* = k_C^*$.

Thus, by summarizing all of these three results derived from comparing the steady-state levels of capital accumulation, our first finding could simply be stated in the following proposition.

Proposition 1. *Suppose that the formulations of the three cases above hold. The comparisons of the level of steady-state capital accumulation in these three cases yield:* $k_A^* > k_B^*; k_A^* > k_C^*; k_B^* = k_C^*$.

The intuition behind this proposition is straightforward. In case A, where the government taxes both young and old generations, there is an incentive to save from young generation to maintain his or her desirable consumption level in the old period, as a response to the head tax being imposed by the government at this period. However, in case B and C, it seems that this saving incentive might be lower than that in case A, since the government will not impose a head tax to the young generation.

4.2. The comparison of social welfare levels

Let V_A, V_B and V_C be the social welfare levels under case A, B and C respectively.

4.1.1. Comparison between V_A and V_B

In order to compare the level of social welfare under case A and B, we utilize the government's objective function as stated in equations (33) and (40). Then, we reformulate them to become

$$V_A - V_B = \frac{2+\rho}{1+\rho}\log\frac{k_A^{*\beta}}{k_B^{*\beta}} + \frac{1}{1+\rho}\alpha\log\left(\frac{1+\beta k_A^{*\beta-1}}{1+\beta k_B^{*\beta-1}}\right) + \frac{1}{1+\rho}(1-\alpha)\log\left(\frac{1+\beta k_A^{*\beta-1}}{1+n}\right). \tag{51}$$

Since from the proposition 1 we know that $k_A^* > k_B^*$, then we can easily observe that the sign of the first and second parts in the right hand side (RHS) of (51) are positive. On the other hand, the last part in the RHS of (51) contains an ambiguous value, except when a certain condition is satisfied. In this case, by recalling (19), we might safely assume that in the steady-state, $\beta k_A^{*\beta-1} = r_A$, in which, it implies that the value of the last part in the RHS of (51) depends on the magnitude of r_A and n. We can easily see that if $r_A \geq n$, then $V_A - V_B > 0$. On the other hand, if $r_A < n$, then $V_A - V_B$ will result in an ambiguous value since the last part in the RHS of (51) yields a negative value. Thus, we conclude that $V_A - V_B > 0$ if $r_A \geq n$ and $V_A - V_B$ will yield an ambiguous value if otherwise.

4.1.2. Between V_A and V_C

By subtracting (33) by (44), we can obtain:

$$\begin{aligned} V_A - V_C = {} & \frac{2+\rho}{1+\rho}\log\frac{k_A^{*\beta}}{k_C^{*\beta}} + (1-\alpha)\log\left(\frac{(1+\rho)(2+n)}{(2+\rho)(1+n)}\right) + \frac{1}{1+\rho}\alpha\log\left(\frac{1+\beta k_A^{*\beta-1}}{1+\beta k_C^{*\beta-1}}\right) \\ & + \frac{1}{1+\rho}(1-\alpha)\log\left(\frac{(1+\beta k_A^{*\beta-1})(2+n)}{(2+\rho)(1+n)}\right). \end{aligned} \tag{52}$$

From (52), we can see that the value of the first and the third parts in the RHS of (52) are positive, while the value of the second and the last parts remain ambiguous. The value of this second part will be positive or zero if $\rho \geq n$ and will be negative if $\rho < n$. On the other hand, by following the previous assumption that $\beta k_A^{*\beta-1} = r_A$, the value of the last part in the RHS of (52) will depend on the magnitude of r_A, ρ and n. In this case, $r_A \geq \rho \geq n$ will make this last part become positive or at least zero. Therefore, based on these two conditions, we can get a condition of $r_A \geq \rho \geq n$ in order to ensure that the value of $V_A - V_C$ to become positive. Needless to say, $V_A - V_C$ will still positive however, if $r_A = \rho = n$. From this consideration, $r_A = \rho = n$ is a sufficient condition to ensure the positive result of $V_A - V_C$. Thus, we might conclude that the social welfare in case A is greater than that in case C if $r_A \geq \rho \geq n$.

4.1.3. Between V_B and V_C

Since we know that $k_B^* = k_C^*$, and by comparing (40) and (44), we can obtain:

$$V_B - V_C = (1-\alpha)\log\left(\frac{(1+\rho)(2+n)}{(2+\rho)(1+n)}\right) + \frac{1}{1+\rho}(1-\alpha)\log\left(\frac{(2+n)}{(2+\rho)}\right). \quad (53)$$

In this equation, the social welfare comparison depends on the value of n and ρ. Then, if and only if $\rho = n$, then $V_B = V_C$. In addition, we could still obtain the sign of $V_B - V_C$ when $\rho \neq n$. [2] By manipulating (53), we can get:

$$V_B < V_C = \left(\frac{(1+\rho)^{(1+\rho)}}{(2+\rho)^{(2+\rho)}}\right) < \left(\frac{(1+n)^{(1+\rho)}}{(2+n)^{(2+\rho)}}\right). \quad (54)$$

Then, we can obtain the derivative of the expression on the left of this inequality that is given by:

$$\left(\frac{(1+\rho)^{(1+\rho)}(2+\rho)^{(2+\rho)}[\log(1+\rho) - \log(2+\rho)]}{(2+\rho)^{2(2+\rho)}}\right), \quad (55)$$

in which, one can see that $[\log(1+\rho) - \log(2+\rho)] < 0$ while the rest is positive. Consequently, the expression on the left of the inequality is decreasing with respect to ρ. Since $V_B = V_C$ if and only if $\rho = n$, then $V_c > V_B$ if and only if $\rho > n$. We summarize these results in the following proposition.

Proposition 2. *The results of social welfare comparisons among the three cases could be clearly obtained if the following conditions are satisfied:*

a. $V_A > V_B$, if $r_A \geq n$;

b. $V_A > V_C$, if $r_A \geq \rho \geq n$;

c. $V_B = V_C$, if and only if $\rho = n$, and $V_C > V_B$, if and only if $\rho > n$.

Thus, the social welfare level under case A is greater than those of case B and C. If the conditions of a, b and c above do not hold, the comparisons of social welfare will yield ambiguous values.

The intuition behind this proposition could be stated as follows. First, recall the equation (19). In the steady state, the marginal product of capital for the case A could be given as

$$r_A = \beta k_A^{*\,\beta-1}. \quad (19)'$$

The golden rule capital stock is defined by $r_{GR} = n$. In this case, after inserting (48) into (19)', the value of r_A can be either more or less than r_{GR}. In particular, for α sufficiently small, $r_A < r_{GR}$, which means that the capital stock in the steady state exceed the golden rule level. In this paper, we assume that the $\alpha's$ value is such that makes $r_A = r_{GR} = n$, in order to rule out the possibility of permanent increase in the consumption. Brueckner (1999) called this condition as golden rule welfare condition. In our formulation, this golden rule welfare condition is a sufficient condition which makes the level of social welfare in the case A greater than that of case B, the result that contradicts Brueckner's (1999).

5.**Conclusion**

The analyses in this paper suggest that the greater steady-state levels of capital accumulation and social welfare may constitute an additional benefit of fiscal federalism, which match the expectation of the most recent empirical researches in this field. These results, deriving from the three possible cases of the government's taxing policy toward individuals, suggest that the level of steady-state capital accumulation and social welfare under fiscal federalism are greater than that of under the unitary system as long as certain conditions are satisfied.

[2] We follow the referee's suggestion in deriving this result.

Appendix A
The Unitary System: taxing on both young and old generations
The budget constraint of government is:

$$g = \frac{1}{a(2+n)}\left((1+n)\tau^{y} + \tau^{o}\right). \tag{A1}$$

The government objective function could be stated as:

$$V^{u} = \left\{\left(\alpha\log\left(\frac{1+\rho}{2+\rho}\left((1-\beta)k^{*\beta} - \tau^{y} - \frac{\tau^{o}}{(1+\beta k^{*\beta-1})}\right)\right) + (1-\alpha)\log\frac{1}{a(2+n)}\left((1+n)\tau^{y}+\tau^{o}\right)\right)\right.$$
$$\left. + \frac{1}{1+\rho}\left(\alpha\log\left(\frac{1}{2+\rho}\left((1+\beta k^{*\beta-1})((1-\beta)k^{*\beta}-\tau^{y})-\tau^{o}\right)\right) + (1-\alpha)\log\frac{1}{a(2+n)}\left((1+n)\tau^{y}+\tau^{o}\right)\right)\right\}. \tag{A2}$$

Solving (A2) to get τ^{y} and τ^{o} will result in:

$$\tau^{y} = (1+\beta k^{*\beta-1})((1-\beta)k^{*\beta}\left(\frac{(1+n)-(1+\beta k^{*\beta-1})}{(1+\beta k^{*\beta-1})\left(2(1+n)-(1+\beta k^{*\beta-1})\right)-(1+n)^{2}}\right), \tag{A3}$$

$$\tau^{o} = (1+\beta k^{*\beta-1})((1-\beta)k^{*\beta}\left(\frac{(1+n)\left((1+\beta k^{*\beta-1})-(1+n)\right)}{(1+\beta k^{*\beta-1})\left(2(1+n)-(1+\beta k^{*\beta-1})\right)-(1+n)^{2}}\right). \tag{A4}$$

Inserting these levels of head taxes into (A1), we then could find g level that is equal to zero.

Appendix B
The Unitary System: taxing on both young and old generations
The budget constraint of government is:

$$\tau = ag. \tag{B1}$$

The government objective function could be stated as:

$$V^{u} = \left\{\left(\alpha\log\left(\frac{1+\rho}{2+\rho}\left((1-\beta)k^{*\beta} - \tau - \frac{\tau}{(1+\beta k^{*\beta-1})}\right)\right) + (1-\alpha)\log\frac{\tau}{a}\right)\right.$$
$$\left. + \frac{1}{1+\rho}\left(\alpha\log\left(\frac{1}{2+\rho}\left((1+\beta k^{*\beta-1})((1-\beta)k^{*\beta}-\tau)-\tau\right)\right) + (1-\alpha)\log\frac{\tau}{a}\right)\right\}. \tag{B2}$$

Solving (B2) to get τ will result in:

$$\tau = \frac{(1-\alpha)}{(2+\beta k^{*\beta-1})}(1+\beta k^{*\beta-1})(1-\beta)k^{*\beta}. \tag{B3}$$

Inserting these levels of head taxes into (26) by replacing the related value of τ^{y} and τ^{o} as τ, we then could find the level of k^{*}:

$$k^{*} = \frac{1}{(1+n)(2+\rho)}\left((1-\beta)k^{*\beta} - \frac{(1-\alpha)}{(2+\beta k^{*\beta-1})}(1+\beta k^{*\beta-1})(1-\beta)k^{*\beta}\right.$$
$$\left. + \frac{1+\rho}{1+\beta k^{*\beta-1}}\frac{(1-\alpha)}{(2+\beta k^{*\beta-1})}(1+\beta k^{*\beta-1})(1-\beta)k^{*\beta}\right). \tag{B4}$$

This might yield:

$$k^{*} = \frac{(1-\beta)k^{*\beta}}{(1+n)(2+\rho)} \frac{(2+\alpha\beta k^{*^{\beta-1}} + \rho - \alpha\rho)}{(2+\beta k^{*^{\beta-1}})}; \tag{B5}$$

which cause the possibility of a multiple value of k^{*}, and accordingly, there is no an exact patterns how k^{*} will evolve over time given its initial value (for more details, please see for instance, Romer, 2001, pp.83-85).

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[1)] Initially, we formulated 4 (four) cases, in which the fourth case was formulated by taxing both young and old generations under the unitary system. We begin the formulation of this case by introducing two possible combinations in which, it differs only on its government's budget constraint. However, the result showed that the level of public goods are zero, deriving from the condition that head tax for the young and old generations are canceling each other (Appendix A). In other case, the steady-state level of capital stock accumulation might be not in clear pattern or there is a possibility of a multiple k^*. (Appendix B). Therefore, we focus our formulation based on 3 (three) cases as mentioned above.