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Privatization and Government's Preference under Mixed Oligopoly: A Generalization

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Abstract

In this paper, we generalize Kato's (*Economics Bulletin*, 2008) model by allowing many private firms in the mixed oligopoly setting, rather than the mixed duopoly framework of Kato (2008). By introducing the government's preference for tax revenues into the theoretical framework of mixed oligopoly, we show that Kato's results are robust when there are many private firms. That is, as the number of private firms increases, both total output and the government's payoff in the mixed oligopoly are larger than those in the private oligopoly if and only the weight of the government's preferences on tax revenues increases and vice versa.

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1 Introduction

Within the literature of mixed oligopoly, a strand examines government subsidies designed to limit the reduction in quantity. For example, White (1996), Poyago-Theotoky (2001), and Myles (2002) showed that optimal subsidy, profits and social welfare are identical before and after the privatization of a public firm in a mixed oligopoly. That is, irrespective of whether a public firm moves simultaneously with private firms, the public firm acts as a Stackelberg leader, or all firms behave as profit maximizers. On the other hand, Fjell and Heywood (2004) demonstrated that when the public leader is privatized and becomes the private leader, the optimal subsidy, output, and social welfare are all reduced. Moreover, by introducing taxes (ad valorem or specific) into a mixed oligopoly, Mujumdar and Pal (1998) showed that privatization could increase both social welfare and tax revenues, where an increase in tax does not change the total output but increases the output of the public firm and the tax revenue. In all the abovementioned studies that consider both subsidies and taxation in a mixed oligopoly market, the public firm as well as the government maximize social welfare, which is defined as the sum of the tax revenue or subsidy, consumers' and producers' surplus.

Although some theoretical studies have been successful in explaining mixed oligopolies, Matusumura (1998), Saha and Sensarma (2008) and Kato (2008) have explicitly investigated the different objective functions existing between a public firm and the government in mixed duopolies. Contrary to previous findings that of the different objective functions existing between the government and a public firm in a mixed duopoly is relatively new because, up until the present, the literatures on the objective functions between the government and a public firm have found various robust results in a mixed oligopoly. In this sense, Matusumura (1998), Saha and Sensarma (2008) and Kato (2008) have made a significant contribution to the literature. Contrary to the results of previous studies (Poyago-Theotoky (2001), Fjell and Heywood (2004), Mujumdar and Pal (1998) and Myles (2002)), if a public firm takes full account of consumer and producer surplus, while the government attaches greater priority to both its social welfare and preference for tax revenues, Kato (2008) showed that a government's readiness to privatize a public firm would depend on its preference for raising greater tax revenues.

To study the effects that arise when the objective functions of the government and a public firm are different, we generalize Kato's (2008) model by allowing n private firms in the mixed oligopoly setting, rather than the mixed duopoly framework used in Kato (2008). In this paper, we show that Kato's (2008) results are robust when there are more private firms. Hence, our result is a generalization of the result of Kato (2008), which investigated the government preference for tax revenue when there is only one private firm. Here, according to our results derived from general formulation, we find that as the number of private firms increase, both total output and the government's payoff in a mixed oligopoly are larger than those in the privatized oligopoly if and only if the weight of the government's preference for tax revenues would increase and vice versa.

2 The Model

Consider a mixed oligopoly situation for a homogeneous good that is supplied by one public firm and n private firms. Firm i ($i = 1, \dots, n$) is a profit-maximizing private firm and firm 0 is a public firm that maximizes social welfare. Assume that the inverse demand is characterized by

$$p = 1 - x_0 - \sum_{i=1}^n x_i,$$

where x_0 is the output level of the public firm and x_i is the output level of the private firm i .

Given all firms share the same production technology represented by quadratic cost function $\frac{x_j^2}{2}$, each firm's profit is as following function

$$\pi_j = px_j - \frac{x_j^2}{2} - tx_j, \quad j = 0, 1, \dots, n, \quad (1)$$

where t is the specific tax. On the other hand, the public firm's objective is to maximize welfare, which is defined as the sum of the consumer surplus, the profits of individual firms. Thus, the public firm aims to maximize its objective function as follows;

$$W = \frac{X^2}{2} + \sum_{i=1}^n \pi_i + \pi_0 = \frac{X^2}{2} + (1 - X)X - \frac{(x_0^2 + \sum_{i=1}^n x_i^2)}{2} - T, \quad (2)$$

where X is the total output and $T = t(x_0 + \sum_{i=1}^n x_i)$ is tax revenue.

In the manner of Kato (2008), we also assume that the government's payoff is given by $G = W + (1+a)T$, where a is the parameter that represents the weight of the government's preference for tax revenues. Here $a \geq 0$, i.e., the government values tax revenues T more than social welfare W .

Finally, a two-stage game is conducted. The timing of the game is as follows. In the first stage, the government sets the specific tax. In the second stage, each firm simultaneously chooses its quantity x_j to maximize its respective object knowing specific tax of the government.

2.1 The Mixed Oligopoly

Given tax t in the second stage, by the first-order condition to maximize (1) and (2), we obtain,

$$x_0 = \frac{1 - t - nx_i}{2}, \quad x_i = \frac{1 - t - x_0}{n + 2}. \quad (3)$$

By solving the first-order conditions (3), we obtain,

$$x_0 = \frac{2(1 - t)}{4 + n}, \quad x_i = \frac{1 - t}{4 + n}. \quad (4)$$

We now move to the first stage of the game. From (4), the government's payoff, G^m , in the mixed oligopoly can be rewritten as follows:

$$\max_t G^m = \frac{(1-t)^2(8+7n+n^2) + 2t(1-t)(1+a)(4+n)(2+n)}{2(4+n)^2}.$$

Straightforward computation yields the optimal tax rate as follows:

$$t^m = \frac{a(4+n)(2+n) - n}{2a(4+n)(2+n) + 8 + 5n + n^2}. \quad (5)$$

Thus, by using (5), we have the equilibrium output levels as follows:

$$x_0^m = \frac{2(1+a)(2+n)}{2a(4+n)(2+n) + 8 + 5n + n^2}, \quad x_i^m = \frac{(1+a)(2+n)}{2a(4+n)(2+n) + 8 + 5n + n^2}. \quad (6)$$

Using (6) and noting that $G^m = W^m + (1+a)T^m$, we can compute the government's payoff G^m as follows:

$$G^m = \frac{(1+a)^2(2+n)^2}{2[2a(4+n)(2+n) + 8 + 5n + n^2]}. \quad (7)$$

2.2 The Privatized Oligopoly

Consider the case of a privatized oligopoly where the public firm is privatized without cost.

Given tax t in the second stage, the first-order condition to maximize (1) is

$$x_i = \frac{1-t - \sum_{j \neq i} x_j}{3}$$

when there are $n+1$ private firms. Symmetry across private firms implies that each output level is given by

$$x_i = \frac{1-t}{3+n}. \quad (8)$$

Turning to the first stage and using the equilibrium output, the government's payoff G^p in the privatized oligopoly can be rewritten as follows:

$$\max_t G^p = \frac{(1+n)(1-t)[(n+4)(1-t) + 2t(1+a)(3+n)]}{2(3+n)^2}.$$

Straightforward computation yields optimal tax rate in the privatized oligopoly as follows:

$$t^p = \frac{n+4 - (1+a)(3+n)}{n+4 - 2(1+a)(3+n)}. \quad (9)$$

Thus, by using (9), we have each equilibrium output level as follows:

$$x_i^p = \frac{-(1+a)}{n+4 - 2(1+a)(3+n)}. \quad (10)$$

Similar to previous subsection, using (10) and noting that $G^p = W^p + (1+a)T^p$, we can compute the government's payoff G^p as follows:

$$G^p = \frac{-(1+a)^2(1+n)}{2[n+4 - 2(1+a)(3+n)]}. \quad (11)$$

3 Comparisons

The difference in the optimal tax rates can be calculated as

$$t^m - t^p = \frac{(1+a)(8+3n)}{F(a)} \quad (12)$$

where $F(a) \equiv a^2(96 + 104n + 36n^2 + 4n^3) + a(80 + 86n + 32n^2 + 4n^3) + 16 + 18n + 7n^2 + n^3$. Note that $F(a) > 0$ for all $a \geq 0$, $F(0) > 0$ and $F'(0) > 0$ since it is a second-order polynomial of a . Hence, we can obtain

$$t^m - t^p > 0 \quad \text{when} \quad n \geq 1.$$

Next, the differences in the government's payoffs and total outputs can be calculated as

$$X^m - X^p = \frac{(1+a)[8a - n(1-4a)]}{2F(a)}, \quad (13)$$

$$G^m - G^p = \frac{(1+a)^2[8a - n(1-4a)]}{4F(a)} \quad (14)$$

for which the signs change according to the numerator of (13) and (14) since each denominator consists of which $F(a) > 0$ for all $a \geq 0$. Therefore, if

$$\frac{8a}{1-4a} > n, \quad (15)$$

the numerator of (13) and (14) always has positive values; otherwise, it always has negative values. Hence, the sign of (13) and (14) always becomes

$$G^m - G^p > 0 \quad \text{and} \quad X^m - X^p > 0 \quad \text{for all} \quad \frac{8a}{1-4a} > n.$$

This means that when the number of private firms increases, both total output and the government's payoff in the mixed oligopoly are larger than those in the privatized oligopoly if and only if the weight of the government's preference for tax revenues increases and vice versa. As a result, having derived the equilibrium values for taxes, total outputs, and government preference, we can state the following:

Proposition 1: *Suppose that there exists the government's preference for tax revenues. Then, as the number of private firms increases, both total output and the government's payoff in the mixed oligopoly are larger than those in the privatized oligopoly if and only if the weight of the government's preference for tax revenues increases and vice versa.*

This proposition suggests that when both total output and the government's payoff in the mixed oligopoly are larger than those in the privatized oligopoly, the number of private firms is proportional to the government's preference for tax revenues, which is simultaneously determined by the equilibrium values.

In sum, except for the difference in the optimal tax rates, the difference in a government's payoffs and total outputs depends on both the number of private firms and

a government's preference for tax revenues. Furthermore, we show that even though a government sufficiently prefers tax revenue, it does have an incentive to privatize a public firm when the number of private firms is sufficiently small. This result is different from that of DeFraja and Delbono (1989), who found that the privatization of the public firm is desirable in terms of social welfare when the number of existing private firms is large. Finally, Kato (2008) showed that both total output and the government's payoff in a mixed duopoly are larger than those in a privatized duopoly if the government's preference is sufficiently large. In contrast, our paper demonstrates that even though the government's preference for tax revenues is sufficiently large, both total output and the government's payoff in the mixed oligopoly are smaller than those in the privatized oligopoly if the number of existing firms is sufficiently small.

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