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Estimation of consumption-capital asset pricing model (C-CAPM) with two clusters of consumption expenditures

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Abstract

In this paper, we develop a new model that explicitly considers two endogenous consumption items and investigates its applicability to consumption-capital asset pricing model (C-CAPM) by testing it with various sets of instruments. We found that our model is not rejected with reasonable values for both risk aversion and time preference parameters.

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1. Introduction

In this paper, we extend the standard Consumption-Capital Asset Pricing Model (C-CAPM) for a single consumption item by explicitly modeling two endogenous consumption items, namely non-durables and services. The introduction of two consumption items in the inter-temporal consumption-investment problem makes it possible to investigate the role of distinct consumption items and their prices in both inter-temporal and intra-temporal optimizations. This paper fills a gap noted by Pennacchi (2008, p7) between microeconomics that primarily analyzes consumers' optimal choice among multiple goods and financial economics that mainly focuses on the optimal choice between a single consumption item and financial assets at different points in time under varying conditions and states of nature.

Our model is close to Mankiw, Rotemberg and Summers (1985) and Eichenbaum, Hansen and Singleton (1988) that introduced two endogenous variables, namely consumption and leisure, in the utility function of agents.

We also consider the effect of a difference of an instrument set on the estimation and testing of the model. If we test a model with only a few sets of instruments and reject it, the rejection may be due to the particular choice of instruments. Thus, we form a large set of instruments and see whether or not our model is rejected. We found that our model is not rejected with reasonable values for both the risk aversion and time preference parameters involving many sets of instruments.

2. Models with two consumption items

In our model, we explicitly classify total consumption into two items, namely non-durables and services. Both variables are endogenous. We consider three types of financial assets: stocks, bonds and risk-free assets. The consumer in the economy develops a consumption and portfolio plan so as to maximize the expected utility subject to the budget constraint. There is both an intra-temporal choice between non-durables and services and an inter-temporal choice between consumption and investment. The lifetime utility function and the budget constraint at time t are given by:

$$\max_{\{c_{n,t},c_{s,t}\},\{N_t^F,N_t^B,N_t^S\}} E_t[\textstyle\sum_{k=0}^{\infty}\beta^k U(c_{n,t+k},c_{s,t+k})] \tag{1}$$

subject to:

$$Y_t + 1 * N_{t-1}^F + (P_t^B + D_t^B)N_{t-1}^B + (P_t^S + D_t^S)N_{t-1}^S = c_t^n p_t^n + c_t^s p_t^s + P_t^F N_t^F + P_t^B N_t^B + P_t^S N_t^S \tag{2}$$

where β is the time preference parameter, E_t is the expectation conditioned on the information set at time t, Y_t is nominal disposable income, 1 is the payoff of the riskless asset which is a discount bond, N_t^F is the number of shares of the riskless asset, P_t^B and D_t^B are the nominal price and coupon payment of bonds, respectively, N_t^B is the number of shares of bonds, P_t^S and D_t^S are the nominal price and dividend payment of stocks, respectively, and N_t^S is the number of shares of stocks. Also, $p_{n,t}$ and $p_{s,t}$ are the price deflators for non-durables and services, respectively, and $c_{n,t}, c_{s,t}$ are per capita real consumption for non-durables and services, respectively. The time separable utility function $U(c_{n,t}, c_{s,t})$ is specified as:

$$U(c_{n,t}, c_{s,t}) = \frac{c_{n,t}{}^{1-\gamma} c_{s,t}{}^{1-\delta} - 1}{(1-\gamma)(1-\delta)} \tag{3}$$

where γ and δ are the risk aversion parameters for non-durables and services, respectively. Both parameters take positive values from a theoretical restriction.

Note that we have two ways to deflate nominal series into real series in our model. One is to employ the price deflator for non-durables and the other is to use that for services. Thus we have two pairs of real rates of return for each nominal rate of return from financial assets. We denote the nominal rates of return from time t to t+1 for risk-free assets, bonds and stocks by $R_{f,t+1}$, $R_{b,t+1}$ and $R_{s,t+1}$. Then the real rates of returns for these three financial assets are given by:

$$1 + r^n_{j,t+1} = (1 + R_{j,t+1}) / (p_{n,t+1} / p_{n,t}) \qquad (j = f, b, s) \tag{4}$$

$$1 + r^s_{j,t+1} = (1 + R_{j,t+1}) / (p_{s,t+1} / p_{s,t}) \qquad (j = f, b, s) \tag{5}$$

The first order conditions for the inter-temporal optimization between consumption and investment are given by:

$$E_t[\beta(\frac{c_{n,t+1}}{c_{n,t}})^{-\gamma}(\frac{c_{s,t+1}}{c_{s,t}})^{1-\delta}(1 + r^n_{j,t+1})] = 1 \qquad (j = f, b, s) \tag{6}$$

$$E_t[\beta(\frac{c_{n,t+1}}{c_{n,t}})^{1-\gamma}(\frac{c_{s,t+1}}{c_{s,t}})^{-\delta}(1 + r^s_{j,t+1})] = 1 \qquad (j = f, b, s) \tag{7}$$

The first order condition for the intra-temporal optimization between non-durables and services is as follows:

$$E_t[\frac{c^s_t}{c^n_t} \cdot \frac{p^s_t}{p^n_t} \cdot \frac{1-\gamma}{1-\delta}] = 1 \tag{8}$$

Following Mankiw, Rotemberg and Summers (1985), we estimate and test all the seven equations simultaneously.

3. Data

For the consumption series, we employ monthly household-level consumption data derived from the *Family Income and Expenditure Survey* (FIES) from January 1980 through April 2002 compiled by the Statistics Bureau of Japan. The total number of observations is 279. For the financial returns, we employ nominal stocks, bonds and risk free returns from *Stocks,* *Bonds, Bills and Inflation* (SBBI) compiled by Ibbotson Associates Japan. The mean real returns on the stock index, long-term government bond and money market instruments calculated by the price deflator of non-durables are 1.0035, 1.0057 and 1.0024 respectively, and those evaluated by the price deflator of services are 1.0025, 1.0047 and 1.0014, respectively.

We apply seasonal adjustment to the seasonally unadjusted consumption series published by the Statistics Bureau because seasonal fluctuations such as an outlay for Christmas shopping are not specified in our model. We follow Ghysels and Osborn (2001) who argue that seasonal adjustment should be applied at the last stage. Note that both seasonally unadjusted consumption series and the price deflators for households are available from FIES compiled by the Statistics Bureau. We divide the nominal seasonally unadjusted consumption item by the corresponding seasonally unadjusted price deflator. Then we divide the real consumption item by the number of household members. Finally, we apply the X-12ARIMA method developed by the Bureau of Labor Statistics to obtain the seasonally adjusted real per capita consumption series. The average growth rates between t and t+1 period on non-durables and that on services are 1.00084 and 1.00145, respectively.

4. Instruments and GMM estimation results

In order to estimate the model: (6), (7) and (8) simultaneously, we employ the GMM method. We use the Newey-West HAC estimator with a Bartlett kernel (MA=4). We selected the following set of instruments including constant, consumption for non-durables, services; returns of stocks, bonds and risk-free assets evaluated by the price deflator for non-durables or services:

$$(1, c_{n,t-1}, c_{s,t-1}, r_{s,t}^{n}, r_{s,t}^{s}, r_{b,t}^{n}, r_{b,t}^{s}, r_{f,t}^{n}, r_{f,t}^{s}) \tag{7}$$

We take every combination of instrument sets from the above variables and have a total of 255 sets of instruments. When we test the model, we impose the following criteria on the magnitude of parameters and test statistics:

(1) P-value of the model > 0.05
(2) *t*-values for β, γ and δ > 1.96
(3) β < 1
(4) γ, δ > 0

The first criterion is on the magnitude of the J-test statistics at the significance level of five percent, the second is on the *t*-statistics for β, γ and δ at the significance level of five percent, the third and fourth are the theoretical restrictions regarding preference parameters.

Out of 255 instrument sets, we obtain 221 sets that converge for the distinct estimation and testing results. Out of these 221 results, the model is not rejected in 195 cases by J-test. In these 195 cases, 156 cases are not rejected statistically judging from the *t*-statistics. The theoretical restrictions indicated in (3) and (4) are fully satisfied by the above 156 cases. Thus, 156 out of 255 combinations satisfy the above four conditions.

The magnitude of γ is between 0.250 and 0.490 and that of δ is between 0.332 and 0.559 in 152 cases. In the remaining four cases, the maximum magnitude of both γ and δ coefficients are l6.023 and 5.60 respectively. The time preference parameter is between 0.9907 and 0.9991 in all 156 cases. Table I indicates the distribution of the number of parameter estimates for γ and δ.

Table I: The distribution of the number of parameter estimates for γ and δ

Range	γ	δ
0.25 – 0.50	152	117
0.50 – 0.75	0	35
0.75 – 1.0	0	0
1.0 – 2.0	0	0
2.0 – 5.0	1	3
5.0 – 7.0	3	1
Total	156	156

Note: the minimum values for γ and δ are 0.2501 and 0.3322, respectively and the maximum values for γ and δ are 6.023 and 5.602, respectively.

5. Conclusion

In this paper, we proposed an extension of the standard one-commodity C-CAPM model into a multiple-commodity C-CAPM by explicitly modeling non-durables and services separately. As a working hypothesis, we set the null hypothesis that our multiple-commodity model is correct. Based on this hypothesis, we estimated the parameters of the time preference, and the two sets of the risk aversion parameters. As the present model is robust, we have a possibility to extend the scope for SDF (stochastic discount factor).

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