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Endogenous participation costs and equilibrium abstention in voting with complete information: A three-player case

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Abstract

This note examines the endogenous determination of participation costs in a costly voting game with complete information when there are three voters. I find that there are two types of equilibria: (1) one where a voter who has a minority opinion definitely abstains, and (2) where he or she votes with some positive probability. In either equilibrium, the voter never invests to reduce his or her participation costs. Thus, inefficiency arises solely from a free-rider problem among voters in the majority.

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1. Introduction

The purpose of this note is to analyze endogenous abstention from a small group meeting for decision-making (e.g., voting in parent-teacher associations, committee meetings, faculty meetings). In contrast to my previous analysis in this *Bulletin* (Adachi, 2004) where voters are assumed to have the *same* exogenous participation cost, I consider the situation where participations costs are endogenously determined: each voter can invest to reduce his or her participation cost, and as a result voters may have *different* participation costs.[1]

I focus on the case where there are two alternatives to avoid problems such as the Condorcet paradox. I call one of these two alternatives the "*majority* opinion" of a society if the number of supporters for that alternative is larger than that of supporters for the other alternative, which I call the "*minority* opinion." By allowing a voter to determine the level of his participation cost to go to a meeting place, I analyze how each member in equilibrium behaves differently depending on whether they fall in the majority of minority group.

In an influential paper, Palfrey and Rosenthal (1983) assume exogenous participation costs (which are common to all voters) and complete information.[2] They show that there are typically two types of equilibria: (1) one where all voters choose mixed strategies (which differ depending on whether a voter is in the majority or the minority) over whether he votes or abstains, and (2) this includes some voters who definitely abstain and those who definitely vote. Their main focus is on the asymptotic properties of these equilibria: when the number of voters is very large, the probability of going to vote tends to be zero for all the voters in the former type of equilibrium, which seems a natural consequence. However, the second type of equilibrium does not have that property.[3]

To the best of my knowledge, Xu (2002) is the first author who considers endogenous voting costs, and investigates the effect of the costs on voters' welfare and their incentive to reduce the costs. Xu (2002) assumes that there are *two* voters and that there is *incomplete* information: each voter's benefit of winning is his or her private information. Xu's (2002) main result is that the cost reduction has two opposite effects on each voter's welfare; the *direct* effect and the *indirect* (*strategic*) effect. Given the probability that another voter appears to vote, the first voter's expected utility increases as the cost of voting decreases. This is the direct effect. However, another effect of reducing the voting cost exists. Since the other voter is also reducing the cost, he or she is more likely to participate, reducing the probability that the voter wins the election. Depending on parameter values, either the direct effect or the indirect effect dominates. In the latter case, as the cost of voting decreases, each voter is worse off ex ante.

Contrary to Xu's (2002) setting, this note assumes *complete* information but that there are *three* voters. A new issue arising here is a problem of coordination among voters in the

[1] See Feddersen and Pesendorfer (1996, 1999) for endogenous abstention without participation cost. The driving force of endogenous abstention in their analysis is the assumption that there are two types of citizens all of whom have a common benefit: one is the well-informed type and the other is uninformed type. The latter type has an incentive to abstain and not influence the voting outcome (or to avoid the "Swing Voter's Curse").

[2] In a companion paper, Palfrey and Rosenthal (1985) consider a model with incomplete information on the costs and/or preferences of other members. Levine and Palfrey (2007) is an experimental study of Palfrey and Rosenthal's (1985) model.

[3] Adachi (2004) shows that a unique mixed Nash equilibrium obtains when there are three players in Palfrey and Rosenthal's (1983) model, a result which Palfrey and Rosenthal (1983) do not find or at least do not mention. The intuitive reason for the uniqueness is that a voter in the minority group does not dare to definitely abstain because he is the only voter who can be pivotal.

majority. I find that there are two types of equilibria: (1) one where a voter who has a minority opinion definitely abstains, and (2) where he or she participates with some positive probability. In either equilibrium, the voter never invests to reduce his voting cost. Thus, inefficiency arises solely from a free rider problem among voters in the majority.

2. The Model

A social group exists, consisting of three members who are risk neutral. They face a problem of choosing one of two alternatives, A and B. Suppose that the number of supporters for A is *two* (players 1 and 2), and the number of supporters for B is *one* (player 3). In this sense, alternative A (resp., B) represents a *majority* (resp., *minority*) opinion in this group. We assume that these alternatives are fixed.[4]

A supporter for alternative $x \in \{A, B\}$ gains 1 if alternative x wins, and gains 0 if alternative $y \in \{A, B\}$, $y \neq x$ wins.[5] We denote by $c \in (0, 1/2)$ each voter's initial common opportunity cost of voting (for example, the possible earnings a voter can acquire when he goes to work instead of participating in voting).[6] Before deciding to participate or not, however, each of voters can simultaneously reduce his own voting cost. For example, the voter might commit to work less. As in Xu (2002), we assume that voter $i = 1, 2, 3$ can reduce his voting cost to $(c - e_i)$ by incurring a sunk investment $e_i \in [0, c]$ with disutility of $-ke_i^2/2$, where k is a positive constant greater than $2/c^2$. This restriction implies $k > 8$, and is made to exclude a boundary solution $e_i = c$ in the following first best case.

After every voter observes (e_1, e_2, e_3), each voter decides whether he will participate in voting or abstain. What he does at the meeting place is to write "A" or "B" on a ballot and cast a vote. Majority rule decides the winning alternative. In a tie or when nobody votes, the winner is decided by a fair lottery. The conditions described so far are common knowledge among the three members.

Notice here that after a voter decides to show up, it is a weakly dominated strategy to write against his preferred alternative for any event. Thus, supposing a weakly dominated strategy is not played, we focus on endogenous determination of what will be a Nash equilibrium in each subgame (e_1, e_2, e_3).

Note also that the maximum social welfare is $2 - c + 1/(2k)$, which is achieved by making only one player between players 1 and 2 (say, player 1) invests to reduce his cost ($e_1 = 1/k$, $e_2 = e_3 = 0$) and participate in voting.[7] In particular, voter 3 should not invest because it is a socially wasteful activity.

3. Equilibria in Participation Subgames

[4] Osborne, Rosenthal and Turner (2000) consider a model where alternatives are not fixed, but the final decision is a function (which they call a compromise function) of how many and what types of members have participated. Turner and Weninger (2005) is an empirical analysis based on the results of Osborne, Rosenthal and Turner (2000).

[5] This normalization is the same as in Palfrey and Rosenthal (1983, 1985), which validates the main points of the note.

[6] Adachi (2004) shows that in the three-player model with $c = 0$ and no room for cost reduction, there are two Nash equilibria; $(\sigma_1, \sigma_2, \sigma_3) = (1, 1, 0)$ and $(1, 1, 1)$, and that trembling-hand perfection eliminates the equilibrium $(\sigma_1, \sigma_2, \sigma_3) = (1, 1, 0)$.

[7] If the social planner can directly choose one alternative between the two, however, the maximized social welfare is 2.

Let $S \equiv \{(e_1, e_2, e_3) \in [0, c]^3\}$ be a set of participation subgames. Also let $\sigma_i(e_1, e_2, e_3) \in [0, 1]$ be the probability of player i's participation of voting in subgame $(e_1, e_2, e_3) \in S$. The first observation is that for some subgames there are two Nash equilibria: one is where voter 3 definitely abstains, and the other is where he or she participates with some positive probability. However, for all subgames, there always exist the latter type of Nash equilibrium.[8]

Proposition 1. For all subgames (e_1, e_2, e_3), there is a Nash equilibrium,

$$\begin{aligned}
& \sigma^I \equiv (\sigma_1(e_1,e_2,e_3), \sigma_2(e_1,e_2,e_3), \sigma_3(e_1,e_2,e_3)) \\
= \; & (\sqrt{\frac{[1-2(c-e_2)][1-2(c-e_3)]}{1-2(c-e_1)}}, \\
& \sqrt{\frac{[1-2(c-e_3)][1-2(c-e_1)]}{1-2(c-e_2)}}, \\
& 1 - \sqrt{\frac{[1-2(c-e_1)][1-2(c-e_2)]}{1-2(c-e_3)}}),
\end{aligned} \tag{1}$$

which is unique except for subgames (e_1, e_2, e_3) such that

$$(c-e_1) + (c-e_2) - (c-e_3) \leq 2(c-e_1)(c-e_2)$$

hold, where there is an additional equilibrium,

$$\sigma^{II} \equiv (1-2(c-e_2), 1-2(c-e_1), 0).$$

Proof. Fix an arbitrary subgame (e_1, e_2, e_3) and fix any mixed strategy for player 3 in that subgame, $\sigma_3(e_1, e_2, e_3) = \sigma_3 \in [0, 1]$. Then, the best response correspondence for player $i = 1, 2$ is as follows:

$$BR_i(\sigma_j, \sigma_3) = \begin{cases} 1 & \text{if } 1 > \sigma_j(1-\sigma_3) + 2(c-e_i) \\ [0,1] & \text{if } 1 = \sigma_j(1-\sigma_3) + 2(c-e_i) \\ 0 & \text{if } 1 < \sigma_j(1-\sigma_3) + 2(c-e_i) \end{cases} \text{ for } j = 1,2 \text{ and } j \neq i,$$

because his or her expected payoff from abstention is

$$\begin{aligned}
& \sigma_j\sigma_3(1/2) + \sigma_j(1-\sigma_3) + (1-\sigma_j)(1-\sigma_3)(1/2) \\
= \; & (1+\sigma_j-\sigma_3)(1/2),
\end{aligned} \tag{2}$$

while his or her expected payoff from participation is

$$\begin{aligned}
& \sigma_j\sigma_3 + \sigma_j(1-\sigma_3) + (1-\sigma_j)\sigma_3(1/2) + (1-\sigma_j)(1-\sigma_3) - (c-e_i) \\
= \; & (2-\sigma_3+\sigma_j\sigma_3)(1/2) - (c-e_i).
\end{aligned}$$

Similarly, if player 3 abstains from voting, then his or her expected payoff is

$$(1-\sigma_1)(1-\sigma_2)(1/2), \tag{3}$$

[8] Because each subgame is finite, the existence of a Nash equilibrium is obvious.

while if he participates in voting, it is

$$\begin{aligned} & [(1-\sigma_1)\sigma_2 + \sigma_1(1-\sigma_2)](1/2) + (1-\sigma_1)(1-\sigma_2) - (c-e_3) \\ = \; & (2-\sigma_1-\sigma_2)(1/2) - (c-e_3). \end{aligned}$$

Now, fix $e_1 \geq e_2$ (the analogous argument holds for the other case). The candidates for Nash equilibria are as follows:

(i) If $0 \leq \sigma_3 < \min[2(c-e_1), 2(c-e_2)] = 2(c-e_1)$, then they are $(1,0,\sigma_3)$, $(0,1,\sigma_3)$ and $([1-2(c-e_2)]/(1-\sigma_3), [1-2(c-e_1)]/(1-\sigma_3), \sigma_3)$.

(ii) If $2(c-e_1) \leq \sigma_3 \leq 2(c-e_2)$, then the only candidate is $(\sigma_1, 1, \sigma_3)$ where $\sigma_1 \in [0, [1-2(c-e_2)]/[1-2(c-e_1)]]$.

(iii) If $2(c-e_2) \leq \sigma_3 \leq 1$, then $(1,1,\sigma_3)$ is the only candidate. For subgames $(e_1,e_2,e_3) \in \{(e_1,e_2,e_3) : 0 \leq e_1 \leq c, 0 \leq e_2 \leq c, e_3 = c\}$, $(1,1,\sigma_3)$ is a Nash equilibrium where $\sigma_3 \geq 2(c-e_2)$.

Consider the case where $\sigma_3 = 0$. Then, the the candidates for Nash equilibrium are $(1,0,0)$, $(0,1,0)$ and $(1-2(c-e_2), 1-2(c-e_1), 0)$. In the first two cases, we have $(2-\sigma_1-\sigma_2)(1/2)-(c-e_3) = 1/2-(c-e_3) > 0 = (1-\sigma_1)(1-\sigma_2)(1/2)$. Thus, $\sigma_3 = 0$ is not a best response. In the last case, we have $(2-\sigma_1-\sigma_2)(1/2)-(c-e_3) = (c-e_1)+(c-e_2)-(c-e_3)$ and $(1-\sigma_1)(1-\sigma)(1/2) = 2(c-e_1)(c-e_2)$. Thus, for $\sigma_3 = 0$ to constitute an equilibrium, it must be the case that

$$(c-e_1)+(c-e_2)-(c-e_3) \leq 2(c-e_1)(c-e_2). \tag{4}$$

Next, consider the case where $0 < \sigma_3 < 1$. In this case, it must be

$$1-\sigma_1\sigma_2 = 2(c-e_3),$$

which also requires that $0 < \sigma_i < 1$ for $i = 1,2$, meaning that the only surviving candidate is $([1-2(c-e_2)]/(1-\sigma_3)$, $[1-2(c-e_1)]/(1-\sigma_3)$, $\sigma_3)$ for $\sigma_3 \in (0, 2(c-e_1))$.

The indifference conditions are

$$\begin{cases} 1-\sigma_2(1-\sigma_3) = 2(c-e_1) \\ 1-\sigma_1(1-\sigma_3) = 2(c-e_2) \\ 1-\sigma_1\sigma_2 = 2(c-e_3). \end{cases}$$

By solving these, we have (1) as a unique solution. Note that this σ_3 is verified to be less than $2(c-e_1) < 1$. ■

Inequality (4) shows that multiple equilibria arise for low or high values of c. This occurs because for *high* c, voter 3 could be too discouraged by the *direct* effect of his or her cost to participate, which in turn makes voters 1 and 2 behave more aggressively. For *low* c, the voter is now adversely affected by this *indirect* (*strategic*) effect of voters 1 and 2's aggressiveness.

4. Determination of Voting Costs

Now, we verify what level of investment e_i voter i chooses in equilibrium.[9] The following proposition is obtained.

[9] We confine our attention to pure strategies of investment, and assume symmetry between voters 1 and 2.

Proposition 2. For all $c \in (0, 1/2)$ (and for all $k > 8$),

$$(e_1, e_2, e_3) = \min[e^*(c,k), c], \min[e^*(c,k), c], 0)$$

is an equilibrium choice of cost reduction, where

$$e^*(c,k) = \frac{1 - k(1-2c)\sqrt{1-2c} + \sqrt{1 + k^2(1-2c)^3}}{2k\sqrt{1-2c}}.$$

It is a unique equilibrium except for

$$c \in (0, \frac{k+4-\sqrt{k(k-8)}}{4k}] \cup [\frac{k+4+\sqrt{k(k-8)}}{4k}, 1/2),$$

where there is an additional equilibrium choice,

$$(e_1, e_2, e_3) = (1/k, 1/k, 0).$$

Proof. First, suppose that all voters anticipate that equilibrium σ^I occurs in any subgame. Given this and any choice of voters 1 and 2, (e_1, e_2), voter 3's expected payoff is

$$\frac{1}{2}\left(1 - \sqrt{\frac{[1-2(c-e_2)][1-2(c-e_3)]}{1-2(c-e_1)}}\right)\left(1 - \sqrt{\frac{[1-2(c-e_3)][1-2(c-e_1)]}{1-2(c-e_2)}}\right) - \frac{k}{2}e_3^2$$

from (3), and it can be verified that the optimal choice is $e_3 = 0$.

Voter $i = 1, 2$ maximizes

$$\frac{1}{2}\left(\sqrt{\frac{[1-2(c-e_i)][1-2(c-e_3)]}{1-2(c-e_j)}} + \sqrt{\frac{[1-2(c-e_i)][1-2(c-e_j)]}{1-2(c-e_3)}}\right) - \frac{k}{2}e_i^2$$

with respect to e_i, where $j \neq i$, $j = 1, 2$. Under symmetry, the equilibrium choice for voters 1 and 2 is $e_1 = e_2 = \max[e^*(c,k),\, c] > 0$.

Next, suppose that players anticipate that equilibrium σ^{II} occurs in some subgame $(e_1, e_2, e_3) \in \{(e_1, e_2, e_3) \in [0, c]^3 : (c - e_1) + (c - e_2) - (c - e_3) \leq 2(c - e_1)(c - e_2)\ \}$, and also suppose that this subgame is reached. Given this, voter 3's optimal choice should imply $e_3 = 0$. From (2), voter $i = 1, 2$ solves

$$\max_{e_i \in [0,c]} \frac{1}{2}[1 + 1 - 2(c - e_i)] - \frac{k}{2}e_i^2,$$

whose unique solution is $e_i = 1/k$. Thus, from inequality (4), we know that as long as

$$c^2 - \frac{k+4}{2k}c + \frac{k+1}{k^2} \geq 0,$$

equivalently,

$$0 < c \leq \frac{k+4-\sqrt{k(k-8)}}{4k} \text{ or } \frac{k+4+\sqrt{k(k-8)}}{4k} \leq c < \frac{1}{2}$$

holds, the above subgame can be reached. ■

Propositions 1 and 2 show that there exists an equilibrium path,

$$\begin{aligned} & (\sigma_i(e^*(c,k), e^*(c,k), 0))_{i=1,2,3} \\ = \ & (\sqrt{1-2c}, \sqrt{1-2c}, 1 - \frac{1-2(c-\min[e^*(c,k), c])}{\sqrt{1-2c}}), \end{aligned}$$

and there exists another equilibrium path,

$$\begin{aligned} & (\sigma_i(e^*(c,k), e^*(c,k), 0))_{i=1,2,3} \\ = \ & (1-2(c-1/k), 1-2(c-1/k), 0) \end{aligned}$$

for

$$c \in (0, \frac{k+4-\sqrt{k(k-8)}}{4k}] \cup [\frac{k+4+\sqrt{k(k-8)}}{4k}, 1/2).$$

The first equilibrium is similar to the one obtained by Adachi (2004): members in the majority group are too discouraged, and those in the minority group are too encouraged to participate in voting from the viewpoint of social welfare In the second type, the voter in the minority "perfectly saves" the cost of voting: inefficiency arises from a problem of coordination among voters in the majority, namely, which voter should represent the majority.

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