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On the definition of nonessentiality

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Abstract

Nonessentiality of a good is often used in welfare economics, cost-benefit analysis and applied work. Various definitions of this property are presented in the literature on public and environmental economics. This note clarifies their relationship.

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1. Introduction

A good is nonessential (for a consumer) if there are situations under which the consumer is willing to do without it. E.g., luxury goods and goods which are not vitally necessary can be nonessential. In the literature three different definitions of this concept have been presented and are used independently. This note investigates the connections between them.

Two definitions are based on demand functions. In this case a good is called nonessential if the demand for it can be zero. The demand function can be either uncompensated (Marshallian) or compensated (Hicksian). A third definition proposed by Willig requires that the consumer can attain any (feasible) utility level without consuming the nonessential good.

Nonessentiality of a good is often introduced and discussed in theoretical and applied welfare economics. If demand can be zero, the demand curve intersects the price axis and the corresponding consumer surplus (CS) measure is always well defined. The property is therefore important whenever welfare measures are to be calculated since it guarantees[1] the finiteness of the Marshallian CS and, respectively, the Hicksian measures. Moreover, the property of nonessentiality is employed in public and environmental economics when the concept of weak complementarity is used. A private good and a nonmarket good are weakly complementary if the marginal willingness to pay for the nonmarket good is zero when the private good is not consumed (cf. e.g. Freeman 2003). In this case the private good must be nonessential. The concept allows to evaluate changes in nonmarket goods (like environmental and public goods) by observations on the related private good.

This note reconsiders the alternative definitions of nonessentiality and completely clarifies their relationship. It turns out that the definitions are not equivalent. The strongest one is based on the compensated demand function. It implies the other ones which are in turn independent from one another.

2. Alternative definitions of nonessentiality

The framework and the notation are introduced first. We assume that there are only two (private) goods, *X* and *Y*. Here we denote by *X* the good which is nonessential and by *Y* a Hicksian composite which could be replaced by $m \geq 2$ commodities without difficulties in the analysis below: A commodity bundle is denoted by $(X,Y) \in \mathbb{R}^2_+$. We consider one consumer possessing a preference ordering $\succsim$ over bundles (X,Y). Its domain *D* is a subset of $\mathbb{R}^2_+$ and contains $\mathbb{R}_+ \times \mathbb{R}_{++}$, i.e. the ordering is also defined for bundles $(0,Y)$ where $Y \in \mathbb{R}_{++}$. It is assumed that $(0,0) \notin D$. The ordering $\succsim$ is supposed to be continuous and convex on *D* and increasing in both commodities on $\mathbb{R}^2_{++}$. Furthermore it is assumed that the demand correspondence is single-valued and the ordering is represented by a (direct) utility function $U(X,Y)$ which is continuous, increasing and concave on *D*. Prices are given by $(p_X, p_Y) \in \mathbb{R}^2_{++}$ and the range of *U* by $\mathcal{R}(U) := \{U(X,Y) | (X,Y) \in D\}$. Let $M \in \mathbb{R}_{++}$ be the consumer's (exogenous) income. The ordinary (Marshallian) and, respectively, compensated (Hicksian) demand functions are denoted by $X(p_X, p_Y, M)$, $Y(p_X, p_Y, M)$ and $X^c(p_X, p_Y, u)$, $Y^c(p_X, p_Y, u)$ where $u \in \mathcal{R}(U)$ is a utility level.

[1] If the demand curve does not intersect the price axis CS can be finite or infinite depending on the underlying preference ordering.

Given this framework we are able to introduce the concept of nonessentiality precisely. We present three different definitions:[2]

Definition

a) *Good X is Marshall-nonessential (M-nonessential)*
$:\Leftrightarrow$ *For all* $\left(p_Y, M\right) \in \mathbb{R}^2_{++}$ *there is a minimal price* $\tilde{p}_X\left(p_Y, M\right)$ *such that* $X\left(\tilde{p}_X\left(p_Y, M\right), p_Y, M\right) = 0$. (M)

b) *Good X is Hicks-nonessential (H-nonessential)*
$:\Leftrightarrow$ *For all* $\left(p_Y, u\right) \in \mathbb{R}_{++} \times \mathcal{R}\left(U\right)$ *there is a minimal price* $\tilde{p}^c_X\left(p_Y, u\right)$ *such that* $X^c\left(\tilde{p}^c_X\left(p_Y, u\right), p_Y, u\right) = 0$. (H)

c) *Good X is Willig-nonessential (W-nonessential)*
$:\Leftrightarrow$ *For all* $\left(X, Y\right) \in D$ *there is a quantity* $\tilde{Y}\left(X, Y\right)$ *such that* $U\left(X, Y\right) = U\left(0, \tilde{Y}\left(X, Y\right)\right)$. (W)

The minimal price $\tilde{p}_X$ and, respectively, $\tilde{p}^c_X$ for which consumption of *X* is zero is often called choke price. The alternative definitions are suggested in the literature: A good is M-nonessential if one can find a choke price under which its consumption is driven to zero (Just, Hueth, and Schmitz 2004, p. 212). The choke price $\tilde{p}_X$ corresponds to the price where the (Marshallian) demand curve intersects the price axis. A good *X* is called H-nonessential if there is a (minimum) price for it such that the Hicksian demand equals zero (Bockstael and Kling 1988, p. 656 and Freeman 2003, p. 112). Then the compensated demand curve for the nonessential good intersects the price axis at $\tilde{p}^c_X$. H-nonessentiality of a good *X* is equivalent to a geometrical property of the preference ordering: all indifference curves intersect the $X = 0$-axis. W-nonessentiality of a good requires ‘that it can be omitted from the consumption bundle without completely decimating the consumer’ (Willig 1978, p. 230), i.e. that any bundle including the nonessential good can be matched by a bundle excluding it (Johansson (1987), p. 46). In this case there is a quantity of the other good(s) which will compensate the consumer for the absence of (or loss of access to) the nonessential good (Bockstael, McConnell, and Strand 1991, p. 239, and Bockstael and McConnell 1993, p. 1248). The references mentioned demonstrate that nonessentiality is a relevant property in welfare economics and environmental economics.

3. Discussion and conclusion

Now we want to investigate the relationship between the alternative definitions. At first we show that these properties can be satisfied simultaneously:

(1) **Existence**: *There exist preference orderings satisfying (M), (H), and (W).*

Proof: Consider Example 1: The preference ordering $\succsim$ is represented by $U\left(X, Y\right) = \left(X + 1\right)^{1/2} Y^{1/2}$ and implies the demand functions $X\left(p_X, p_Y, M\right) = \left(M/p_X - 1\right)/2$ and $X^c\left(p_X, p_Y, u\right) = \left(p_Y/p_X\right)^{1/2} u - 1$. It is obvious that $\tilde{p}_X\left(p_Y, M\right) = M$, $\tilde{p}^c_X\left(p_Y, u\right) = p_Y u^2$ and $\tilde{Y}\left(Y, X\right) = \left(X + 1\right)Y$. □

But the definitions presented are not equivalent as the following analysis demonstrates.

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[2] In order to distinguish between the alternative concepts we extend ‘nonessential’ by an appropriate term.

(2) **Implications of (M)**: *(M) does not imply (H) or (W).*

Proof: We introduce Example 2: In this case the preference ordering $\succsim$ is represented by $U(X,Y) = \ln(X+1) - 1/Y$. It leads to the demand function $X(p_X, p_Y, M) = \left[M + p_Y\left(0.5 - \sqrt{0.25 + (M + p_X)/p_Y}\right)\right] / p_X$ and the choke price $\tilde{p}_X(p_Y, M) = M^2 / p_Y$. Thus (M) is satisfied. On the other hand $U(0,Y) = -1/Y$ is always strictly negative. Therefore it is obvious that (W) is violated since there is no $\tilde{Y}(X,Y)$ whenever $U(X,Y) \geq 0$. For the same reason, for $u \geq 0$ the (compensated) demand for *X* cannot become zero. Thus (H) is also not satisfied. □

In Example 2 some indifference curves cut the $X = 0$-axis (those corresponding to a strictly negative utility level), others do not (the remaining ones). Therefore (H) and (W) are not fulfilled.

(3) **Implications of (W)**: *(W) does not imply (M) or (H).*

Proof: We present Example 3: Consider $U(X,Y) = \sqrt{X} + \sqrt{Y}$. We get $\tilde{Y}(X,Y) = \left(\sqrt{X} + \sqrt{Y}\right)^2$, i.e. (W) is satisfied. The corresponding demand functions are given by $X(p_X, p_Y, M) = (p_Y / p_X) M / (p_X + p_Y)$ and $X^c(p_X, p_Y, u) = u^2 / (1 + p_X / p_Y)^2$. Demand for good *X* is always strictly positive since $p_Y, M \in \mathbb{R}_{++}$ and $\mathcal{R}(U) = \mathbb{R}_{++}$. Therefore the properties (M) and (H) are violated. □

In Example 3 every indifference curve cuts the $X = 0$-axis, but the marginal willingness to pay for the first unit of *X* is infinite if *X* is equal to zero (the marginal rate of substitution between *X* and *Y* is equal to $MRS_{XY}(X,Y) = \sqrt{Y} / \sqrt{X}$). Thus there is no finite price system (p_X, p_Y) for which the consumer chooses $X = 0$.

Finally we examine the implications of (H):

(4a) **Implication of (H)**: *(H) implies (M).*

Obviously (H) and (M) are not independent. One would expect this result for the case in which *X* is a normal good. Since then the Hicksian demand curve intersects the Marshallian demand curve from above. But normality is not used in the proof: The result is also true if *X* is (locally) inferior.

Proof: Choose any *U* satisfying (H). Then define $\hat{X} := 0$, $\hat{Y} := M / p_Y$, and $\hat{u} := U\left(\hat{X}, \hat{Y}\right)$ for given $(p_Y, M) \in \mathbb{R}^2_{++}$. (H) implies that there is $\tilde{p}^c_X(p_Y, \hat{u})$ such that $X^c\left(\tilde{p}^c_X(p_Y, \hat{u}), p_Y, \hat{u}\right) = 0 = \hat{X}$ and $Y^c\left(\tilde{p}^c_X(p_Y, \hat{u}), p_Y, \hat{u}\right) = \hat{Y} = M / p_Y$. Then $\tilde{p}_X(p_Y, M) = \tilde{p}^c_X(p_Y, \hat{u})$. □

(4b) **Implication of (H)**: *(H) implies (W).*

The properties (H) and (W) are also related.

Proof: In order to prove this claim we choose any *U* satisfying (H). Then define $\hat{u} := U(X,Y)$ for any $(X,Y) \in D$. Setting $\hat{p}_X := 1$ and $\hat{p}_Y := MRS_{XY}(X,Y)$ we obtain $U\left(X^c(\hat{p}_X, \hat{p}_Y, \hat{u}), Y^c(\hat{p}_X, \hat{p}_Y, \hat{u})\right) = \hat{u}$. Because of (H) there exists $\tilde{p}^c_X(\hat{p}_Y, \hat{u})$ leading to

$\hat{X} := X^c\left(\tilde{p}_X^c\left(\hat{p}_Y,\hat{u}\right),\hat{p}_Y,\hat{u}\right) = 0$ and $\hat{Y} := Y^c\left(\tilde{p}_X^c\left(\hat{p}_Y,\hat{u}\right),\hat{p}_Y,\hat{u}\right)$. Then $U\left(\hat{X},\hat{Y}\right) = U\left(0,\hat{Y}\right) = \hat{u} = U\left(X,Y\right)$. Thus $\tilde{Y}\left(X,Y\right) = \hat{Y}$. □

These investigations clarify the relationship between (M), (H), and (W). Collecting the results we obtain

Proposition

(H) implies (M) and (W). Neither (M) nor (W) implies (H). (M) and (W) are independent.

It turns out that Hicks-nonessentiality is the strongest property. It has to be emphasized that the proposition holds for all preference orderings satisfying the basic assumptions. No further properties, like e.g. normality or inferiority of good X, are required.

Given the main result Hicks-nonessentiality seems to be preferable to Marshall- and Willig-nonessentiality since it implies (M) and (W). But more important is the fact that – given (H) – any kind of welfare measurement with respect to changes of the nonessential good can be performed: The areas behind the Marshallian and Hicksian demand functions which measure the compensation required by the consumer for an elimination of this good are always finite. Thus consumer surplus *and* the Hicksian welfare measures are also well-defined and finite for an arbitrary change in (the price of) the nonessential good, i.e. they can be employed without any difficulties in applied work.

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